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A Field-Theoretic View of Logic Functions

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Abstract Logic functions of several variables are considered as mappings from a Cartesian product of several copies of a finite (Galois) field into the field itself. Based on the well-known fact that such mappings are always polynomial, it is shown how the corresponding polynomial expressions for given logic functions are obtained through several concrete computational examples.

1. Preliminaries

Let $K=GF(q)$, $q=p^n$, where p is prime. We consider the case $q=p=3, 5$ here. Refer to [NZK] for the case $q=2, 4$. We make computation only for $p=3$ because an extension to the case $p=5$ is routine. The following are some relevant results from field theory:

(1) $K^*=K-\{0\}$ is a cyclic group of order p-1.

(2) $\forall\ x \in K^*,\ x^{p-1}=1$ (Fermat's Theorem).

(3) $\forall\ x \in K,\ x^p=x$.

(4) $K=\{x: x \text{ is a solution of } x^p-x=0\}$.

(5) $x^{p-1}-1=\prod_{i=1}^{p-1}(x-i)$, so $\prod_{i=1}^{p-1} i=(p-1)!=-1$ (Wilson's Theorem).

(6) $\forall\ x \in K,\ \exists$ a unique $\sqrt[p]{x}$

$K=GF(3)=\{0, 1, 2\}$ has the field structure given by the following addition (+) and multiplication (* or juxtaposition) tables:

+	0	1	2
0	0	1	2
1	1	2	0
2	2	0	1

*	0	1	2
0	0	0	0
1	0	1	2
2	0	2	1

2. Formulas for Polynomial Representation

We let $K=GF(3)$ once for all. There are several formulas known for polynomial representation of functions; the following results quoted from [T] are particularly useful for

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our purpose:

(1) If $f: K \longrightarrow K$ is a function, then f can be expressed in a polynomial form over K:

$$f(x) = a_0 + a_1 x + a_2 x^2,$$

where $a_0 = f(0)$, $a_i = -\sum_{x \in K} x^{2-i} f(x)$, $(i = 1, 2)$

(2) If $f: K^2 \longrightarrow K$ is a mapping, then f can be realized as a polynomial mapping over K in two variables:

$$f(x, y) = \sum_{0 \le i, j \le 2} a_{ij} x^i y^j,$$

where $a_{00} = f(0, 0)$, $a_{i0} = -\sum_{x \in K} x^{2-i} f(x, 0)$, $a_{0i} = -\sum_{y \in K} y^{2-i} f(0, y)$, $a_{ij} = \sum_{x, y \in K} x^{2-i} y^{2-j} f(x, y)$, $(i, j = 1, 2)$

(3) If $f: K^3 \longrightarrow K$ is a mapping, then f can be realized as a polynomial mapping over K in three variables:

$$f(x, y, z) = \sum_{0 \le i, j, k \le 2} a_{ijk} x^i y^j z^k,$$

where $a_{000} = f(0, 0, 0)$, $a_{i00} = -\sum_{x \in K} x^{2-i} f(x, 0, 0)$, $a_{0j0} = -\sum_{y \in K} y^{2-j} f(0, y, 0)$, $a_{00k} = -\sum_{z \in K} z^{2-k} f(0, 0, z)$, $a_{ij0} = \sum_{x, y \in K} x^{2-i} y^{2-j} f(x, y, 0)$, $a_{i0k} = \sum_{x, z \in K} x^{2-i} z^{2-k} f(x, 0, z)$, $a_{0jk} = \sum_{y, z \in K} y^{2-j} z^{2-k} f(0, y, z)$, $a_{ijk} = -\sum_{x, y, z \in K} x^{2-i} y^{2-j} z^{2-k} f(x, y, z)$, $(i, j, k = 1, 2)$

It is to be noted that all calculations in the formulas above must be done in the Galois field $K = GF(3)$, and also that the mapping can be generalized to the one from K^n into K, i.e., of n variables.

3. Examples and Computations

We pick several examples of logic functions and express them as polynomial in one, two, or three variables. Refer to the Appendix for the BASIC programs used.

