Neutron Penumbral Imaging : 2. Distortion-Free Reconstruction by Genetic Algorithms

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Abstract

In this paper, a new nonlinear technique based on genetic algorithm (GA) is proposed for reconstruction of penumbral images. In GA, the reconstruction problem is modeled as an optimization problem, whose cost function is to be minimized. The technique allows distortion-free reconstruction over a large field of view. Furthermore, because in GA the complicated *a priori* constraints can be easily incorporated by the appropriate modification of the cost function, the algorithm is also very tolerant of the noise.

Key words: penumbral imaging, point spread function, non-isoplanatic (space-variant), genetic algorithm, constraint

1. Introduction

Penumbral imaging [1] is a powerful technique for imaging the penetrating radiations suchas neutrons and γ rays. The technique uses the facts that spatial information can be recovered from the shadow or penumbra that an unknown source casts of simple large circular aperture. In the previous paper of this series [2], we presented that the straightforward image reconstruction will introduce some significant distortion for a large field of view because of non-isoplanaticity of the aperture point spread function. In this paper, a new nonlinear technique based on genetic algorithm (GA) is proposed for reconstruction of penumbral images. In GA, the reconstruction problem is modeled as an optimization problem, whose cost function is to be minimized. The technique allows distortion-free reconstruction over a large field of view. Furthermore, because in GA the complicated *a priori* constraintscan be easily incorporatedby the appropriate modific-ation of the costfunction, the algorithm is also very tolerant of the noise.

2. The genetic algorithm

The genetic algorithm [3] is an iterative random search algorithm for nonlinear problem based on mechanics of natural selection and natural genetics. It uses probabilistic transition rules to guide itself toward an optimum solution. In this method the reconstruction problem is modeled as an optimization problem, whose cost function is to be minimized. The typical flowchart of the genetic algorithm for reconstruction of penumbral images is shown in Fig.1.

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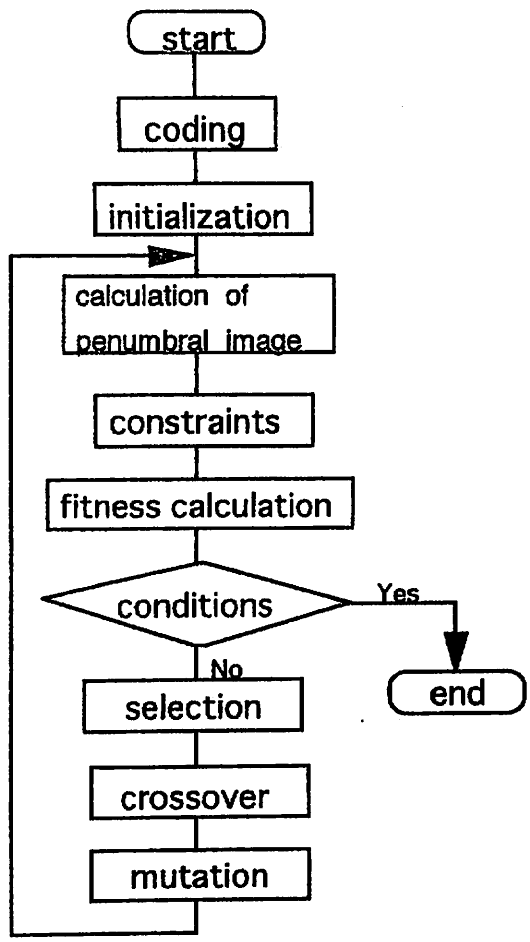

Fig. 1 A typical flowchart of genetic algorithms for reconstruction of penumbral image.

2-1 Image coding

We use a binary matrix as a chromosome to represent the image. In this paper, we focus our study on reconstruction of binary images for simplicity. Thus, the size ($N \times N$) of the matrix is the same as the pixel size of the image and the allele value (1 or 0) of the chromosome corresponds to the pixel intensity of the image. Figure 2 shows a typical example of "E".

