Finite Strain Analysis on Geology

Bull. Coll. Sci., Univ. Ryukyus, No.54 : 123－148 (1992)

Finite Strain Analysis on Geology

Daigoro HAYASHI*

*Department of Marine Sciences, University of the Ryukyus, Okinawa 903-01, Japan

Abstract

Basic concept of deformation and strain is described. Strain ellipse and reciprocal strain ellipse are then introduced as the interface between deformation and strain. Mohr's circle is explained as the magnificent tool to obtain axial ratio and direction of axis of strain using graph instead of calculation. Practical techniques in field are described to measure axial ratio and direction of axis of strain. Practical techniques are the methods that use the strain grid, glass shard and fossils. The famous R_f/ϕ technique is explained that the method is invented from applying the theory of superposition of homogeneous strain.

Introduction

There are two variations in finite deformation theory, one is called general finite deformation theory, the other is called homogeneous finite deformation theory. They are also called non-affine deformation theory and affine deformation theory, respectively.

The homogeneous finite deformation is, in a word, the deformation in which a line is transformed into a line and a plane is transformed into a plane. Other deformations except homogeneous finite deformation belong to general finite deformation.

Deformation that occurs in nature is inhomogeneous finite deformation. Any geological large deformation thus should be treated with the general finite deformation theory. The general theory is, however, too complex to handle the geological strain in practice. I treat therefore of the geological large deformation as homogeneous deformation. Thus, the deformation described in the paper is the homogeneous finite deformation. Theory of 3-dimensional deformation is not described here.

I consider the concept of homogeneous deformation and the definition of technical terms of deformation, and then show how to construct the concept of strain. Several methods are introduced to get strain in field, for example, strain grid and glass shard and so forth.

All the methods explained here are different ones than those described by Ramsay (1967) and Ramsay and Huber (1983) in terms of using strict definition of the sign of angles. The paper describes clearly how to measure angles clockwise or anticlockwise, and from which direction to which direction. The famous R_f/ϕ technique is considered as an applied examples of the theory of superposed finite strain.

I wish to express my thanks to J.C.Jaeger, J.G.Ramsay, M.I.Huber and W.D.Means, because I write the paper by referring their works (Jaeger,1956;Ramsay,1967;Means,1976; Ramsay and Huber,1983).

Special Finite Deformation

Dilatation, contraction, pure extension, pure shear and simple shear are well known as

Accepted : August 4, 1992

A part of this research was presented at the 99th Annual Meeting of the Geological Society of Japan in Kumamoto, April 5, 1992

special deformation.

(1) *Dilatation* is the deformation where an object extends to two directions at the same rate. Representation is,

$$\begin{cases} x_1 = (1+e)\,x \\ y_1 = (1+e)\,y \end{cases} \qquad \text{-----(1.1)}$$

where a point $P(x, y)$ is deformed to $P_1(x_1, y_1)$ and e is a positive constant indicating extension ratio.

(2) *Contraction* is the deformation where an object contracts to two directions at the same rate. Representation is same as that of dilatation, but e is negative.

(3) *Pure extension* is the deformation where an object stretches to one direction but doesn't change in other direction. Its representation is,

$$\begin{cases} x_1 = (1+e)\,x \\ y_1 = y \end{cases} \qquad \text{-----(1.2)}$$

(4) *Pure shear* is the deformation where an object extends to one direction and contracts to the other direction.

$$\begin{cases} x_1 = (1+e_x)\,x \\ y_1 = (1+e_y)\,y \end{cases} \qquad \text{-----(1.3)}$$

where e_x and e_y are the constants which indicate the extension ratio for x and y directions respectively. $(1+e_x)(1+e_y)=1$ is the condition of constant volume.

(5) *Simple shear* is the deformation without volume change and is realized by the next transformation.

$$\begin{cases} x_1 = x + y\tan\phi_x \\ y_1 = y \end{cases} \qquad \text{-----(1.4)}$$

This shows the simple shear parallel to x axis but if it is a simple shear parallel to y direction, its formula is

$$\begin{cases} x_1 = x \\ y_1 = y + x\tan\phi_y \end{cases} \qquad \text{-----(1.5)}$$

Furthermore, when these deformations are superposed each other, many deformations are produced as the cases of general deformation. They are called as *superposed deformation.*

(6) *Superposed deformation* S_1+S_2

After suffering simple shear S_1 parallel to x axis, suffering the other simple shear S_2 parallel to y axis, representation of the deformation is,

$$S_1 \begin{cases} x_1 = x + y\tan\phi_x \\ y_1 = y \end{cases} \qquad \text{-----(1.6)}$$

$$S_1+S_2 \begin{cases} x_2 = x_1 \\ y_2 = y_1 + x_1\tan\phi_y \end{cases} \qquad \text{-----(1.7)}$$

The deformation S_1+S_2 is thus written by original coordinates x and y.

$$S_1+S_2 \begin{cases} x_2 = y\ tan\phi_x + x \\ y_2 = y\ (1 + tan\phi_x\ tan\phi_y) + x\ tan\phi_y \end{cases} \quad \text{-----(1.8)}$$

(7) *Superposed deformation S_2+S_1*

When the order of deformation is changed,

$$S_2 \begin{cases} x_1 = x \\ y_1 = y + x\ tan\phi_y \end{cases} \quad \text{-----(1.9)}$$

$$S_2+S_1 \begin{cases} x_2 = x_1 + y_1\ tan\phi_x \\ y_2 = y_1 \end{cases} \quad \text{-----(1.10)}$$

S_2+S_1 is represented by the original coordinates x and y.

$$S_2+S_1 \begin{cases} x_2 = x\ (1 + tan\phi_x\ tan\phi_y) + y\ tan\phi_x \\ y_2 = x\ tan\phi_y + y \end{cases} \quad \text{-----(1.11)}$$

