An Adaptive Simulated Annealing Applied to Optimization of Phase Distribution of Kinoform

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Shinya NOZAKI, Yen-Wei CHEN and Zensho NAKAO

Dept.of Electrical and Electronic Eng. University of the Ryukyus
1 Senbaru, Nishihara Okinawa, 903-0213, Japan
Phone: (+81) 98-895-8703 Fax:(+81) 98-895-8708
E-mail:nozaki@augusta.eee.u-ryukyu.ac.jp chen@tec.u-ryukyu.ac.jp

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Abstract

Computer Generated Holography (CGH)[1] is a technique which gengerate a hologram by using computers. A kinoform is one of the CGHs. And it has many applications. But its reconstruction includes much noise. Several methods[2][3], like Simulated Annealing (SA)[4], have been used to decrease reconstruction noise. But since these methods take calculation time too much, it is neccesary to improve the large computation cost. In this reason, we propose an adaptive simulated annealing to reduce the noise.

1. Introduction

Kinoform is one of the CGHs, that can manage wave fronts arbitrary with only the phase information of the complex amplitude. Compared with other types of CGHs, its diffraction efficiency is much high. In this reason, it can be used in many applications, such as optical information processing, optical interconnection, and spatial filtering and so on. But the kinoform include the reconstruction noise caused by the amplitude neglection and phase quantization since the amplitude of the transfer function is assumed as a constant in the kinoform, so the kinoform needs optimization.

These have been adopted for phase optimization of the kinoform. One of the methods which decrease the noise is Simulated Annealing. Since these methods are iterative approaches, they take long computation time to generate an optimized phase distribution.

In Section 2 a kinoform synthesis method that uses simulated annealing is discussed. In Section 3 An Adaptive Simulated Annealing (ASA) that can reduce computation time than SA is disscussed.

2. Kinoform Synthesis by Simulated Annealing

Here we consider a Fourier transformed kinoform that reconstructs an image given by

$$u(x,y) = a(x,y)\exp[i\theta(x,y)] , \qquad (1)$$

where $a(x,y)$ and $\varphi(x,y)$ denote the amplitude and the phase, respectively. Then its Fourier tansform is given by

$$U(u,v) = A(u,v)\exp[i\theta(u,v)] = \iint u(x,y)\exp\left[-\frac{2\pi i}{\lambda f}(ux+vy)\right]dxdy , \qquad (2)$$

where λ is the wavelength and f is the focal length of a Fourier transher lens. In the kinoform approximation, the amplitude is set to be constant. Therefore the equation(2) is given as follows;

$$U(u,v) = \exp[i\theta(u,v)] \quad . \qquad (3)$$

The phase $\theta(u,v)$ of the kinoform is designed in the range of $0 \le \theta < 2\pi$. In most cases the phase θ can be quantizated, and θ can be described by a step function written as

$$\theta(u,v) = \frac{2n\pi}{L} , \qquad n=0,1,...,L-1 \qquad (4)$$

where L is the number of the quantization level. The kinoform is then represented by a discrete phase distribution. The kinoform reconstruction intensity I can be derived in the image plane as

$$I = \left|F[U(u,v)]^{-1}\right|^2 \qquad (5)$$

where $F[U(u,v)]^{-1}$ is Inverse Fourier tansform from $U(u,v)$.

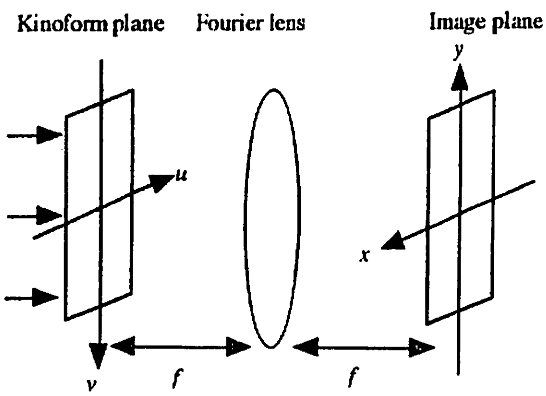
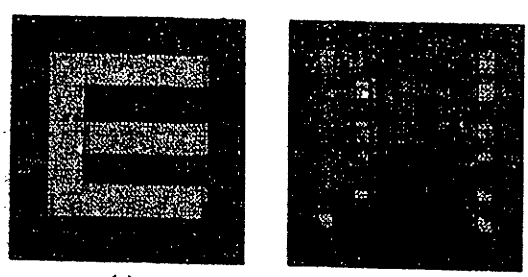


Fig.2. Schematic illustration of the optical reconstruction setup for the synthetic Fourier transform kinoform.

If the kinoform is synthesized by the conventional synthesis method, the reconstruction intensity might contain reconstruction noise. The cause of this noise is the neglect of amplitude information in the kinoform. Therefore we usually optimize the phase of the kinoform to derive a better reconstructed image with less reconstruction noise.



