Reconstruction of CT Images by the Back-Propagation Algorithm

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Abstract

A new and modified neural network model is proposed for CT image reconstruction from four projection data only. The model uses the well known back-propagation delta rule for adaptation of its weights. In addition to the error in projection data of the image being reconstructed, the proposed network makes use of errors in pixels between a filtered image and the reconstructed one. Improved reconstruction was obtained, and the proposed method was found to be very effective in CT image reconstruction when the given number of projection directions is very limited.

1 Introduction

In computed tomography (CT) applications, some projection data of an unknown multidimensional distribution (target image) are assumed to be given. Several techniques are known for reconstructing the original image by manipulating the projection data(Figure 1). When the number of projection directions is limited to three or four, genetic algorithms (GAs), algebraic reconstruction technique (ART)[1, 2, 7, 8], and simulated annealing[6] have been used for reconstruction to date.

Here, a neural network model is applied to gray CT image reconstruction from only four projection data, and is based on the previously proposed back-propagation algorithm based method[3, 4, 5]. The current and newly proposed model has two teaching signals: the given projection data of the unknown image and the reconstructed image which passed a median filter. By using the projection of the unknown image, the error in projection data of a reconstructed image is decreased. On the other hand, the median filter reduces the noise of the reconstructed image.

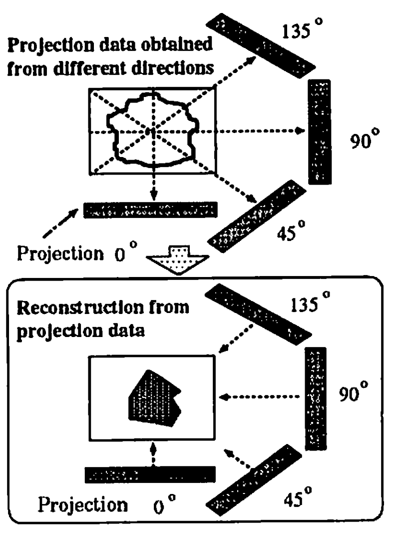

Figure 1: The Reconstruction of the Image from the Projections

2 Back-Propagation System

2.1 Structure of the Network

The system for the reconstruction of the images which we use is structured with three layers of neurons, as is depicted in Figure 2.

The number of the neurons of the first layer is equal to the projection data which we get from an original image. The next layer is the hidden

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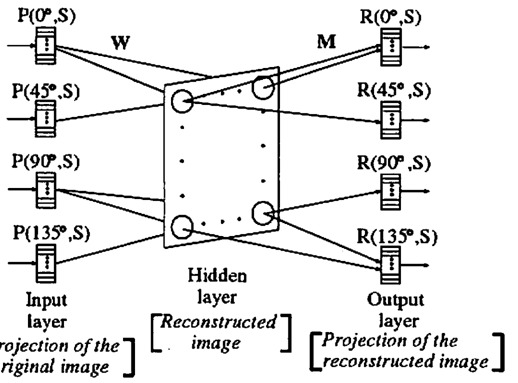

Figure 2: The Structure of the Network

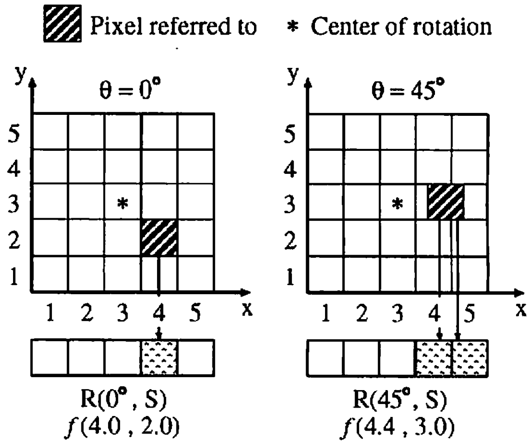

Figure 3: The Initialization of the Weights

layer and has a same number of neurons to the pixels of the image. The number of neurons of the output layer is the same as that of the input layer. The output from the output layer is the projection data from the reconstructed image.

The connection weights between the first and the second layers are denoted by W, which are adapted through the back-propagation learning process. The weights between the second and the third layer are denoted by M, which are kept constant through the learning process. The weight matrix M is determined by the method explained in section 2.2, and the projection data can be computed from the second layer, and inputted to the third layer.