(8) *Superposed deformation $S+P$*

Superposing pure shear P to simple shear S parallel to x axis,

$$S \begin{cases} x_1 = x + y\ tan\phi \\ y_1 = y \end{cases} \quad \text{-----(1.12)}$$

$$S+P \begin{cases} x_2 = (1 + e_x)\ x_1 \\ y_2 = (1 + e_y)\ y_1 \end{cases} \quad \text{-----(1.13)}$$

Representing $S+P$ by the original coordinates x and y,

$$S+P \begin{cases} x_2 = (1 + e_x)\ (x + y\ tan\phi) \\ y_2 = (1 + e_y)\ y \end{cases} \quad \text{-----(1.14)}$$

(9) *Superposed deformation $P+S$*

Exchanging the order of deformation above,

$$P \begin{cases} x_1 = (1 + e_x)\ x \\ y_1 = (1 + e_y)\ y \end{cases} \quad \text{-----(1.15)}$$

$$P+S \begin{cases} x_2 = x_1 + y_1\ tan\phi \\ y_2 = y_1 \end{cases} \quad \text{-----(1.16)}$$

Showing $P+S$ by the original coordinates x and y,

$$P+S \begin{cases} x_2 = (1 + e_x)\ x + (1 + e_y)\ y\ tan\phi \\ y_2 = (1 + e_y)\ y \end{cases} \quad \text{-----(1.17)}$$

It should be noted that the final representation of deformation is different due to the order of superposition.

For example, superposed deformation S_1+S_2 is different to S_2+S_1 shown by (1.8) and (1.11). The rule is right not only for the same kind of superposition of deformation, but for the different kind of superposition. Say pure shear and simple shear, the change of order of deformation affects to the final representation formula. This is clear from the examples of superposition $S+P$ and $P+S$.

General Finite Deformation

General deformation means the deformation that is realized by the affine transformation. The representation is,

$$\begin{cases} \bar{x} = a\,x + b\,y \\ \bar{y} = c\,x + d\,y \end{cases} \qquad \text{-----(2.1)}$$

where quantities after deformation are described with bar above their symbol. Transform matrix of (2.1) is defined as

$$\begin{bmatrix} a & b \\ c & d \end{bmatrix}$$

Reciprocal representation is,

$$\begin{cases} x = \dfrac{d\,\bar{x} - b\,\bar{y}}{ad - bc} \\ y = \dfrac{-c\,\bar{x} + a\,\bar{y}}{ad - bc} \end{cases} \qquad \text{-----(2.2)}$$

Deformation of a line

Transforming a line $y = mx + k$ by (2.1), we have

$$\bar{y} = \frac{c + dm}{a + bm}\,\bar{x} + \frac{ad - bc}{a + bm}\,k \qquad \text{-----(2.3)}$$

Deformation of a circle

Transforming a unit circle $x^2 + y^2 = 1$ by (2.1), we have

$$(c^2 + d^2)\,\bar{x}^2 - 2(ac + bd)\,\bar{x}\,\bar{y} + (a^2 + b^2)\,\bar{y}^2 = (ad - bc)^2 \qquad \text{-----(2.4)}$$

This is called as *strain ellipse.*

Deformation of an ellipse

Transforming an ellipse $lx^2 - 2mxy + ny^2 = 1$ by (2.1), it becomes

$$p\bar{x}^2 - 2q\bar{x}\,\bar{y} + r\bar{y}^2 = 1 \qquad \text{-----(2.5)}$$

where

$$p = \frac{ld^2 + 2mcd + nc^2}{(ad - bc)^2}$$

$$q = \frac{m\,(ad + bc) + lbd + nac}{(ad - bc)^2} \qquad \text{-----(2.6)}$$

$$r = \frac{lb^2 + 2mab + na^2}{(ad - bc)^2}$$

If this ellipse becomes a unit circle, the original ellipse is called as *reciprocal strain ellipse.* The condition for the case is $p = r = 1$ and $q = 0$. Then the reciprocal strain ellipse is written as

$$(a^2 + c^2)\,x^2 + 2(ab + cd)\,xy + (b^2 + d^2)\,y^2 = 1 \qquad \text{-----(2.7)}$$

Principal axis of strain

A principal axis of strain means the direction where the original perpendicular

direction keeps its perpendicularity. The line that is normal to $y=mx$ is $y=-\frac{x}{m}$.
Since the gradient of $y=mx$ after deformation has already given as $\frac{md+c}{mb+a}$ by (2.3), the gradient of $y=-\frac{x}{m}$ after deformation is obtained by replacing $-\frac{1}{m}$ to m in the former formula. The result is $\frac{mc-d}{ma-b}$. If these two lines are perpendicular each other, we have

$$\frac{md+c}{mb+a}\ \frac{mc-d}{ma-b}=-1 \qquad \text{-----(2.8)}$$

Then we get

$$m^2+\frac{a^2-b^2+c^2-d^2}{ab+cd}\,m-1=0 \qquad \text{-----(2.9)}$$

where m should have two real solutions. The directions of m and $-\frac{1}{m}$ are called as *principal axes of strain before deformation*, while the directions of $\frac{md+c}{mb+a}$ and $\frac{mc-d}{ma-b}$ are called as *principal axes of strain after deformation.*

We can now obtain the principal direction of strain from m that is calculated from (2.9).

It is clear, omitting its proof here, that the principal axis of strain before deformation coincides with the principal axis of reciprocal strain ellipse and that of after deformation corresponds to the principal axis of strain ellipse. We thus call the ellipse, which has to be called as deformation ellipse because it was deformed from a unit circle, as a strain ellipse.

