# Image Restoration by a Constrained Genetic Algorithm

# Image Restoration by a Constrained Genetic Algorithm

Yen-Wei CHEN, Zensho NAKAO and Makoto IGUCHI


## Abstract

A genetic algorithm is presented for the deconvolution problem of image restoration. The restoration problem is modeled as an optimization problem, whose cost function is minimized based on mechanics of natural selection and natural genetics. Because the complicated *a priori* constraints can be easily incoporated by the appropriate modification of the costfunction, the algorithm is well suited to the solution of ill-posed problem. The simuation results show that it is possible to obtain a perfect restoration even in the presence of noise by constraining the cost function with some *a priori* information such as smoothness.


## 1. Introduction

Many image processing applications, such as satellite remote sensing, medical and scintific imaging, require a high resolution image. However, currently available optical or imaging systems have certain physical limitations, and the image $g(x,y)$ we usually observe is a degraded one by the convolution of the original image $f(x,y)$ and the point spread function (blurring function) $h(x,y)$ of the optical or imaging system, which can be expressed as

$$g(x,y)=\int\int_{-\infty}^{\infty} f(x',y')\cdot h(x-x',y-y')dx'\cdot dy'+n(x,y)$$
$$=f(x,y)*h(x,y)+n(x,y) \quad (1)$$

where $*$ denotes the convolution operator and $n(x,y)$ is the random noise contained in the blurred image. In order to recover the original image $f(x,y)$ from the blurred image $g(x,y)$, we have to deconvolve the $g(x,y)$ with the blurring function $h(x,y)$. The inverse filter is a typical linear filter used to deconvolution problem, whose point spread function is the inverse of the blurring function. In the absence of noise, $f(x,y)$ is, in principle, perfectly recoverable by the inverse filter. However in the presence of noise, the noise $n(x,y)$ will be amplified to very high levels at spatial frequencies with an amplitude close to zero and the image restoration problem is an ill-posed problem. Although a large number of filters such as Wiener filter [ 1 ], where the mean-square error is minimized, have been proposed to overcome this limitation, it is impossible to avoid reduction of resolution and quality of the recovered image.

In these two decades, a few nonlinear techniques have been proposed for image restoration such as the maximum entropy (ME) method [ 2 ], [ 3 ] and the maximum a posteriori (MAP) method [ 1 ], [ 4 ], which incorporate constraints to reduce the amplification of noise in the restoration.

In this paper we present an alternative method for image restoration based on a genetie algorithm (GA) [ 5 ] under the constraint of a Laplacian operator. In this method the restoration problem is modeled as an optimization problem,whose cost function is minimized based on mechanics of natural selection and natural genetics. Because the complicated *a priori* constraints can be easily incorporated by the appropriate modification of the cost function, the GA method is well suited to the solution of the ill-posed problem. The performance and behavior of the proposed method are demonstrated with two typical blurring functions. The first one is for restoration of imaga blurred by out of focusing (defocusing)and the second one is blurred by an ideal lowpass filter. For the second case the inverse of the blurring function does not exist and it is impossible to recover the original image by the linear techniques. The basis of the genetic algorithm is given in section 2 and the constraint used for the ill-posed problem is discussed in section 3. Simulation results are presented in section 4.

## 2. The genetic algorithm

The typical flowchart of genetic algorithms for image restoration is shown in Fig. 1. We firstly create a population of estimates of the original image as chromosomes randomly in the coding and initialization processes. And then the degraded image is calculated for each chromosome (estimate). The fitness for each chromosome is evaluated by comparing the calculated degraded image of the chromosome with the degraded image of the original image. The cost function for evaluation is shown as Eq. (2):

$$E=\|g-\hat{f}*h\|^2 \quad (2)$$

Received: 6 Nov., 1995
Department of Electrical and Electronic Engineering, Faculty of Engineering.

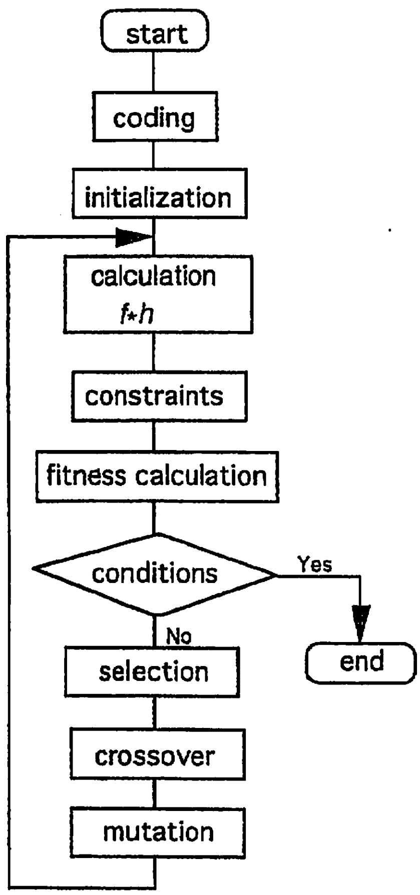



Fig. 1 A typical flowchart of genetic algorithms for image restoration.