This system is based on the idea that if the error between the projections of the original image and of the reconstructed image is small, the reconstructed image will be similar to the original image. The input to the system is the projection data of the original unknown image as an input to the first layer.

2.2 Weight Initialization

The weights M, which connect the second and the third layers, are used for computing the projection data of the reconstructed image as input to the output layer. The method for initialization of the weights is explained with Figure 3.

Suppose that 5×5 square give one image, each square representing one pixel. The image on the left is before rotation and the one on the right is after rotation. For 0 degree projection the coordinates (4.0,2.0) of the pixel marked does not move and is on the same point at 0 degree, and this pixel will project the entire area (1.0) to the 4th box under the image and this value is taken as a weight for the 0 degree projection.

But after the 45 degree rotation, the pixel (4,0,2.0) moves to the coordinates (4.4,3.0). This pixel is projected into two separate boxes with area ratio 0.6 and 0.4. That is to say, the pixel on the coordinate (4,0,2.0) is related to the pixel on the $R(45°,4)$, $R(45°,5)$ (these representing the 4th and the 5th projection data for 45 degree projection direction) with weights 0.6 and 0.4. Likewise, weights of all the pixels are determined and computed for all directions.

2.3 The Learning Method of the Network

The network is to renew the weights with the *delta rule*. Suppose that a_i is the output of the i-th unit of the input layer and W_{ji} is the connection weight between nodes i and j, then the output y_j of the j-th unit in the hidden layer is given by

$$y_j = \frac{u_j}{N}, \qquad u_j = \sum_i a_i W_{ji}$$

where N is the number of the projection data. The output b_k of the k-th unit in the output layer becomes

$$b_k = \sum_j y_j M_{kj}$$

and, the corresponding error function E_k is given by

$$E_k = \frac{1}{2}(b_k - d_k)^2$$

and, the total error becomes

$$E = \sum_k E_k$$

Using the total error E, the weights W are renewed with the formula below, where W'_{ji} is the renewed weight and α is a scaling constant.

$$\begin{aligned} W'_{ji} &= W_{ji} - \alpha \frac{\partial E}{\partial W_{ji}} \\ &= W_{ji} - \alpha \sum_k ((b_k - d_k) M_{kj}) \frac{1}{N} a_i \end{aligned}$$

2.4 The Mechanics of the Back-Propagation System

The mechanics of the BPS is shown below:

- **Step1 Initialization of the Weight**
  The weight M between the input and the hidden layer is determined by the method in section 2.2, and the weight W is initialized to M.

- **Step2 Determination of the Input and the Teaching Signal**
  The projection data of an unknown image is used as the input data as well as a teaching signal.

- **Step3 Computing the Hidden Layer**
  Using the formula in section 2.3, the output of each neuron is computed.

- **Step4 Computing the Error**
  The error between the teaching signal and the output of the output layer $b_0, b_1, \cdots, b_{N-1}$ is computed.

- **Step5 Renewing the Weights**
  The weights W are renewed with the delta rule.

- **Step6 Decision on Termination**
  If the number of steps has reached the pre-determined epoch number or the error has got smaller than the pre-assigned tolerance, the action is terminated, otherwise, the action goes back to Step3.

2.5 Enhancement of the System

2.5.1 Median Filter

A median filter is one kind of filters which eliminate noises in the image. Suppose that a pixel in the center of Figure 4 is selected. The values of the nine surrounding pixels including the center itself are copied out:

4 3 2 6 10* 5 4 4 6

and sorted:

2 3 4 4 4* 5 6 6 10

The fifth value 4 is selected as a new pixel value replacing 10. The original value 10 was too big compared with other pixels, and this action will make the image smoother. The median filter would tend to keep an edge of the image compared with other filters.

An example of a filtering is depicted in Figure 5.

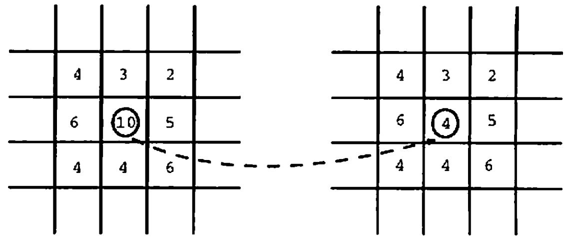

Figure 4: Renewal of the pixel value with the median filter.