Axial length of strain ellipse

In order to seek axial length of strain ellipse, let consider the intersection of a strain ellipse and a circle.

$$\begin{cases}(c^2+d^2)\bar{x}^2-2(ac+bd)\bar{x}\,\bar{y}+(a^2+b^2)\bar{y}^2=(ad-bc)^2\\ \bar{x}^2+\bar{y}^2=r^2\end{cases} \qquad \text{-----(2.10)}$$

Eliminating constant terms in the simultaneous equation (2.10),
we have

$$\left(\frac{c^2+d^2}{h^4}-\frac{1}{r^2}\right)\bar{x}^2-2\frac{ac+bd}{h^4}\,\bar{x}\,\bar{y}+\frac{a^2+b^2}{h^4}\bar{y}^2=0 \qquad \text{-----(2.11)}$$

where

$$h^2=ad-bc. \qquad \text{-----(2.12)}$$

If the strain ellipse and the circle contact at two points each other, the quadratic curve (2.11) denotes a line. Its condition is that the equation (2.11) is a perfect square. If this is the case, r becomes axial length of the strain ellipse. The formula D with which we judge the equation to be perfect square or not is

$$D= \left(\frac{ac+bd}{h^4}\right)^2 - \left(\frac{c^2+d^2}{h^4} - \frac{1}{r^2}\right)\left(\frac{a^2+b^2}{h^4} - \frac{1}{r^2}\right) = 0 \qquad \text{-----(2.13)}$$

Rearranging the equation with regard to r, we have

$$r^4 - (a^2+b^2+c^2+d^2)\,r^2 + h^4 = 0 \qquad \text{-----(2.14)}$$

Supposing the solution of r^2 to be X^2 and Y^2 $(X \geqq Y)$, we have next equations from the relation of root and coefficient.

$$X^2 + Y^2 = a^2+b^2+c^2+d^2 \qquad \text{-----(2.15)}$$

$$X^2 Y^2 = h^4 \qquad \text{-----(2.16)}$$

Then we have $XY = h^2$ from (2.16). It should be noted that h^2 means the ratio of dilatation. From this relation and (2.15), we get

$$\begin{cases} (X+Y)^2 = (a+d)^2 + (b-c)^2 \\ (X-Y)^2 = (a-d)^2 + (b+c)^2 \end{cases} \qquad \text{-----(2.17)}$$

X and Y are given by solving simultaneously these equations.

Transform Matrix Realizing Irrotational Strain is Symmetric

As described in the section "Principal axis of strain", the principal axis before deformation doesn't generally coincide with that of after deformation. Strain is generally rotational. They however coincide together in a special case. This case is called as the state of irrotational strain. The condition for the case is

$$\begin{cases} \dfrac{md+c}{mb+a} = m \\ \dfrac{mc-d}{ma-b} = -\dfrac{1}{m} \end{cases} \qquad \text{-----(2.18)}$$

Rewriting them, we have

$$\begin{cases} bm^2 + (a-d)\,m - c = 0 \\ cm^2 + (a-d)\,m - b = 0 \end{cases} \qquad \text{-----(2.19)}$$

In order to hold these two equations simultaneously, the condition $b=c$ is necessary and enough.

Definition of Finite Strain

There are two kinds of finite strain. One is called as longitudinal strain which corresponds to change of length, the other is named shear strain which corresponds to change of angle.

The longitudinal strain has three variations. When length l_0 changes to l_1, they are defined as follows.

(1) Extension $e = \dfrac{\ell_1 - \ell_0}{\ell_0} = \dfrac{d\ell}{d\ell_0}$

(2) Quadratic elongation $\lambda = \left(\dfrac{\ell_1}{\ell_0}\right)^2 = (1+e)^2$

(3) Natural strain $\varepsilon = \displaystyle\int \frac{1}{\ell}\, d\ell = \log\frac{\ell_1}{\ell_0} = \log(1+e)$

which is also called as logarithmic strain or true strain.

The quadratic elongation is conveniently used to describe large homogeneous finite strain.

On the other hand, shear strain is defined by $\gamma = \tan\phi$ where if right angle is deformed to lose its perpendicularity, the deviation from the right angle is called as angle of shear ϕ.

Mathematical Representation of Finite Strain

Let consider a unit circle $x^2+y^2=1$ which is deformed to an ellipse. If we considert he irrotational deformation, generality doesn't lose, thus we can use Fig.1.

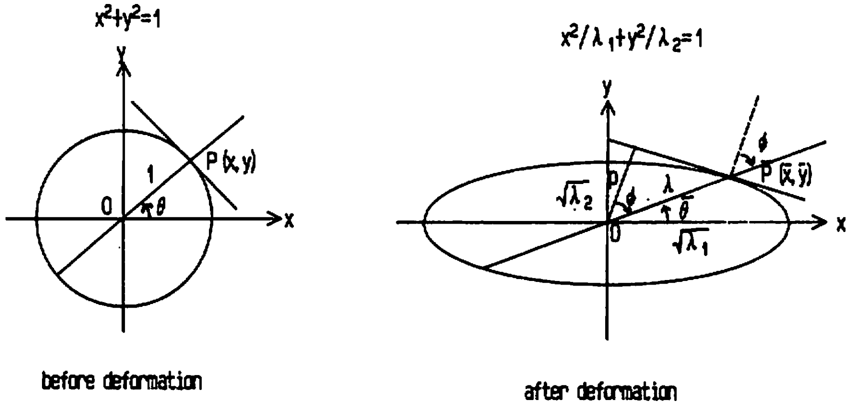

Fig. 1. Deformation from a unit circle to a strain ellipse.