where $\hat{f}$ is the estimate (or chromosome) of $f$. The lower is the cost, the higher is the fitness. The optimum solution (perfect restoration) can be obtained by minimizing the cost function of Eq. ( 2 ).

Three genetic operators (selection, crossover and mutation) are performed on the whole populations to guide the chromosomes toward the optimum solution (minimum cost).

a. Selection

We used an elitist selection scheme [ 5 ] for the selection process as shown in Fig. 2 (a). In this scheme, the chromosomes (estimates) with lower cost (higher fitness) are selected as elitists, which are copied directly in the new generation without any changes, The chromosomes (estimates) with higher cost (lower fitness) are replaced with offsprings (new estimates) produced by the parents, which are selected from the elitist portion randomly, with crossover and mutation.

b. Crossover

A newly developed 2-D uniform crossover operator is used for generation of offspring instead of the normal unifom crossover operator [ 5 ], which is a one-point uniform crossover. The process of the new crossover is shown in Fig. 2 (b). The crossover operation is performed for every row or every column as follows:

Step 1 : Randomly select an integral number between 0 and 1 to determine the row or the column (If the returned value is 0 then the crossover operation is performed for every row, otherwise the crossover operation is performed for every column).

Step 2 : Randomly select a real value from the interval [0.0, 1.0]

Step 3 : If the returned value is equal to or less than the probability of crossover (0.5 in the simulations) then copy the row/column from the parent 1 to the offspring, otherwise copy the row/column from the parent 2 to the offspring.

Step 4 : Step 2 and step 3 are applied to every row or every column.

In the proposed crossover process, the number of operations is N, while in the process of normal uniform crossover, the crossover operation is performed for every pixel and the number of operations is $N^2$, where $N^2$ is the number of pixels of the image. Furthermore,the simulation results show that the convergence toward the optimum solution by the proposed operator is faster than that by the normal uniform crossover [ 6 ]. The calculation time can be largely reduced by using the proposed efficient crossover operator.

c. Mutation

A single mutation scheme [ 5 ] as shown in Fig. 2 (c) is also used for generation of offspings together with the crossover. The process is done as follows:

Step 1 : Randomly select a real value from the interval [0.0, 1.0].

Step 2 : If the returned value is equal to or less than the given probability of mutation (0.02 in the simulations), then the pixel value is mutated from 0 to 1 or from 1 to 0, otherwise the pixel value is kept without mutation.

Step 3 : Step 1 and step 2 are applied to every pixel. The probability of mutation is an important parameter, which will significantly affect the efficiency of the algorithm. It is found to be inversely proportional to the number of pixels of the image.

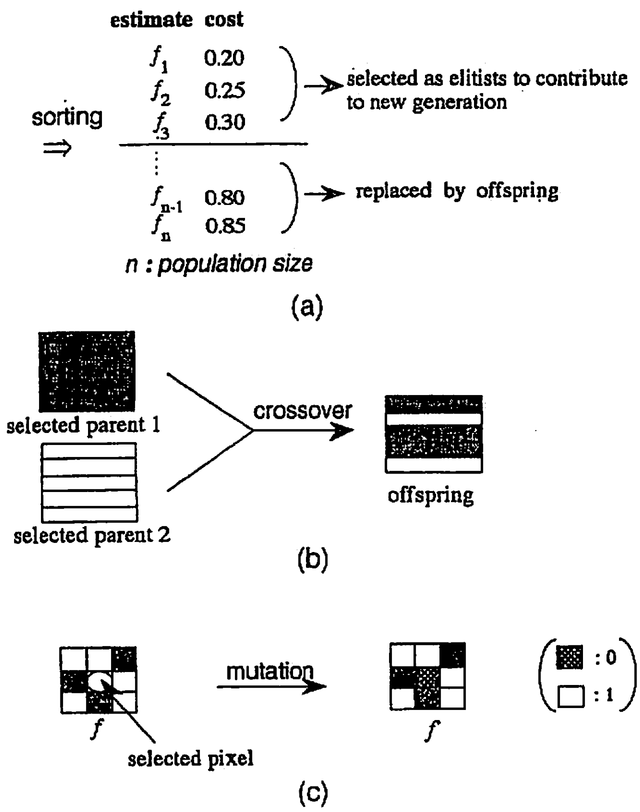



Fig. 2 Evolutionary operators : selection (a), crossover (b), and mutation (c).