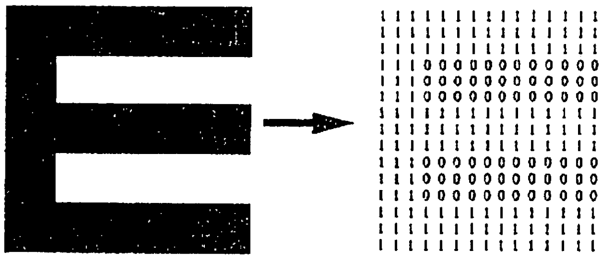

Fig. 2 Image coding.

2-2 Initial population and fitness measure

We firstly create some *pop_size* number of chromosomes (estimates of the original image) randomly in initialization process. And then the penumbral image is calculated for each chromosome (estimate). The fitness for each chromosome is evaluated by comparing the calculated penumbral image of the chromosome with the penumbral image of the original image. The fitness measure or cost function for evaluation is given as :

$$E = \left\| P(\mathbf{r}) - \hat{P}(\mathbf{r}) \right\|^2 \tag{1}$$

and

$$\hat{P}(\mathbf{r}) = \int A(\mathbf{r};\mathbf{r}') \cdot \hat{O}(\mathbf{r}')\, d\mathbf{r}' ; \tag{2}$$

where $\hat{O}$ is the estimate (or chromosome) of the original image O. P is recoded penumbral image and A is aperture function. The lower is the cost, the higheris the fitness. The optimum solution (distortion-freereconstruction) can be obtained by minimizing the costfunction of Eq.(1).

2-3 Genetic operators

Three genetic operators (selection, crossover and mutation) are applied on the whole population to guide the chromosomes toward the optimum solution (minimum cost). The generation process of the new population is shown in Fig.3.

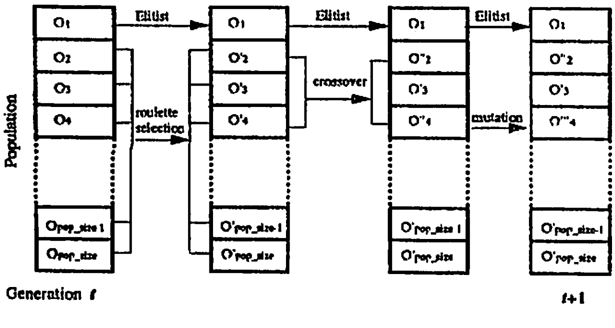

Fig. 3 Generation process of the new population.

a. Selection

We used an elitist selection scheme [3] for selection process. In this scheme, the best chromosome (estimate) with the lowest cost (the highest fitness) is selected as an elitist, which is copied directly into the new population without any changes. Theother chromosomes (estimates) of the new population are selected by using a roulette selection scheme [3]. In this scheme, a roulette wheel with slots size according to fitness is used. We spin the roulette wheel *pop_size-1* times; each time we select one chromosome for the new population. Obviously, some chromosomes would be selected more than once. This is in accordance with the Schema Theorem [3]: the best chromosomes get more copies, the average ones stay even, and the worst ones die off. Furthermore, some chromosomes of this new population undergo alterations by means of crossover and mutation.

b. Crossover

Crossover combines the features of two parent chromosomes to form two similar offspring by swapping corresponding segments of the parents. The intention of the crossover operator is information exchange between different potential solutions.

Two newly developed crossover operators are used.

• *Uniform R/C crossover*

One is called uniform R/C crossover. Two selected parents exchange their row or column information at the same position as shown in Fig.4(a). It performs a local small-scale exchange. It proceeds as follows:

Step 1: Randomly generate an integer value between 0 and 1 to determine the row or the column. If the returned value is 0 then the crossover operation is performed for row, otherwise the crossover operation is performed for column.

Step 2: Randomly generate a real value r from the range [0.0, 1.0]. If $r < Pc$ (probability of crossover), the chromosome is selected for crossover. The process is done for whole population.