A point P is moved to $\bar{P}$ in Fig.1, then with respect to the point $\bar{P}$ we have

$$\lambda = x^2 + y^2 \qquad \text{-----(4.1)}$$

and also

$$\begin{cases} \bar{x} = x\sqrt{\lambda_1} = \cos\theta\sqrt{\lambda_1} \\ \bar{y} = y\sqrt{\lambda_2} = \sin\theta\sqrt{\lambda_2} \end{cases} \qquad \text{-----(4.2)}$$

where λ_1 and λ_2 are principal quadratic elongations $(\lambda_1 \geqq \lambda_2)$.

$$\lambda = \lambda_1\cos^2\theta + \lambda_2\sin^2\theta \qquad \text{-----(4.3)}$$

This represents λ by the angle before deformation θ. It is necessary to indicate λ by the angle after deformation $\bar{\theta}$. Modifying (4.2) to

$$\begin{cases} \cos\theta = \dfrac{\bar{x}}{\sqrt{\lambda_1}} = \dfrac{\sqrt{\lambda}\cos\bar{\theta}}{\sqrt{\lambda_1}} \\ \sin\theta = \dfrac{\bar{y}}{\sqrt{\lambda_2}} = \dfrac{\sqrt{\lambda}\sin\bar{\theta}}{\sqrt{\lambda_2}} \end{cases} \qquad \text{-----(4.4)}$$

Substituting (4.4) to $\sin^2\theta + \cos^2\theta = 1$, we have

$$\begin{cases} \dfrac{1}{\lambda} = \dfrac{\cos^2\overline{\theta}}{\lambda_1} + \dfrac{\sin^2\overline{\theta}}{\lambda_2} \\ \lambda' = \lambda'_1\cos^2\overline{\theta} + \lambda'_2\sin^2\overline{\theta} \end{cases} \qquad \text{-----(4.5)}$$

where λ', λ'_1 and λ'_2 are called as reciprocal quadratic elongation and principal reciprocal quadratic elongation that are defined as $\lambda' = \dfrac{1}{\lambda}$, $\lambda'_1 = \dfrac{1}{\lambda_1}$ and $\lambda'_2 = \dfrac{1}{\lambda_2}$.

Next, we will write shear strain in mathematical form. The contact line at the point $\overline{P}$ is

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1 \qquad \text{-----(4.6)}$$

The equation of the contact line at the point $\overline{P}(\overline{x}, \overline{y})$ is

$$\frac{x\overline{x}}{a^2} + \frac{y\overline{y}}{b^2} = 1 \qquad \text{-----(4.7)}$$

It is written as

$$\frac{x\overline{x}}{\lambda_1} + \frac{y\overline{y}}{\lambda_2} = 1 \qquad \text{-----(4.8)}$$

Substituting (4.2) to (4.8), we have

$$\frac{x\cos\theta}{\sqrt{\lambda_1}} + \frac{y\sin\theta}{\sqrt{\lambda_2}} = 1 \qquad \text{-----(4.9)}$$

Thus the length of p is

$$p = \sqrt{\frac{1}{\dfrac{\cos^2\theta}{\lambda_1} + \dfrac{\sin^2\theta}{\lambda_2}}} \qquad \text{-----(4.10)}$$

From Fig.1 we have

$$\sec\phi = \frac{\sqrt{\lambda}}{p} \qquad \text{-----(4.11)}$$

Substituting (4.11) to (4.10), we have

$$\sec\phi = \sqrt{\lambda\left(\frac{\cos^2\theta}{\lambda_1} + \frac{\sin^2\theta}{\lambda_2}\right)} \qquad \text{-----(4.12)}$$

While from the definition of γ

$$\gamma^2 = \tan^2\phi = \sec^2\phi - 1 \qquad \text{-----(4.13)}$$

Substituting (4.13) to (4.12)

$$\gamma^2 = \lambda\left(\frac{\cos^2\theta}{\lambda_1} + \frac{\sin^2\theta}{\lambda_2}\right) - 1 \qquad \text{-----(4.14)}$$

Substituting (4.3) and $(\cos^2\theta + \sin^2\theta)^2 = 1$ to (4.14), we get

$$h^2\gamma = (\lambda_1 - \lambda_2)\cos\theta \ \sin\theta \qquad \text{-----(4.15)}$$

where

$$h^2=\sqrt{\lambda_1 \lambda_2} \qquad \text{-----(4.16)}$$

This is the representation of shear strain γ with respect to the angle before deformation θ.

In order to describe γ by the angle after deformation $\bar{\theta}$, substituting (4.4) to (4.15), we have

$$\frac{\gamma}{\lambda}=\left(\frac{1}{\lambda_2}-\frac{1}{\lambda_1}\right) \sin\bar{\theta} \cos\bar{\theta} \qquad \text{-----(4.17)}$$

This is written as

$$\gamma' = (\lambda'_2-\lambda'_1)\sin\bar{\theta} \cos\bar{\theta} \qquad \text{-----(4.18)}$$

where

$$\gamma' = \frac{\gamma}{\lambda} \qquad \text{-----(4.19)}$$

Representation of Finite Strain by Means of Mohr's Circle

Representing λ' and γ' with respect to the angle after deformation $\bar{\theta}$, we get next equations from (4.5) and (4.18)

$$\begin{cases} \lambda' = \lambda'_1 \cos^2\bar{\theta} + \lambda'_2 \sin^2\bar{\theta} \\ \gamma' = (\lambda'_2-\lambda'_1)\sin\bar{\theta} \cos\bar{\theta} \end{cases} \qquad \text{-----(5.1)}$$

Considering next two relations,

$$\begin{cases} \cos^2\theta = \dfrac{1+\cos 2\theta}{2} \\ \sin^2\theta = \dfrac{1-\cos 2\theta}{2} \end{cases} \qquad \text{-----(5.2)}$$

,the equation for λ' is written as

$$\lambda' = \frac{\lambda'_1+\lambda'_2}{2} + \frac{\lambda'_1-\lambda'_2}{2}\cos 2\bar{\theta} \qquad \text{-----(5.3)}$$