## 3. Constraint

If the degraded image is with noise, the cost function of Eq.( 2 ) must satisfy

$$\left\| g - \hat{f} * h \right\|^2 = \varepsilon \tag{3}$$

where $\varepsilon$ is a parameter determined by the variance of noise. Eq.( 3 ) is an ill-posed equation. There are many solutions $\hat{f}$ such that ( 3 ) is satisfied. In order to select the optimum solution, we have to use some *a priori* knowledge as constraints such as smoothness and so on. In GA, the constraints are introduced in a simple manner. They are just associated as additional terms in the cost function. The newly proposed constrained cost function is shown in Eq.( 4 ).

$$E = \left\| g - \hat{f} * h \right\|^2 + \lambda \cdot \left\| \mathbf{P} \cdot \hat{f} \right\|^2 \tag{4}$$

where the second term is the constraint. P is the constraint operator, and the Laplacian operator [ 1 ] is used here to obtain a "smooth" estimate. $\lambda$ is a parameter used to balance the first term and the second term. In our simulation $\lambda = 0.02$.

## 4. Simulation results

We have carried out computer simulations to validate the applicability of GA for image restoration. A simple binary phantom as shown in Fig. 3 is used in simulations. The pixel size is 15×15. The optimum parameters for the simulations, which are obtained through several testing runs, are as follows:

| | |
|---|---|
| Population size | 300 |
| Elitist selection rate | 10% |
| Probability of crossover | 0.5 |
| Probability of mutation | 0.02 |

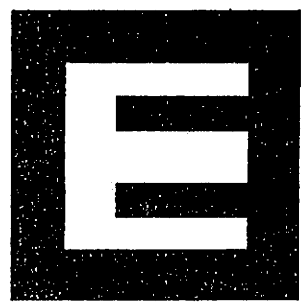


Fig. 3 Phantom used in simulations.

The simulations are done with two different blurring functions. For the first case, the image is degraded by the out of focusing (defocusing). The PSF of the defocusing is shown in Eq.( 5 ),

$$h(x,y) = \begin{cases} 1/\pi a^2, & (x^2+y^2) \le a^2 \\ 0, & otherwise \end{cases} \tag{5}$$

where $a$=3.

Figure 4 shows the simulation results in the absence of noise. Figure 4 (a) is the degraded image without noise. The restorations by normal GA (without constraint) after 1st, 50th and 520th generation are shown in Figs. 4 ( b ), 4 (c) and 4 (d), respectively. It can be seen that the restored image is improved with increasing the generation. The improvement of the cost function is shown in Fig. 5 . The costs of Figs. 4 (c) and 4 ( d ) also shown in Fig.5 with "●". The cost=0 means the perfect restoration.

Figure 6 shows the simulation results in the presence of noise. Figure 6 ( a ) is the degraded image with an additive Gaussian noise ( $\sigma$=0.03, $\mu$=0). The restorations by normal GA (without *constraint*) after 620th and 2000th generation are shown in Figs. 6 (b) and 6 (c), repectively. It can be seen that in the presence of noise, even after 2000th generation we can not obtain an optimum solution (perfect restoration) because of ill-posed