(a) (b)

Fig.3. Reconstructed images from CGH.
(a); with amplitude (b); without amplitude:

A flow chart of the procedure for a simulated annealing for phase optimization is shown in Fig.3. The main process is performed by using of the probability corresponding to a Boltzmann distribution with a temperature parameter. The annealing is started with a high probability for perturbation acceptance, and the temperature is decreased as increasing the number of annealing, resulting in low probability of acceptance.

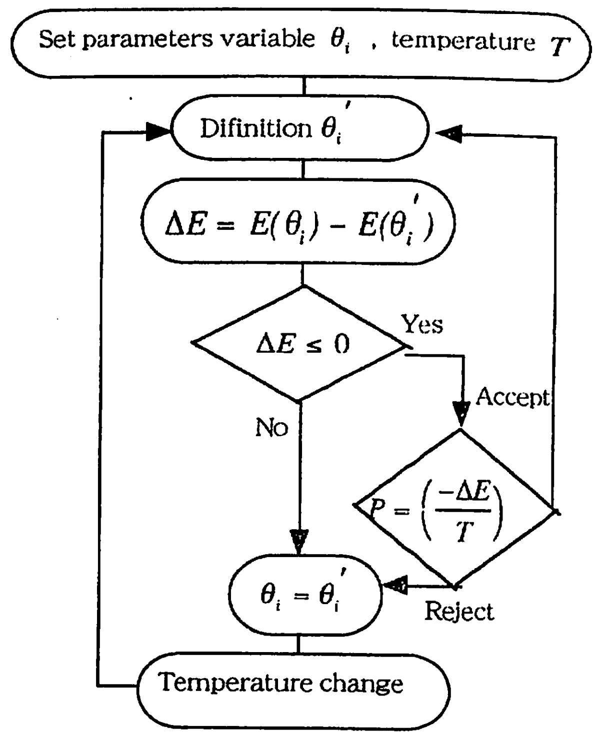


Fig.4. Flow chart of a SA for phase optimization of the kinoform

In Fig.4, E is cost function which is defined as a mean square error between the reconstructed image and input image.
It is shown in Eqs(5)-(6):

$$E(\theta) = \iint \left|I_0(x,y) - \alpha I(x,y)\right|^2 dxdy \ , \qquad (5)$$

$$\alpha = \frac{\iint I_0(x,y)dxdy}{\iint I(x,y)dxdy} \ , \qquad (6)$$

where I_0 is intensity of the input image and α is a scale factor.
The temperature is defined as

$$T = \frac{T_0}{1+t} \qquad (7)$$

where T_0 is initial temperature and t is the numner of annealing cycle.

The kinoforms with optimization and without optimizatyion are shown Fig.5(a') and (b') respectively. Their reconstructed images are shown in Fig.2(a) and (b) respectively. The input image is the latter A with 64 × 64 pixels, and the intensity of the input image is unity.

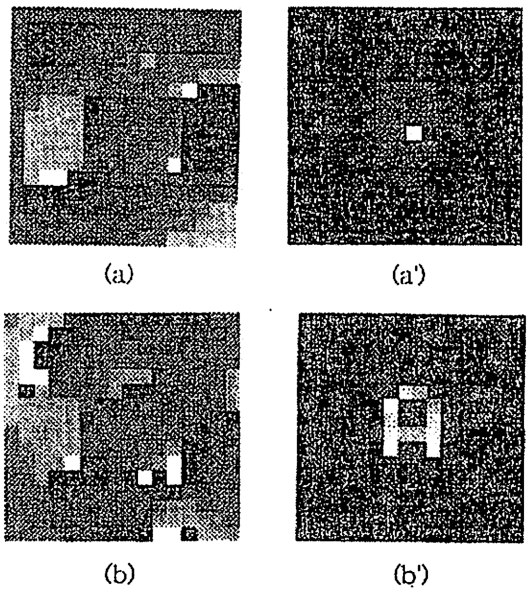


Fig.5. The process which optimizes the phase distribution.

We can see that the reconstructed image with optimizaotion is much better than that without optimization.

3. Adaptive Simulated Annealing(ASA)

Consider the case of a large number of the input data. In this case, the time of optimization of the phase distribution is too large. This reason can be given as follows;

1. Inverse Fourier transform: if the number of the input increase, the calculation time of inverse Fourier Transform increase. Since SA's algorithm must be repeat the caluculation of the inverse Fourier Transform, as a result, the computation time increase.

2: Annealing cycle: One annealing cycle is needed that all the pixels are changed. Because of this, the calculation time increase.

Therefore, we propose Adaptive Simulated Annealing to reduce calculation time.

The difference between the proposed ASA and the conventional SA is that in the conventional SA the perturbation is done for only one pixel and then repeat the perturbation process to each pixel, while in the proposal ASA, the perturbationa are done for gr pixels at the same time. That means the computation time can be improved by factor of gr for one annealing cycle. We adaptively decrease number gr as increasing the annealing cycle. We use a large number of gr in the latter cycle for a local search.