Step 3: Mate selected chromosomes randomly.

Step 4: For each pair of coupled chromosomes (P_1 and P_2) we generate a random integer value *pos* from the range [1, *N*]. (*N* is the size of row or column). The value *pos* indicates the position of the crossing.

Step 5: Generate a pair of offsprings (C_1 and C_2) as shown in Fig.4(a). And the parents (P_2 and P_2) are replaced by the generated offsprings (C_1 and C_2).

• *Random R/C crossover*

Another one is called random R/C crossover. The crossing positions for P_1 and P_2 are randomly selected. They can exchange their information at the different position as shown in Fig.4(b). Thus it performs a large-scale exchange. It proceeds as follows:

Step 1: Randomly generate an integer value between 0 and 1 to determine the row or the column. If the returned value is 0 then the crossover operation is performed for row, otherwise the crossover operation is performed for column.

Step 2: Randomly generate a real value r from the range [0.0, 1.0]. If $r < P_c$ (probability of crossover), select given chromosome for crossover. The process is done for whole population.

Step 3: Mate selected chromosomes randomly.

Step 4: For each pair of coupled chromosomes (P_1 and P_2) we generate two random integer values pos_1 and pos_2 from the range [1, N]. The values pos_1 and pos_2 indicate the positions of the crossing for P_1 and P_2, respectively.

Step 5: Generate a pair of offsprings (C_1 and C_2) as shown in Fig. 4(b). And the parents (P_1 and P_2) are replaced by the generated offspring (C_1 and C_2).

The combination of the two crossover operators is important [4].

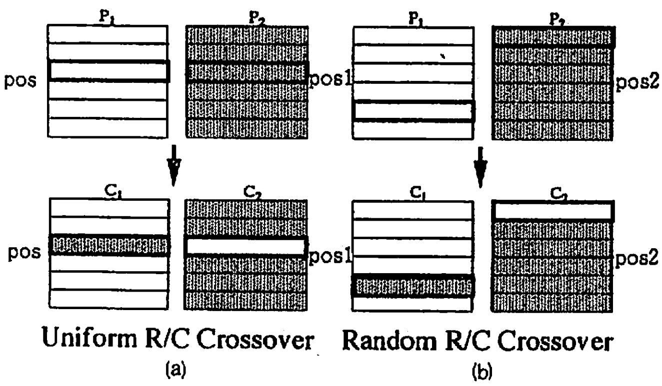

Fig. 4 Crossover operators.

c. Mutation

A weighted mutation scheme as shown in Fig.5 is also used for generation of offsprings together with the crossover. The intention of the mutation operator is the introduction of some extra variability into the population. The proposed mutation operator performs a small-scale survey of the area surrounding the pixel selected for mutation and mutates it in the direction that enhances the smoothing of the image. The process is done as follows:

Step 1: Randomly generate a real value r_1 from the interval [0.0, 1.0].

Step 2: If $r_1 < Pm$ (probability of mutation), the pixel is selected for mutation.

Step 3: Calculate the average value avg of the surrounding 8 pixels as shown in Fig4. The value avg indicates the probability of the selected pixel value=1.

Step 4: Randomly generate a real value r_2 from the interval [0.0, 1.0].

Step 5: If $r_2 < avg$, the selected pixel value = 1, otherwise the selected pixel value = 0.

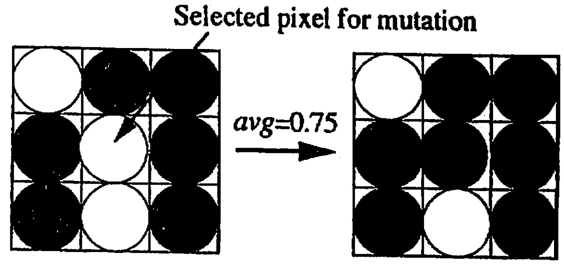

Fig. 5 Mutation operators.