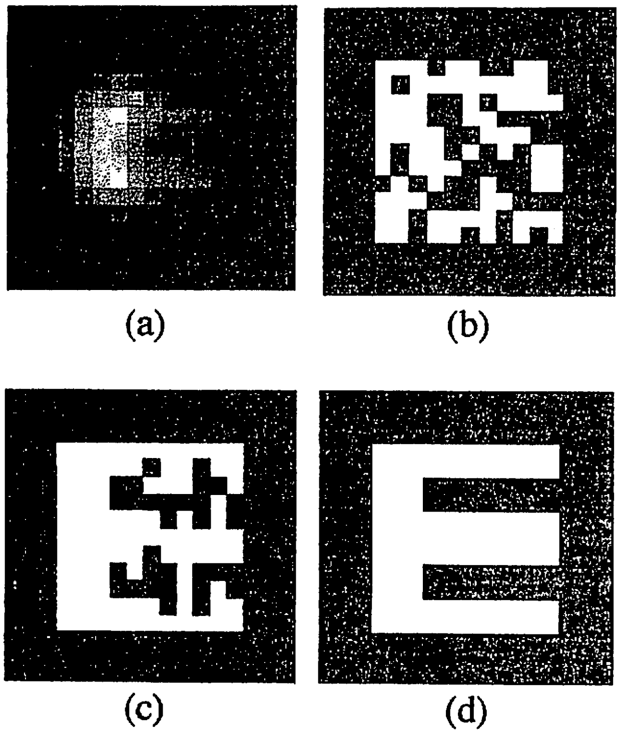



Fig. 4 (a) Degraded image by out of focusing without noise, (b) restred images by the normal GA after lst generation, (c) same as (b) but after 50th generation, (d) same as (b) but after 520th generation.

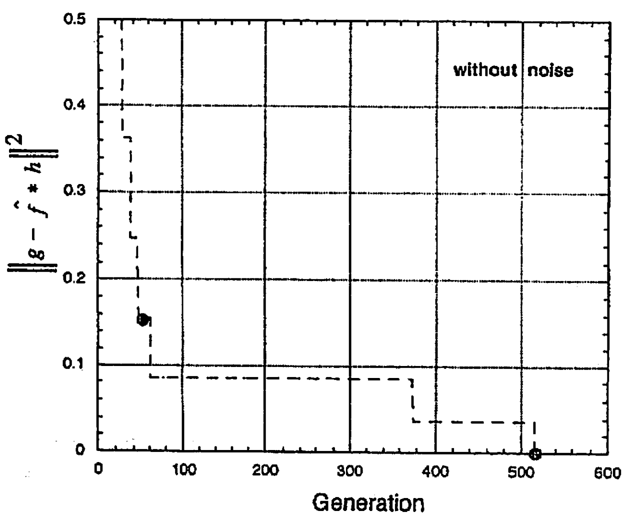



Fig. 5 Improvement of cost function with generation.

condition as shown in Eq.( 3 ). The restoration by the proposed constrained GA after 620th generation is shown in Fig. 6 ( d ). A perfect restoration is obtained. In order to make a comparison, the retorations by inverse filter and Wiener filter are also shown in Figs. 6 (e) and 6 (f), respectively. The qualities of restored image are poor.