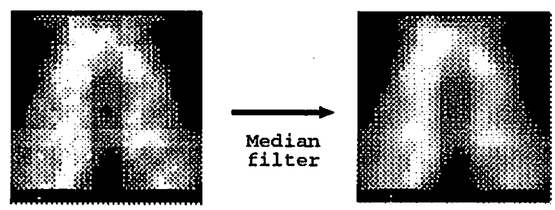

Figure 5: An example median filtering

2.5.2 The Median Filter and the Back-Propagation System

The new network which is proposed here is explained with Figure 6. This network adopts the median filter.

In the older version of the system, only the projection data of the unknown image was used as a teaching signal. The present system has two teaching signals, not only for the output layer but also for the hidden layer on which the reconstructed image appears. The reconstructed image passes the median filter and the resultant image is used as a teaching signal.

The learning method with the median filter is explained below. Suppose that y_j is the the j-th node in the hidden layer (a pixel of the reconstructed image) and x_j is a j-th pixel value of the reconstructed image which passed the median filter, and that the total error is given by:

$$E_{(m)j} = \frac{1}{2}(y_j - x_j)^2$$

The sum of $E_{(m)j}$ is the error of the image:

$$E_{(m)} = \sum_j E_{(m)j}$$

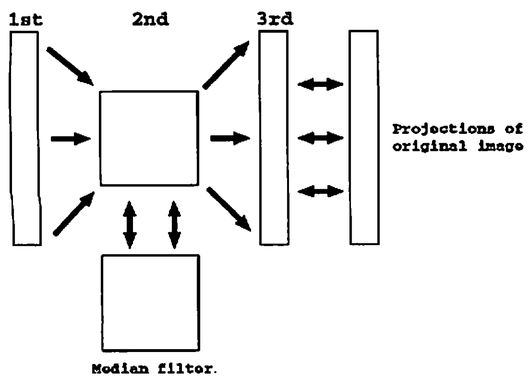

Figure 6: The system with the median filter

and, W is renewed as follows:

$$
\begin{aligned}
W'_{ji} &= W_{ji} - \alpha \frac{\partial E}{\partial W_{ji}} - \beta \frac{\partial E_{(m)}}{\partial W_{ji}} \\
&= W_{ji} - \alpha \sum_{k} ((b_k - d_k) M_{kj}) \frac{1}{N} a_i \\
&\quad -\beta (y_j - x_j) a_i
\end{aligned}
$$

where the parameter β is a scaling constant.

3 Experiments and the Results

Three images are used in simulation. The number of the projections is four at angles, $0°$, $45°$, $90°$, and $135°$.

Figure 7 shows results, and the simulation was done with the parameter values in Table 1. The proposed system reconstructs the images satisfactorily despite the smaller number of projection directions.

Table 1: Parameters

	Image 1	Image 2	Image3
α	0.00005	0.00005	0.00005
β	0.00005	0.00005	0.00005
Image Size	32x32	32x32	32x32
Learning Steps	500	500	500
Filter Starting Step	300	300	300

4 Conclusions and Future Work

Effectiveness of the new back-propagation system with median filter was demonstrated. The newly

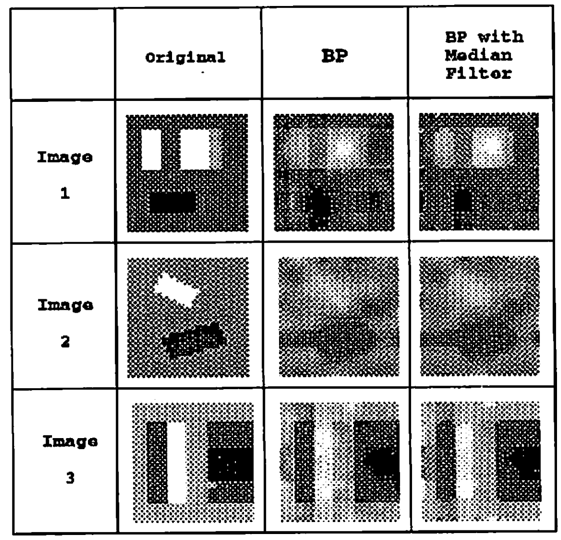

Figure 7: Results

proposed system, however, is not necessarily effective for all types of images. Tuning parameters and adopting other kinds of filters are considered at present.

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