This is further changed to be

$$\lambda' = \frac{\lambda'_1+\lambda'_2}{2} = \frac{\lambda'_1-\lambda'_2}{2}\cos 2\bar{\theta} \qquad \text{-----(5.4)}$$

While, we have the equation for γ' as

$$\gamma' = \frac{\lambda'_2-\lambda'_1}{2}\sin 2\bar{\theta} \qquad \text{-----(5.5)}$$

which is equal to (4.18). After squaring (5.4) and (5.5), adding them for each side

$$\left(\lambda' - \frac{\lambda'_1+\lambda'_2}{2}\right)^2 + \gamma'^2 = \left(\frac{\lambda'_1-\lambda'_2}{2}\right)^2. \qquad \text{-----(5.6)}$$

This realizes the circle shown in Fig.2. As the equations (5.4) and (5.5) are modified to

$$\begin{cases} \lambda' = \dfrac{\lambda'_1 + \lambda'_2}{2} + \dfrac{\lambda'_1 - \lambda'_2}{2} \cos(-2\bar{\theta}) \\ \gamma' = \dfrac{\lambda'_1 - \lambda'_2}{2} \sin(-2\bar{\theta}) \end{cases} \quad \text{-----}(5.7)$$

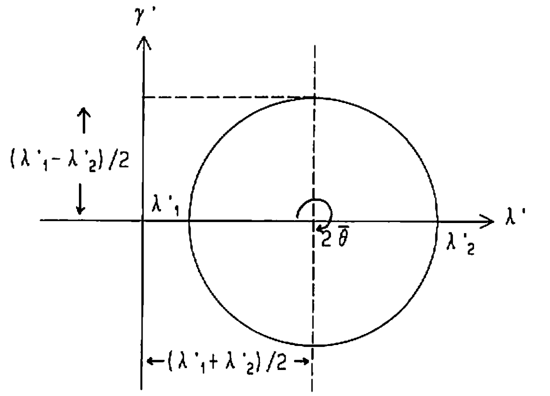

Fig. 2. Mohr's circle with which we get values of strain parameters for after deformation.

,the sense of $2\bar{\theta}$ is such that clockwise is positive. While, when λ and γ are written by the angle before deformation θ, from (4.3) and (4.15) we obtain

$$\begin{cases} \lambda = \lambda_1 \cos^2\theta + \lambda_2 \sin^2\theta \\ h^2\gamma = (\lambda_1 - \lambda_2)\cos\theta \ \sin\theta \end{cases} \quad \text{-----}(5.8)$$

Modifying them similarly as former case

$$\begin{cases} \lambda - \dfrac{\lambda_1 + \lambda_2}{2} = \dfrac{\lambda_1 - \lambda_2}{2} \cos 2\theta \\ h^2\gamma = \dfrac{\lambda_1 - \lambda_2}{2} \sin 2\theta \end{cases} \quad \text{-----}(5.9)$$

Squaring them and adding them for each side, we have

$$\left(\lambda - \frac{\lambda_1 + \lambda_2}{2}\right)^2 + \left(\frac{\gamma}{h^{-2}}\right)^2 = \left(\frac{\lambda_1 - \lambda_2}{2}\right)^2 \quad \text{-----}(5.10)$$

The quadratic curve (5.10) shows the ellipse illustrated in Fig.3. The angle 2θ is here measured such that anticlockwise is positive. If ordinate is calibrated as $h^2\gamma$, the ellipse is drawn as a circle. When ordinate is calibrated as γ, we have an ellipse, where length of axis is given from $\gamma_{max} = \dfrac{R}{h^2}$ $(h^2\gamma_{max} = R)$, where $R = \dfrac{\lambda_1 - \lambda_2}{2}$ is the radius of a circle.

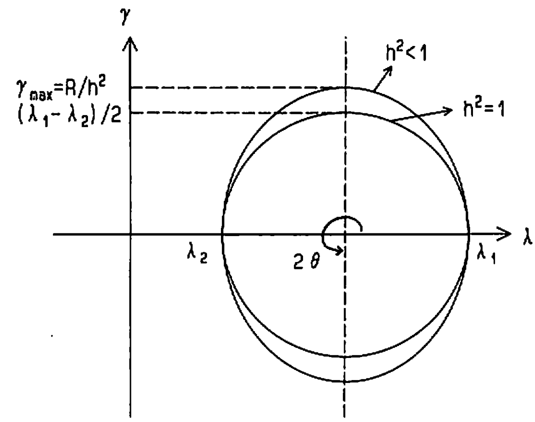

Fig. 3, Mohr's circle with which we get values of strain parameters for before deformation.

Relation between Strain Ellipse and Mohr's Circle

We will consider the case that a circle is deformed to an ellipse where $\lambda_1=1.44$ and $\lambda_2=0.49$ as shown in Fig.4.

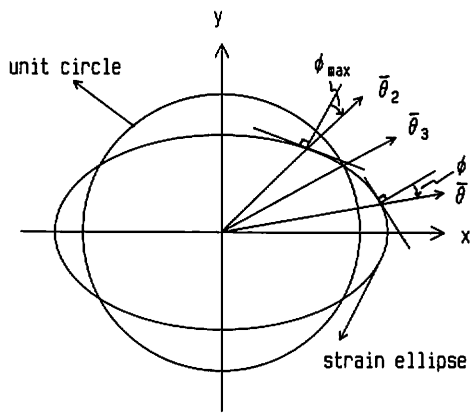

Fig. 4, Strain parameters are illustrated on a strain ellipse where $\lambda_1=1.44$ and $\lambda_2=0.49$.