(1) Monadic operators

Define logic functions of one variable $x^{[0,2]}$ and $x^{[1]}$ as in [HK] by

$$x^{[02]} = \begin{cases} 2, & x \in \{0, 2\} \\ 0, & \text{otherwise} \end{cases}$$

$$x^{[1]} = \begin{cases} 2, & x = 1 \\ 0, & \text{otherwise} \end{cases}$$

By formula (2.1), we obtain the coefficients for $x^{\{0,2\}}$ and $x^{[1]}$, respectively, as follows:

$$a_0 = 2,\ a_1 = a_2 = -1 = 2;$$
$$a_0 = 0,\ a_1 = a_2 = -2 = 1$$

Hence, their polynomial representation now becomes:

$$x^{\{0,2\}} = 2\ (x^2 + x + 1) = 2\ (x+2)^2,$$
$$x^{[1]} = x + x^2 = x\ (x+1)$$

(2) Dyadic Operators

Define logic functions of two variables Max(x,y) and Min(x,y) by

$$\mathrm{Max}(x, y) = \begin{cases} x, & x \geqq y \\ y, & \text{otherwise} \end{cases}$$

$$\mathrm{Min}(x, y) = \begin{cases} x, & x \leqq y \\ y, & \text{otherwise} \end{cases}$$

By formula (2.2), we obtain the coefficients for Max(x,y) and Min(x,y), respectivly, as follows:

$$a_{00} = a_{20} = a_{02} = 0,\ a_{10} = a_{01} = -2 = 1,\ a_{11} = 2,\ a_{21} = a_{12} = a_{22} = 1;$$

$$a_{00} = a_{10} = a_{01} = a_{20} = a_{02} = 0,\ a_{11} = 1,\ a_{21} = a_{12} = a_{22} = 2$$

Thus, their polynomial representation is given by:

$$\mathrm{Max}(x, y) = x + y + 2\,xy + x^2y + xy^2 + x^2y^2 = (x+y) + xy\,[2 + (x+y) + xy],$$
$$\mathrm{Min}(x, y) = xy + 2x^2y + 2xy^2 + 2x^2y^2 = xy\,[1 + 2\,(x + y + xy)]$$

(3) Triadic Operator

Define a logic function of three variables T(x,y,z) by the following table [HK] :

z \ y \ x	0			1			2		
	0	1	2	0	1	2	0	1	2
0	1	0	1	1	2	2	1	0	1
1	1	2	2	2	2	0	0	2	2
2	1	0	1	2	0	2	0	2	0

By formula (2.3) its coefficients are given by the following:

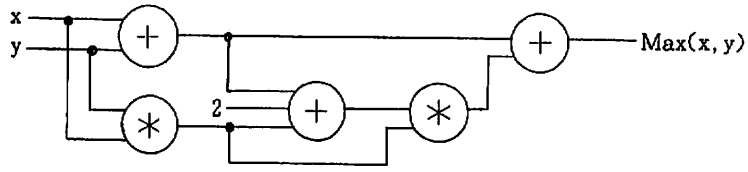
$$a_{000}=a_{110}=a_{011}=a_{210}=a_{102}=a_{012}=1,\ a_{100}=a_{001}=a_{200}=a_{002}=a_{101}=a_{111}=a_{201}=a_{120}=a_{021}$$
$$=a_{220}=a_{202}=a_{022}=a_{211}=a_{222}=0,\ a_{010}=a_{020}=a_{112}=a_{221}=a_{212}=a_{122}=-2=1,\ a_{121}=-1=2$$

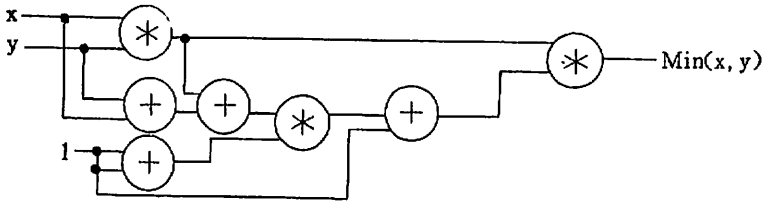
Therefore, the polynomial expression for T(x,y,z) is given by:

$$T(x,y,z)=1+y+y^2+xy+yz+x^2y+xz^2+yz^2+2xy^2z+xyz^2+x^2y^2z+x^2yz^2$$
$$+xy^2z^2=1+y+y(x+y+z+x^2+z^2)+xyz(xy+xz+yz+2y+z)+xz^2$$