2-4 Constraint

If the penumbral image is with noise, the cost function of Eq.(1) must satisfy

$$\left\| P(\mathbf{r}) - \hat{P}(\mathbf{r}) \right\|^2 = \varepsilon \tag{3}$$

where ε is a parameter determined by the variance of noise. Eq.(3) is an ill-posed equation. There are many solutions such that (3) is satisfied. In order to select the optimum solution, we have to use some *a priori* knowledge as constraints such as smoothness and so on. In GA, the constraints are introduced in a simple manner. They are just associated asadditional terms in the cost function. The newly proposed constrained cost function is shown in Eq.(4).

$$E = \left\| P(\mathbf{r}) - \hat{P}(\mathbf{r}) \right\|^2 + \lambda \left\| \mathbf{L} \cdot \hat{O} \right\|^2 \tag{4}$$

where the second term is the constraint. $\mathbf{L}$ is the constraint operator, and the Laplacian operator [5] is used here to obtain a "smooth" estimate. λ is a parameter used to balance the first term and the second term.

3. Genetic reconstruction of penumbral images

We have used the genetic algorithm for reconstruction of penumbral image. The imaging geometry is the same as that depicted in the previous paper of this series [2]. The encoding process of Eq.(2) is performed by the ray tracing of the estimate through the aperture. Fixed parameters, as well as the optimum parameters for the genetic algorithm, which are obtained through several testing runs, are as follows:

Population size (*pop_size*)	30
Probability of crossover 1 (P_{c1})	0.8
Probability of crossover 2 (P_{c2})	0.4
Probability of mutation (P_M)	0.04

Image size $(N \times N)$ 15×15

The resolution of the genetic reconstruction, which is estimated by reconstructing a source-point, is shown in Fig.5 with a dashed line. In order to make a comparison, the resoution [2] obtained by the linear deconvolution technique is also shown in Fig.6 with a solied line. We can see that the linear reconstruction method will introduce some significant distortion, while by the useof the proposed GA there is no reduction of the resolution even for a larger displacement. The resolutionis just determined by the pixel size.

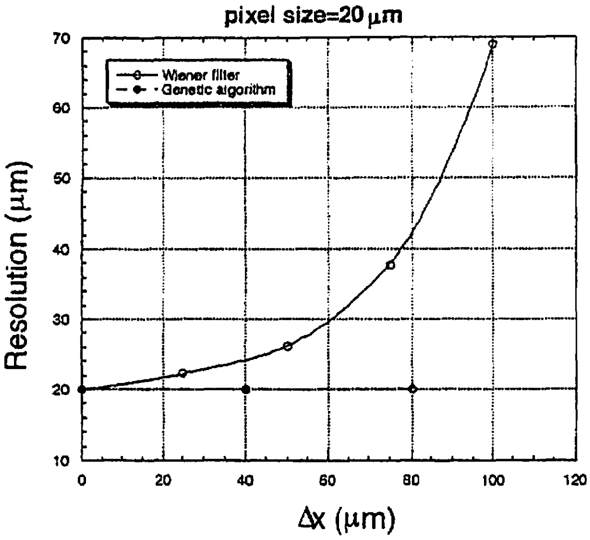

Fig. 6 Resolution of the toroidal-segment aperture as a function of radial displacement of the source point in the source plane.

The genetic reconstruction of the larger object with a size of 300 μm, which is larger than that shown in Ref.2, is shown in Fig.7. Figure 7(a) is the phantom used in the simulations with a pixel number of 15x15. The recorded penumbral image is shown in Fig.7(b) as an input image. The genetic reconstructions is shown in Figs.7(c) - 7(e). It can be seen that the reconstruction is improved as the generation increases. The perfect reconstruction is obtained at 521th generation. The improvement of the cost function is shown in Fig.8 with a solid line. The costs of the reconstructed images shown in Fig.7 are also shown in Fig.8 with "●". The cost=0 means the distortion-free reconstruction. In order to make a comparison, we also show the result using V/H crossover [6], [7]. It can be seen that the convergence toward the optimum solution by the newly proposed crossover operators is much faster than that by the V/H operator.