(A) Quantities after deformation, which are obtained from the Mohr's circle.

$$\lambda' = \frac{\lambda'_1+\lambda'_2}{2} + \frac{\lambda'_1-\lambda'_2}{2}\cos 2\bar{\theta} \qquad \text{-----(5.3)}$$

$$\gamma' = \frac{\lambda'_2-\lambda'_1}{2}\sin 2\bar{\theta} \qquad \text{-----(5.5)}$$

(1) Considering the definition $\gamma = \frac{\gamma'}{\lambda'}$, then if $\lambda'=1$, we have $\gamma'=\gamma$. The direction $\bar{\theta}_2$ to which γ is maximum after deformation is thus the direction of a contact point between the circle and a contact line. As we know $2\bar{\theta}_2=\pm 60.5°$ from Fig.5, we get $\bar{\theta}_2=\pm 30.25°$.

(2) As $\phi_{max}=29.5°$ from Fig.5, $\gamma_{max}=0.57$

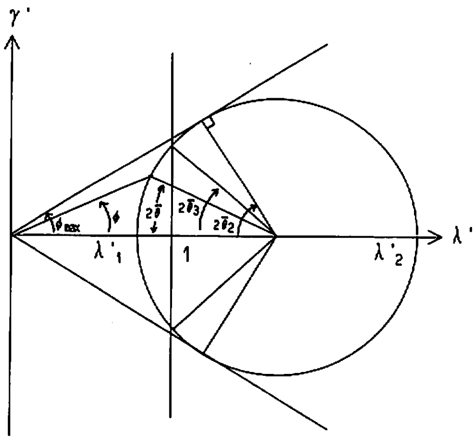

Fig. 5, Mohr's circle for after strain.

(3) The direction $\overline{\theta}_3$ to which no length changes is the direction of the contact point of the circle and the line

$\lambda'_1=1$, direction of which is same as that of $\lambda_1=1$. As $2\overline{\theta}_3=\pm 57°$ from Fig.5, $\overline{\theta}_3=\pm 28.5°$.

(4) Strain of any direction after deformation, say $\overline{\theta}=10°$, is obtained as follows. From Fig.5 we have $\lambda'=0.73$ and $\gamma'=0.23$, then we can get $\lambda=1.37$ and $\gamma=0.315$. Also we obtain the length of the present direction is $\sqrt{1.37}=1.17$ and $\phi=arctan$ $(0.315)=17.5°$.

(B) Quantities before deformation calculated from the Mohr's ellipse

$$\begin{cases} \lambda-\dfrac{\lambda_1+\lambda_2}{2} = \dfrac{\lambda_1-\lambda_2}{2}\cos 2\theta \\ h^2\gamma=\dfrac{\lambda_1-\lambda_2}{2}\sin 2\theta \end{cases} \qquad \text{-----(5.9)}$$

(1) The direction θ_2 to which γ is maximum before deformation. From Fig.6 we have $2\theta_2=\pm 90°$, that is, $\theta_2=\pm 45°$. It should be noted that the value is constant.

(2) We obtain $\phi_{max}=29.5°$ and $\gamma_{max}=0.565$ from Fig.6.

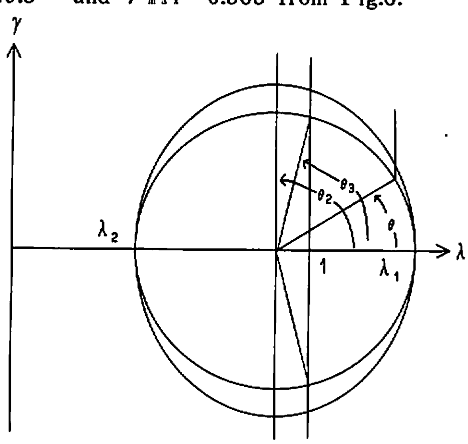

Fig. 6, Mohr's ellipse for before strain.

(3) The direction θ_3 to which no length changes is the direction of intersections of the circle and the line $\lambda_1=1$, that is, R_1 and R_2. We have $2\theta_3=\pm 86^\circ$ from Fig.6, that is, $\theta_3=\pm 43^\circ$.

(4) Strain of any direction before deformation is easily obtained. Say $\theta=16.8^\circ$, we have $\lambda=1.36$ and $\gamma=0.312$ from Fig.6. Thus the length of the direction is $\sqrt{1.36}=1.17$and $\phi=arctan(0.312)=17.3^\circ$. All the quantities obtained are shown on a physical space of Fig.4.

Superposed Finite Strain

Superposed homogeneous finite deformation becomes again homogeneous deformation. Superposition of deformation is usual in geological phenomena.

Axial ratio of initial marker ellipse is described R_i as shown in Fig.7.If the marker's long axis intersects with an angle θ to x axis as shown in Fig.8, and if we transform $\lambda'_{1i}x_1^2+\lambda'_{2i}y_1^2=1$ by rotation Ω, angle of which is θ, we have

$$\lambda'_{1i}(x_2\cos\theta+y_2\sin\theta)^2+\lambda'_{2i}(-x_2\sin\theta+y_2\cos\theta)^2=1 \qquad \text{---(6.1)}$$

where $\lambda'_{1i}$and $\lambda'_{2i}$are the principal reciprocal quadratic elongations $\left(R_i^2=\dfrac{\lambda'_{2i}}{\lambda'_{1i}}\right)$.