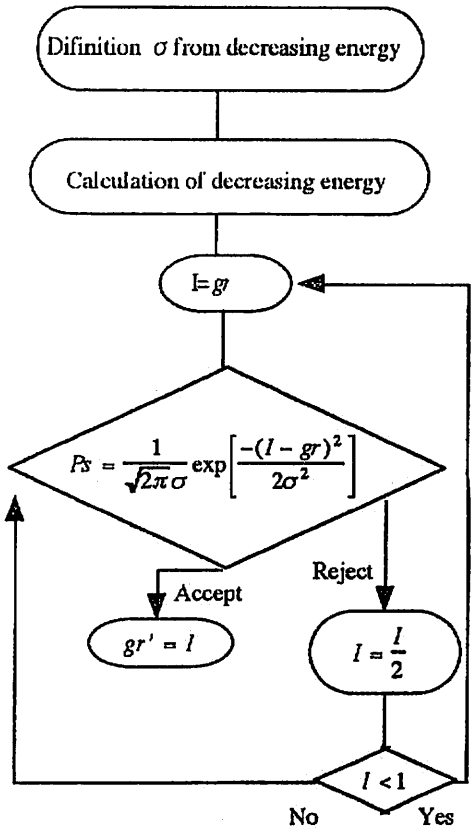


Fig.6. A flow chart of difinition of grouping pixle gr

4.Digital Simulations

The improvement of computation time by the ASA is shown in Fig.7. The results show that the convergence toward the optimun solution by the proposed ASA is much faster than that by the conventional SA.

The processe of the optimizaiotn by ASA and SA is shown in Fig.8. To see the process of the ASA more easily, the reconstructed images are shown in Fig.9. These indicate that ASA could optimize more speedy than SA.

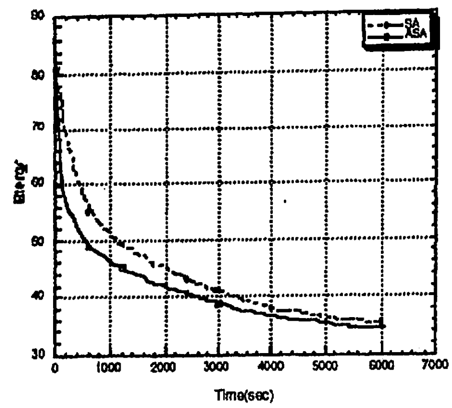


Fig.7 The comparison the calculation time between ASA and SA.

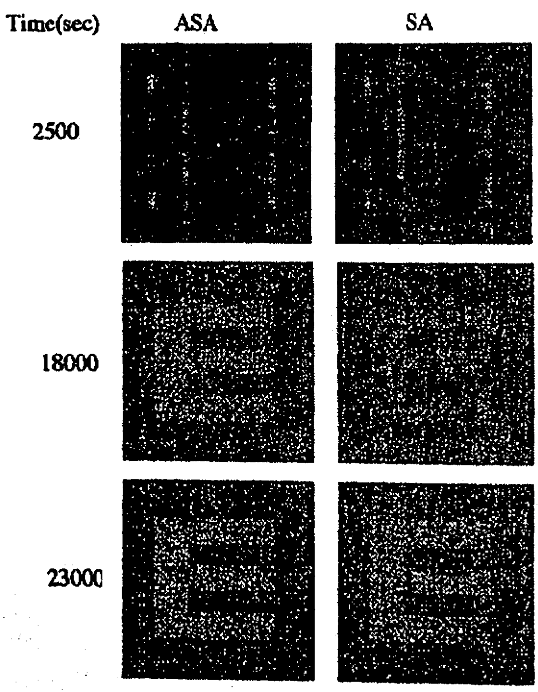


Fig.8. The comparison reconstructed images with ASA and SA.

The gr which is grouping pixel and the energy are shown in Fig.9. The *gr* decrease same like the energy.

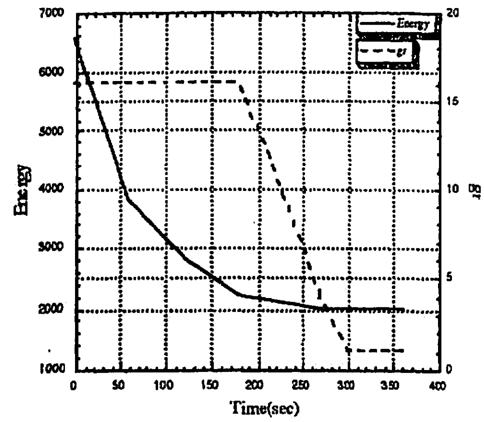


Fig.9. The decrease of gr at each decrease of energy

5.Summary

In summary, we propose an adaptive simulated annealing for optimizating a kinoform. Comparing with SA, the computation time is significantly reduced.

Refernces

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