4. Conclusions

If we introduce two logic gates =⊕– and =⊛– , realizing the two field operators (+) and (*), respectively, then we can easily depict the functions in logic diagrams. Take, for example, Max(x,y) and Min(x,y) ; we get the following logic diagrams:





An obvious advantage for having polynomial expressions is that all calculations can be made efficiently on {0, 1, 2} with the field structure of GF(3), i.e., with the four arithmetic operations of GF(3) itself as we normally do with the real (or complex) number field, thereby turning the manipulation of logic functions into ordinary computation of polynomials. Also, the more familiar polynomial form reveals the configuration of strings of symbols more concretely than otherwise.

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Appendix (BASIC programs)

```
THE PROGRAM EXPRESSES TERNARY LOGIC FUNCTIONS IN POLYNOMIAL FORM.
10  '--------- GF(3) - ONE VARIABLE
20  '--------- THIS PROGRAM EXPRESSES LOGIC FUNCTIONS IN ONE VARIABLE
30  '--------- IN POLYNOMIAL FORM OVER GALOIS FIELD GF(3)
40  DIM F(3), A(3)
50  '--------- DEFINE (+) AND (*) MODULO 3
60  DEF FNM (X*Y) =X*Y-3*INT ((X*Y)/3)
70  DEF FNA (X*Y) =X+Y-3*INT ((X+Y)/3)
80  '--------- DEFINE F(X)
90  CLS 1
100 PRINT" THE INPUTS MUST BE 0, 1, 2. TYPE IN THREE NUMBERS FOR
    F(0),F(1),F(2), "
110 PRINT" SEPARATED BY RETURN. "
120 FOR I=0 TO 2
130       INPUT F(I)
140 NEXT I
150 CLS I
160 PRINT:LPRINT
170 PRINT" POLYNOMIAL EXPRESSIONS FOR LOGIC FUNCTIONS "
180 LPRINT" POLYNOMIAL EXPRESSIONS FOR LOGIC FUNCTIONS "
190 PRINT STRING$ (42, "-")
200 LPRINT STRING$ (42, "-")
210 PRINT:LPRINT
220 PRINT" X" ; " -----> " : " F (X)"
230 LPRINT" X" ; " -----> " : " F (X)"
240 '--------- TABLE FOR X---> F (X)
250 FOR I=0 TO 2
260       RINT I; " -----> " ;F (I)
270       LPRINT I; " -----> " ;F (I)
280 NEXT I
290 PRINT STRING$ (20, "-")
300 LPRINT STRING$ (20, "-")
310 '--------- INITIALIZE
320 N=0 : N1=0
330 '--------- DEFINE YF(X)
340 DEF FND (X)=FNM (X, F (X))
350 FOR I=0 TO 2
360       N=FNA (N, FND (I)) : 'A (I)
370 NEXT I
```

```
380  FOR I=0 TO 2
390        N1=FNA (N1 ,F (I)):'A (2)
400  NEXT I
410  '--------- OUTPUT THE FORMULA FOR F(X)
420  PRINT"  F (X) = " ;F (0); "-" ;N; "X-" ;N1 ; "XX. "
430  LPRINT"  F (X) = " ;F (0); "-" ;N; "X-" ;N1 ; "XX. "
440  PRINT STRING$ (20, "-")
450  LPRINT STRING$ (20, "-")
460  PRINT:LPRINT
470  PRINT"  CONT=RETURN:STOP=0 " ; :A$ =INPUT$ (1)
480  IF A$ <> CHR$ (13) AND A$ <> " 0 " THEN 470
490  IF A$ =CHR$ (13) THEN 90 ELSE 500
500  END
```