Expanding and rearranging (6.1) for x_2 and y_2

$$(\lambda'_{1i}\cos^2\theta+\lambda'_{2i}\sin^2\theta)x_2^2-2(\lambda'_{2i}-\lambda'_{1i})\sin\theta\cos\theta\, x_2y_2$$
$$+(\lambda'_{1i}\sin^2\theta+\lambda'_{2i}\cos^2\theta)y_2^2=1 \qquad \text{-----(6.2)}$$

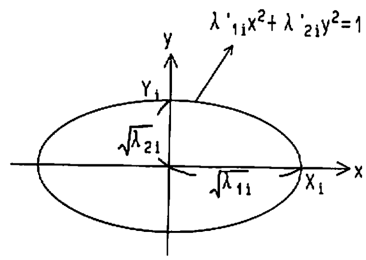

Fig. 7, Marker ellipse described by λ'_{1i} and λ'_{2i}

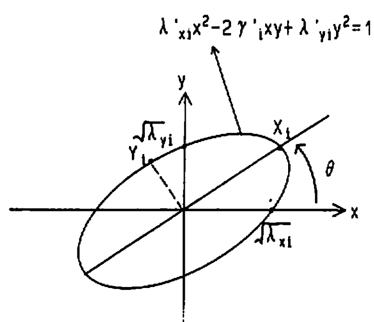

Fig. 8, Marker ellipse rotated its long axis to θ anticlockwise from x axis.

Replacing each coefficient as

$$\lambda'_{xi} = \lambda'_{1i}\ \cos^2\theta + \lambda'_{2i}\sin^2\theta \qquad \text{-----(6.3)}$$

$$\lambda'_{yi} = \lambda'_{1i}\ \sin^2\theta + \lambda'_{2i}\cos^2\theta \qquad \text{-----(6.4)}$$

$$\gamma'_i = (\lambda'_{2i} - \lambda'_{1i})\sin\theta\cos\theta \qquad \text{-----(6.5)}$$

we get

$$\lambda'_{xi}x_2^2 - 2\gamma'_i\ x_2 y_2 + \lambda'_{yi}y_2^2 = 1 \qquad \text{-----(6.6)}$$

This is the equation denoting the ellipse where the axial ratio is R_i, and long axis of which directs θ from x axis as shown in Fig.8.

When a circle, radius of which is r, and which encloses the ellipse of Fig.9, suffers irrotational finite deformation R_s as shown in Fig.10, the principal direction of the

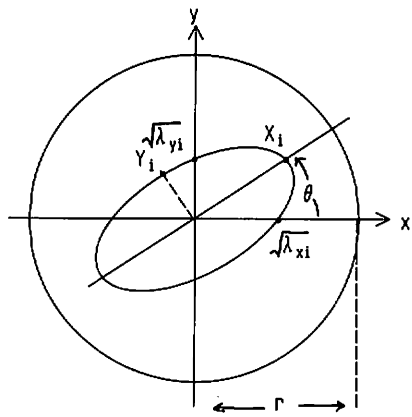

Fig. 9, A rotated marker ellipse and a circle of which the radius is r and which encloses the rotated ellipse.

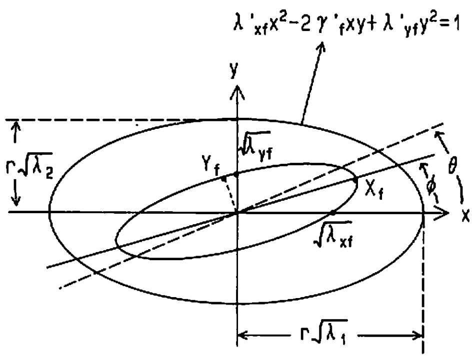

Fig. 10, Ellipses which are deformed from a rotated ellipse and a circle.

ellipse changes from θ to ϕ and the axial ratio is modified from R_i to R_f. The problem is how to represent ϕ and R_f by the known values R_i, R_s and θ.

We deform the ellipse (6.6) irrotationally by replacing x_2 by $\sqrt{\lambda'_{1s}}\ x_3$ and y_2 by $\sqrt{\lambda'_{2s}}\ y_3$ where $R_s^2 = \dfrac{\lambda_{1s}}{\lambda_{2s}} = \dfrac{\lambda'_{2s}}{\lambda'_{1s}}$. We have a deformed ellipse as follows.

$$\lambda'_{xi}\lambda'_{1s}x_3^2-2\gamma'_i\sqrt{\lambda'_{1s}\lambda'_{2s}}\;x_3y_3+\lambda'_{yi}\lambda'_{2s}y_3^2=1 \qquad \text{-----(6.7)}$$

Replacing each coefficient as

$$\lambda'_{xf}=\lambda'_{xi}\lambda'_{1s} \qquad \text{-----(6.8)}$$

$$\lambda'_{yf}=\lambda'_{yi}\lambda'_{2s} \qquad \text{-----(6.9)}$$

$$\gamma'_f=\gamma'_i\sqrt{\lambda'_{1s}\lambda'_{2s}} \qquad \text{-----(6.10)}$$

we have

$$\lambda'_{xf}x_3^2-2\gamma'_f\;x_3y_3+\lambda'_{yf}y_3^2=1 \qquad \text{-----(6.11)}$$

The equation (6.11) is that of the ellipse shown in Fig.10.
If we use the Mohr's circle of Fig.11, 2ϕ, λ'_{1f} and λ'_{2f} are graphically obtained, and R_f is given from $R_f^2=\dfrac{\lambda'_{2f}}{\lambda'_{1f}}$. We however try to get these values by calculation hereafter.

(1) Introducing 2ϕ

It is clear from Fig.11 that

$$tan2\phi=\frac{\gamma'_f}{\left(\dfrac{\lambda'_{yf}-\lambda'_{xf}}{2}\right)} \qquad \text{-----(6.12)}$$

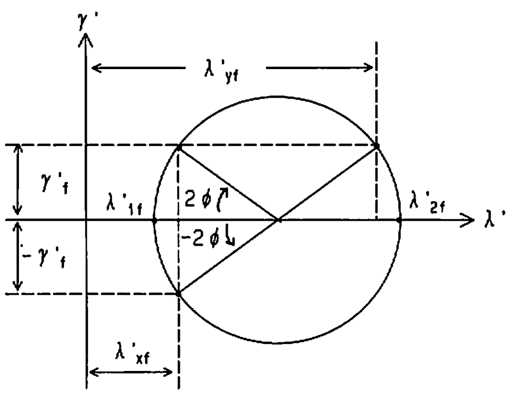

Fig. 11, Mohr's circle.