On the other hand, in real experiments the neutron penumbral images are always degraded by the noise and statistical error [1]. Figure 9 shows the simulation results in the presence of noise. A Gaussian noise

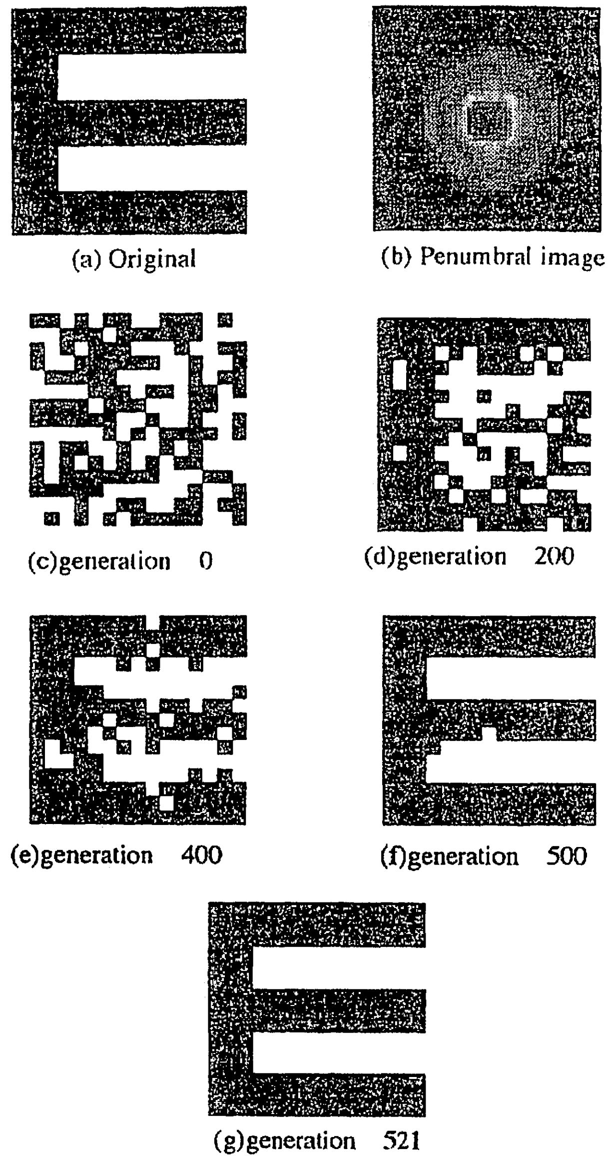

Fig. 7 Genetic reconstruction in the absence of noise.

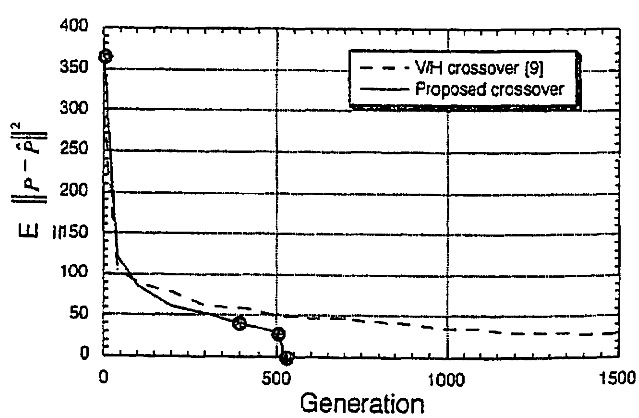

Fig. 8 Improvement of cost function with generation.