THE PROGRAM EXPRESSES TERNARY LOGIC FUNCTIONS IN POLYNOMIAL FORM.

```
10   '-------- GF (3)-TWO VARIABLES
20   '-------- THIS PROGRAM EXPRESSES LOGIC FUNCTIONS OF TWO
     VARIABLES
30   '-------- IN POLYNOMIAL FORM OVER GALOIS FIELD GF(3).
40   DIM F(3,3), A(3,3)
50   '-------- DEFINE (+) AND (*) MODULO 3
60   DEF FNM (X,Y) =X*Y-3*INT ( (X*Y) /3)
70   DEF FNA (X,Y) =X+Y-3*INT ( (X+Y) /3)
80   CLS 1
90   '-------- DEFINEF (X,Y)
100  PRINT" THE INPUTS MUST BE 0,1,2. TYPE IN 9 NUMBERS F(0,0),F
     (0,1),…, F(2,2), "
110  PRINT" SEPARATED BY RETURN. "
120  FOR I=0 TO 2
130        FOR J=0 TO 2
140              INPUT F (I,J)
150        NEXT J
160  NEXT I
170  CLS 1
180  LPRINT:LPRINT
190  PRINT " POLYNOMIAL EXPRESSIONS FOR LOGIC FUNCTIONS "
200  LPRINT " POLYNOMIAL EXPRESSIONS FOR LOGIC FUNCTIONS "
210  LPRINT STRING$ (42, "-")
220  PRINT STRING$ (42, "-")
```

```
230 LPRINT : PRINT
240 '---------- TABLE FOR X, Y -----> F ( X, Y )
250  PRINT"  X              " ; "Y" ; "  ----->   " ; "F (X, Y)"
260  LPRINT"  X              " ; "Y" ; " ----->   " ; "F (X, Y)"
270 FOR I=0  TO  2
280        FOR J=0  TO  2
290              PRINT I; "            " ; J ; " -----> " ; F (I, J)
300              LPRINT I; "            " ; J ; " -----> " ; F (I, J)
310        NEXT J
320 NEXT I
330 PRINT STRING $ (42, " - ")
340 LPRINT STRING $ (42, " - ")
350 '--------- DEFINE XF (X, 0 )
360 DEF FND (X) =FNM (X, F (X, 0) )
370 '--------- DEFINE XF ( 0,X)
380 DEF FNE (X) =FNM (X, F ( 0 , X) )
390 '--------- DEFINE XYF (X,Y)
400 DEF FNT (X, Y) =FNM (X, FNM (Y, F (X, Y) ) )
410 '--------- DEFINE XF (X,Y)
420 DEF FNR (X, Y) =FNM (X, F (X, Y) )
430 '--------- DEFINE YF (X,Y)
440 DEF FNP (X, Y) =FNM (Y, F (X, Y) )
450 '--------- INITIALIZE
460 N=0 : N1 = 0 : N2 = 0 : N3 = 0 : S=0 : R=0 : P=0 : Q=0
470 FOR I=0 TO 2
480        N=FNA (N, FND ( I ) )                  : 'A (1 , 0)
490        N1 =FNA (N1 , F (I, 0) )               : 'A (2 , 0)
500        N2 =FNA (N2 , F (0 , I) )              : 'A (0 , 2)
510        N3 =FNA (N3 , FNE ( I ) )              : 'A (0 , 1)
520 NEXT I
530 FOR I=0  TO  2
540        FOR J=0  TO  2
550              S=FNA (S, FNT (I, J) )                  : 'A (1 , 1)
560              R=FNA (R, FNR (I, J) )                  : 'A (1 , 2)
570              P=FNA (P, FNP (I, J) )                  : 'A (2 , 1)
580              Q=FNA (Q, F (I, J) )                    : 'A (2 , 2)
590        NEXT J
600 NEXT I
610 '--------- OUTPUT THE FORMULA FOR F (X,Y)
620 LPRINT" F (X, Y) = " ; F (0 , 0) ; " - " ; N; "X- " ; N3 ; "Y+ " ; S; "XY- " ; N1 ; "XX- " ;
    N2 ; " YY+ "
```

```
630 LPRINT P; "XXY+" ;R; "XYY+" ;Q; "XXYY."
640 PRINT" F(X,Y)=" ;F(0,0) ; "-" ;N; "X-" ;N3 ; "Y+" ;S; "XY-" ;N1 ; "XX-" ;
    N2 ; "YY+"
650 PRINT P; "XXY+" ;R; "XYY+" ;Q; "XXYY."
660 PRINT STRING$ (42, "-")
670 LPRINT STRING$ (42, "-")
680 PRINT:PRINT
690 '-------- CONTINUE?
700 PRINT"CONT=RETURN, STOP=0" ; :A$ =INPUT$ (1)
710 IF A$ <> CHR$ (13) AND A$ <> "0" THEN 700
720 IF A$ =CHR$ (13) THEN 60
730 END
```