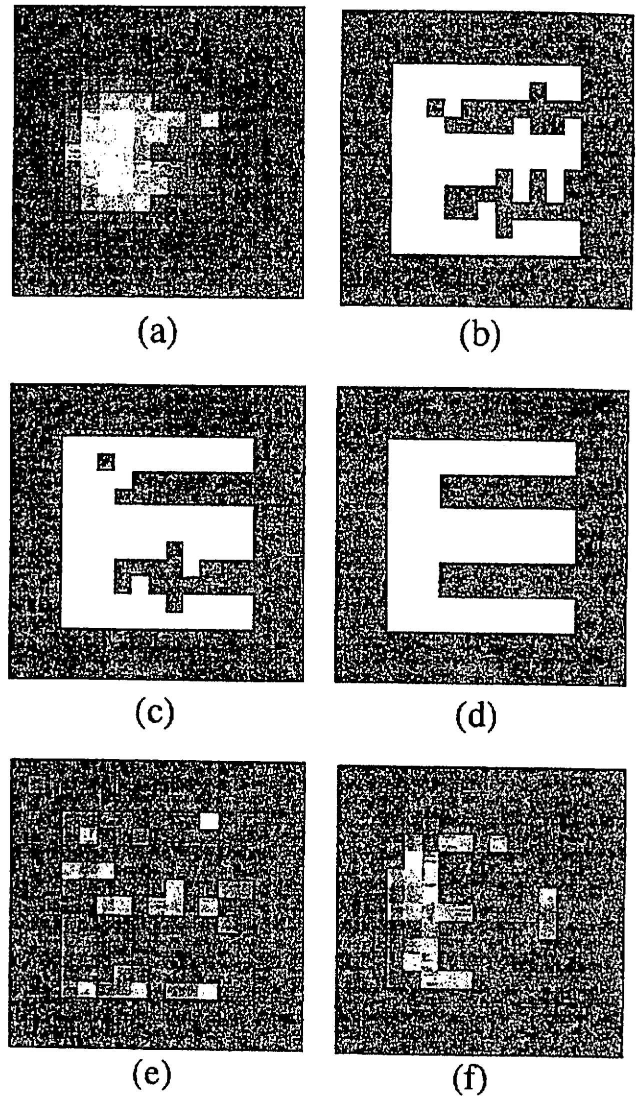


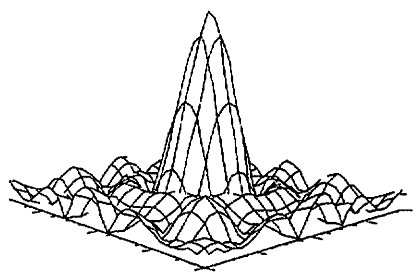

Fig. 6 (a) Degraded image by out of focusing with an additive Gaussian noise, (b) restored image by the normal GA after 620th generation, (c) same as (b) but after 2000th generation, (d) restored image by the constrained GA after 620th generation, (e) restored image by an inverse filter, (f) restored image by a Wiener filter.

For the second case, the image is blurred by an ideal lowpass filter. The modulation transfer function (MTF) $H(u, v)$ of the lowpass filter is shown in Eq.( 6 ).

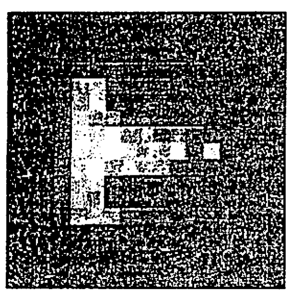
$$H(u,v)=\begin{cases}1, & (u^2+v^2)\le f_c^2 \\ 0, & otherwise\end{cases} \qquad (6)$$

where u and v are spatial frequencies, $f_c$ is the cutoff frequency. In our simulations, $f_c=f_{max}/2$ ($f_{max}$ *is the Nyquist frequency*). The PSF of the lowpass filter is shown in Fig. 7 (a). Figure 7 (b) is the degraded image by the lowpass filter with an additive Gaussian noise ( $\sigma=$

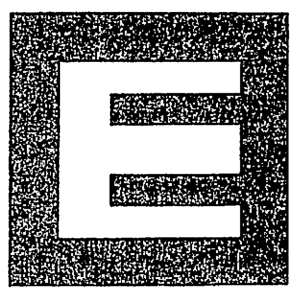
0.03, $\mu = 0$). Since the inverse of the blurring function does not exist in this case, it is impossible to recover the original image from the degraded image using the linear techniques even in the absence of noise. Figure 7 (c) is the restored image by the constrained GA after 610th generations. In this case, a perfect restoration is also obtained.



(a)



(b)



(c)

Fig. 7 (a) PSF of lowpass filter, (b) degraded image by the lowpass filter with an additive Gaussian noise, (c) restored image by the constrained GA.

## 5. Summary

We have applied a genetic algorithm (GA) for image restoration and a Laplacian-constrained cost function has been proposed for restoration of image in the presence of noise. The performance and potential of the proposed method have been demonstrated with computer simulations. The computer simulation results show that it is possible to recover the original image from the degraded image even in the presence of noise. The disadvantage of the proposed algorithm is its large computational cost (the simulation results shown in section 4 required several hours of CPU time on a 32-bit workstation IBMRS/6000). Compared to other nonlinear techniques, GA has an excellent parallelism. We can expect much speedup by using a parallel or multi-processor computing system. A further investigationis also under way to develop more efficient genetic operators in order to reduce the calculation time and to realize the restoration of large-size images.

## Acknowledgments

The authors would like to thank Mr. F. Ali and Ms. M. Takashibu for their helpful discussions on genetic algorithm and Mr. K. Oe for his support on computer calculations.

## References:

[ 1 ] A. Rosenfeld and A. C. KaK, *Digital Picture Processing*, 2 nd Edition, Academic Press, New York, 1982.

[ 2 ] B. R. Frieden, "Restoring with maximum likelihood and maximum entropy", *J. Opt. Soc. Am*, Vol.62, pp.511-518 (1972).

[ 3 ] S. F. Burch, S. F. Gull and J. Skilling, "Image restoration by a powerful maximum entropy method", *Computer Graphics Image Processing*, Vol.12, pp. 113-128 (19839).

[ 4 ] S. Geman and D. Geman, "Stochastic relaxation, Gibbs distribution, ans the Bayesian restoration of images", *IEEE Trans.*, Vol.PAMI-6, pp.721-741 (1984).

[ 5 ] Z. Michalewics, *Genetic Algorithms + Data Structures = Evolution Programs*, Springer-Verlag, Berlin,1992.

[ 6 ] M. Iguchi, Graduation thesis, University of the Ryukyus, 1996