($\sigma = 3\% \cdot I_{max}$, $\mu = 0$) is added to the penumbral image. In this case, ε in Eq.(3) is 141.8. Figures 9(a) and 9(b) show the calculated difference error between $\hat{P}$ and P, and Laplacian value of the reconstructed images. Solid line is the result with constraint (Eq.(4)) and dashed line is the result without constraint (Eq.(1)). Typical reconstructed images by the normal GA (without constraint) and the constrained GA are shown in Figs.9(c) and 9(d), respectively. The brackets indicate the difference error and Laplacian value of the reconstructed image. As shown in Figs.9 (a) and 9(b), both methods guide the difference error to ε, while the Laplacian values of the reconstructed images are quite different. Only the case with constraint, the Laplacian value is guided to that of the original image. As shown in Fig.9(c), if we do not use the constraint, there are many solutions such that Eq.(3) is satisfied. The optimum solution can be selected by using the Laplacian constraint as shown in Fig.9(d). Thus we can see that the constrained GA method is very tolerant of the noise.

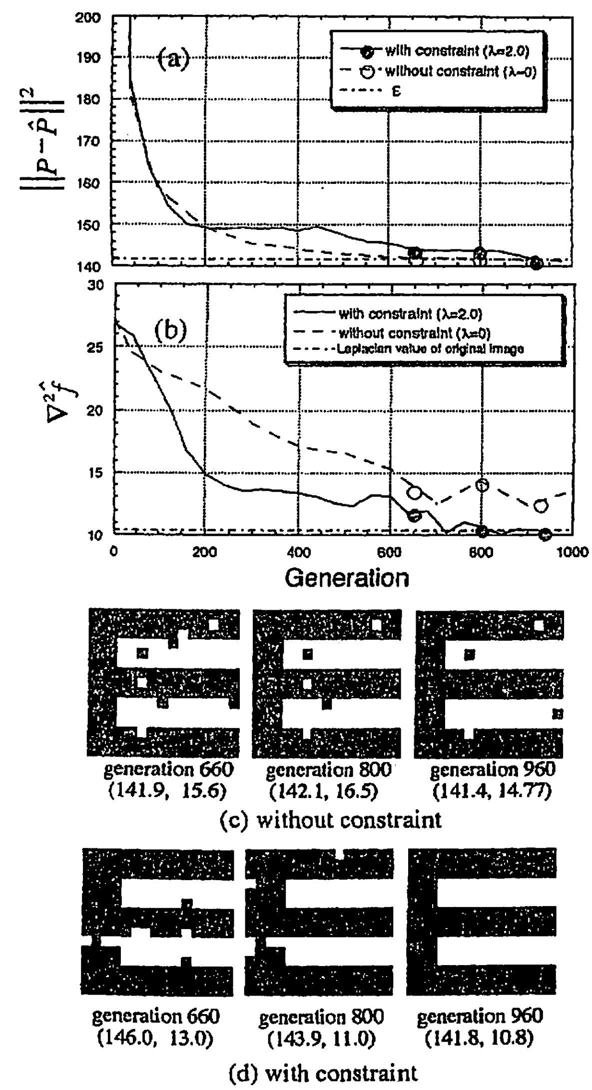

Fig. 9 Genetic reconstruction in the presnece of noise.

5. Summary

We have applied a genetic algorithm (GA) for reconstruction of penumbral image. The performance and potential of the proposed method has been demonstrated. The computer simulation results show that the linear reconstruction method will introduce some significant distortion, while by using the proposed method it is possible to obtain a distortion-free reconstruction even over a large field of view. Furthermore, by using the Laplacian operator as a constraint, the method is very tolerant of the noise contained in penumbral image.

The disadvantage of the proposed algorithm is its large computational cost (the simulation results shown in section 3 required more than 2 hours of CPU time on a 32-bit Sun workstation). Compared to other nonlinear techniques, GA has an excellent parallelism. We can expect much speedup by using a parallel or multi-processor computing system [8]. A further investigation is also under way to develop more efficient genetic operator in order to reduce the calculation time and to realize the reconstruction of real neutron image.

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