THE PROGRAM EXPRESSES TERNARY LOGIC FUNCTIONS IN POLYNOMIAL FORM.

```
10  '-------- GF (3)-THREE VARIABLES
20  '--------- THIS PROGRAM EXPRESSES LOGIC FUNCTIONS OF THREE
    VARIABLES
30  '-------- IN POLYNOMIAL FORM OVER GALOIS FIELD GF(3).
40  DIM F(3,3,3),A(3,3,3),B(27)
50  '--------- DEFINE (+) AND (*) MODULO 3
60  DEF FNM (X,Y) =X*Y-3*INT((X*Y)/3)
70  DEF FNA (X.Y) =X+Y-3*INT((X+Y)/3)
80  CLS 1
90  '--------- DEFINE F(X,Y,Z)
100 PRINT " THE INPUTS MUST BE 0,1,2. TYPE IN 27 NUMBERS F(0,0,0)
    F(0,0,1),"
110 PRINT" F(0,0,2),…,F(2,2,2), SEPARATED BY RETURN."
120 FOR I=0 TO 2
130       FOR J=0 TO 2
140             FOR K=0 TO 2
150                   INPUT F(I,J,K)
160             NEXT K
170       NEXT J
180 NEXT I
190 CLS 1
200 LPRINT:LPRINT
210 PRINT " POLYNOMIAL EXPRESSIONS FOR LOGIC FUNCTIONS "
220 LPRINT " POLYNOMIAL EXPRESSIONS FOR LOGIC FUNCTIONS "
230 LPRINT STRING$ (42, "-")
```

```
240 PRINT STRING$ (42, "-")
250 LPRINT : PRINT
260 '--------- TABLE FOR X,Y,Z----> F(X,Y,Z)
270 PRINT" X        " ; " Y          " ; " Z" ; " ----> " ; " F (X,Y,Z)"
280 LPRINT" X        " ; " Y          " ; " Z" ; " ----> " ; " F (X,Y,Z)"
290 FOR I=0 TO 2
300       FOR J=0 TO 2
310             FOR K=0 TO 2
320                   PRINT I; "      " ;J; "      " ;K; "----> " ;F (I,J,K)
330                   LPRINT I; "     " ;J; "      " ;K; "----> " ;F (I,J,K)
340             NEXT K
350       NEXT J
360 NEXT I
370 PRINT STRING$ (79, "-")
380 LPRINT STRING$ (79, "-")
390 '--------- DEFINE XF (X,0,0)
400 DEF FNX1 ( X ) =FNM (X, F (X, 0 , 0) )
410 '--------- DEFINE YF ( 0,Y,0 )
420 DEF FNY1 ( Y ) =FNM (Y, F (0 , Y, 0) )
430 '-------- INITIALIZE
440 FOR L=1 TO 26
450       B ( L ) = 0
460 NEXT L
470 '--------- DEFINE ZF (0,0,Z)
480 DEF FNZ1 ( Z ) =FNM ( Z, F (0 , 0 , Z ) )
490 FOR I=0 TO 2
500       B (1) =FNA ( B (1), FNX1 ( I ) )                    'A (1 , 0 , 0)
510       B (2) =FNA ( B (2), F ( I, 0 , 0) )                 'A (2 , 0 , 0)
520       B (3) =FNA ( B (3), FNY1 ( I ) )                    'A (0 , 1 , 0)
530       B (4) =FNA ( B (4), F (0 ,I, 0) )                   'A (0 , 2 , 0)
540       B (5) =FNA ( B (5), FNZ1 ( I ) )                    'A (0 , 0 , 1)
550       B (6) =FNA ( B (6), F (0 , 0 ,I ) )                 'A (0 , 0 , 2)
560 NEXT I
570 '--------- DEFINE XYF (X,Y,0 ),YF (X,Y,0 ),XF (X,Y,0 )
580 DEF FNX2 (X , Y) =FNM (X, FNM (Y, F (X , Y , 0) ) )
590 DEF FNX3 (X , Y) =FNM (Y, F (X , Y , 0) )
600 DEF FNX4 (X , Y) =FNM (X, F (X , Y , 0) )
610 FOR I=0 TO 2
620       FOR J=0 TO 2
630             B (7) =FNA (B (7), FNX2 (I , J) )             'A (1 , 1 , 0)
640             B (8) =FNA (B (8), FNX3 (I , J) )             'A (2 , 1 , 0)
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```
650             B(9) =FNA (B(9),FNX4 (I.J))                    'A(1,2,0)
660             B(10) =FNA (B(10),F(I,J,0))                    'A(2,2,0)
670       NEXT J
680  NEXT I
690  '--------- DEFINE XZF (X,0,Z), ZF (X,0,Z), XF (X,0,Z)
700  DEF FNZ1 (X,Z) =FNM (X,FNM (Z,F(X,0,Z)))
710  DEF FNZ2 (X,Z) =FNM (Z,F(X,0,Z))
720  DEF FNZ3 (X,Z) =FNM (X,F(X,0,Z))
730  FOR I=0 TO 2
740       FOR J=0 TO 2
760             B(11) =FNA (B(11),FNZ1 (I,J))                  'A(1,0,1)
770             B(12) =FNA (B(12),FNZ2 (I,J))                  'A(2,0,1)
780             B(13) =FNA (B(13),FNZ3 (I,J))                  'A(1,0,2)
790             B(14) =FNA (B(14),F(I,0,J))                    'A(2,0,2)
810       NEXT J
820  NEXT I
830  '--------- DEFINE YZF(0,Y,Z),ZF(0,Y,Z),YF(0,Y,Z)
840  DEF FNY1 (Y,Z) =FNM (Y,FNM (Z,F(0,Y,Z)))
850  DEF FNY2 (Y,Z) =FNM (Z,F(0,Y,Z))
860  DEF FNY3 (Y,Z) =FNM (Y,F(0,Y,Z))
870  FOR I=0 TO 2
880       FOR J=0 TO 2
890             B(15) =FNA (B(15),FNY1 (I,J))                  'A(0,1,1)
900             B(16) =FNA (B(16).FNY2 (I,J))                  'A(0,2,1)
910             B(17) =FNA (B(17),FNY3 (I.J))                  'A(0,1.2)
920             B(18) =FNA (B(18),F(0,I.J))                    'A(0,2.2)
930       NEXT J
940  NEXT I
950  '--------- DEFINE XYZF(X,Y,Z), YZF(X,Y,Z), XZF(X,Y,Z), XYF
     (X,Y,Z),ZF(X,Y,Z),
960  '--------- YF(X,Y,Z),XF(X,Y,Z)
970  DEF FNZ1 (X,Y,Z) =FNM (X,FNM (Y,FNM (Z,F(X,Y,Z))))
980  DEF FNZ2 (X,Y,Z) =FNM (Y,FNM (Z,F(X,Y,Z)))
990  DEF FNZ3 (X,Y,Z) =FNM (X,FNM (Z,F(X,Y,Z)))
1000 DEF FNZ4 (X,Y,Z) =FNM (X,FNM(Y,F(X,Y,Z)))
1010 DEF FNZ5 (X,Y,Z) =FNM (Z,F(X,Y,Z))
1020 DEF FNZ6 (X,Y,Z) =FNM (Y,F(X,Y,Z))
1030 DEF FNZ7 (X,Y,Z) =FNM (X,F(X,Y,Z))
1040 FOR I=0 TO 2
1050      FOR J=0 TO 2
1060            FOR K=0 TO 2
```

```
1070                    B (19) =FNA (B(19),FNZ 1 (I,J,K))          'A(1,1,1)
1080                    B (20) =FNA (B(20),FNZ 2 (I,J,K))          'A(2,1,1)
1090                    B (21) =FNA (B(21),FNZ 3 (I,J,K))          'A(1,2,1)
1100                    B (22) =FNA (B(22),FNZ 4 (I,J,K))          'A(1,1,2)
1110                    B (23) =FNA (B(23),FNZ 5 (I,J,K))          'A(2,2,1)
1120                    B (24) =FNA (B(24),FNZ 6 (I,J,K))          'A(2,1,2)
1130                    B (25) =FNA (B(25),FNZ 7 (I,J,K))          'A(1,2,2)
1140                    B (26) =FNA (B(26),F(I,J,K))               'A(2,2,2)
1150              NEXT K
1160        NEXT J
1170 NEXT I
1180 '--------- OUTPUT THE FORMULA FOR F (X,Y,Z)
1190 LPRINT"  F (X , Y , Z) = " ; F ( 0 , 0 , 0 ) ; " - " ; B ( 1 ) ; " X- " ; B ( 3 ) ;
     " Y- " ; B ( 5 ) ; " Z- " ; B ( 2 ) ; " XX- " ; B ( 4 ) ; " YY- " ; B ( 6 ) ; " ZZ+ "
     ; B ( 7 ) ; " XY+ " ; B (11) ; " XZ+ "
1200 PRINT"  F (X , Y , Z) = " ; F ( 0 , 0 , 0 ) ; " - " ; B ( 1 ) ; " X- " ; B ( 3 ) ;
     " Y- " ; B ( 5 ) ; " Z- " ; B ( 2 ) ; " XX- " ; B ( 4 ) ; " YY- " ; B ( 6 ) ; " ZZ+
     " ; B ( 7 ) ; " XY+ " ; B (11) ; " XZ+ "
1210 LPRINT  B (15) ; " YZ- " ; B (19) ; " XYZ+ " ; B ( 8 ) ; " XXY + " ; B (12)
     ; " XXZ+ " ;  B ( 9 ) ; " XYY + " ; B (16) ; " YYZ+ " ; B (13) ; " XZZ + " ; B
     ( 7 ) ; " YZZ+ "
1220 PRINT  B (15) ; " YZ- " ; B (19) ; " XYZ+ " ; B ( 8 ) ; " XXY + " ; B (12) ;
     " XXZ+ " ; B ( 9 ) ; " XYY + " ; B (16) ; " YYZ+ " ; B (13) ; " XZZ+ " ; B ( 7 )
     " YZZ+ "
1230 LPRINT  B (10) ; " XXYY + " ; B (14) ; " XXZZ + " ; B (18) ; " YYZZ - " ;
     B (20) ; " XXYZ - " ; B (21) ; " XYYZ - " ; B (22) ; " XYZZ - " ; B (23) ; " X
     XYYZ- " ; B (24) ; " XXYZZ- "
1240 LPRINT  B (25) ; " XYYZZ- " ; B (26) ; " XXYYZZ. "
1250 PRINT  B (10) ; " XXYY + " ; B (14) ; " XXZZ + " ; B (18) ; " YYZZ - " ; B
     (20) ; " XXYZ - " ; B (21) ; " XYYZ - " ; B (22) ; " XYZZ - " ; B (23) ; " XX
     YYZ- " ; B (24) ; " XXYZZ- "
1260 PRINT  B (25) ; "XYYZZ- " ; B (26) ; "XXYYZZ. "
1270 PRINT  STRING $ (79, " - ")
1280 LPRINT  STRING $ (79, " - ")
1290 PRINT : PRINT
1300 '-------- CONTINUE ?
1310 PRINT"  CONT=RETURN , STOP= 0 " ; : A $ =INPUT $ ( 1 )
1320 IF  A $ <> CHR $ (13)  AND  A $ <> " 0 "  THEN 1310
1330 IF  A $ =CHR $ (13)  THEN 60
1340 END
```

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