Considering the equations (6.3, 6.4, 6.5) and (6.8, 6.9, 6.10), and replacing each coefficient as

$$\lambda'_{xf}=(\lambda'_{1i}\;cos^2\theta+\lambda'_{2i}\;sin^2\theta)\,\lambda'_{1s} \qquad \text{-----(6.13)}$$

$$\lambda'_{yf}=(\lambda'_{1i}\;sin^2\theta+\lambda'_{2i}\;cos^2\theta)\,\lambda'_{2s} \qquad \text{-----(6.14)}$$

$$\gamma'_f=\sqrt{\lambda'_{1s}\;\lambda'_{2s}}\,(\lambda'_{2i}-\lambda'_{1i})\;sin\theta\;cos\theta \qquad \text{-----(6.15)}$$

we get

$$tan2\phi=\frac{2\sqrt{\lambda'_{1s}\;\lambda'_{2s}}\;\;(\lambda'_{2i}-\lambda'_{1i})\,sin\theta\;cos\theta}{(\lambda'_{1i}\;sin^2\theta+\lambda'_{2i}\;cos^2\theta)\,\lambda'_{2s}-(\lambda'_{1i}\;cos^2\theta+\lambda'_{2i}\;sin^2\theta)\,\lambda'_{1s}} \qquad \text{-----(6.16)}$$

Dividing numerator and denominator by λ'_{1i} and λ'_{1s}, and from the replacement $R_i^2=\dfrac{\lambda'_{2i}}{\lambda'_{1i}}$ and $R_s^2=\dfrac{\lambda'_{2s}}{\lambda'_{1s}}$, we have

$$tan2\phi = \frac{2R_s(R_i^2-1)\sin\theta\cos\theta}{(\sin^2\theta+R_i^2\cos^2\theta)R_s^2-(\cos^2\theta+R_i^2\sin^2\theta)} \quad \text{-----(6.17)}$$

Representing (6.17) by using 2θ

$$tan2\phi = \frac{2R_s(R_i^2-1)\sin 2\theta}{(R_i^2+1)(R_s^2-1)+(R_i^2-1)(R_s^2+1)\cos 2\theta} \quad \text{-----(6.18)}$$

(2) Introducing R_f

We know $\lambda' = \lambda'_1\cos^2\theta + \lambda'_2\sin^2\theta$ as written in (4.5). Then we have next relation for the ellipse (6.11).

$$\lambda'_{xf} = \lambda'_{1f}\cos^2\phi + \lambda'_{2f}\sin^2\phi \quad \text{-----(6.19)}$$

$$\lambda'_{yf} = \lambda'_{1f}\sin^2\phi + \lambda'_{2f}\cos^2\phi \quad \text{-----(6.20)}$$

Dividing (6.19) by (6.20), we get

$$\frac{\lambda'_{xf}}{\lambda'_{yf}} = \frac{\lambda'_{1f}\cos^2\phi + \lambda'_{2f}\sin^2\phi}{\lambda'_{1f}\sin^2\phi + \lambda'_{2f}\cos^2\phi} \quad \text{-----(6.21)}$$

where $R_f^2 = \frac{\lambda'_{2f}}{\lambda'_{1f}}$. Then, dividing denominator and numerator by λ'_{1f}, we have

$$\frac{\lambda'_{xf}}{\lambda'_{yf}} = \frac{\cos^2\phi + R_f^2\sin^2\phi}{\sin^2\phi + R_f^2\cos^2\phi} \quad \text{-----(6.22)}$$

Dividing a denominator and numerator by $\cos^2\phi$, we get

$$\frac{\lambda'_{xf}}{\lambda'_{yf}} = \frac{1+R_f^2\tan^2\phi}{\tan^2\phi + R_f^2} \quad \text{-----(6.23)}$$

Thus, we have

$$R_f^2 = \frac{\lambda'_{yf} - \lambda'_{xf}\ \tan^2\phi}{\lambda'_{xf} - \lambda'_{yf}\ \tan^2\phi} \quad \text{-----(6.24)}$$

Substituting (6.13,6.14,6.15) to (6.24) and considering

$R_i^2 = \frac{\lambda'_{2i}}{\lambda'_{1i}}$ and $R_s^2 = \frac{\lambda'_{2s}}{\lambda'_{1s}}$, we have

$$R_f^2 = \frac{(\tan^2\theta + R_i^2)R_s^2 - (1+R_i^2\tan^2\theta)\tan^2\phi}{1+R_i^2\tan^2\theta - (\tan^2\theta + R_i^2)R_s^2\tan^2\theta} \quad \text{-----(6.25)}$$

Deformation of Strain Grid

Strain grid means a network with which we measure deformation during short period (around 1 year) by using survey instrument. Glacial flow is an example. We set up thest rain grid being a regular triangle as far as possible. After adequate period, we calculate strain by measuring the deformed network. The network shown in Fig.12 is a strain grid before deformation, while that of Fig.13 is a strain grid after deformation.

(1) Define D and E as $CD\perp AB$ and $AE\perp BC$ on Fig.12.

(2) Define D' and E' on Fig.13 to hold the next relations.

$$\frac{AD}{BD} = \frac{A'D'}{B'D'} \text{ and } \frac{BE}{EC} = \frac{B'E'}{E'C'}$$

(3) Then, we have ϕ_{AB} and ϕ_{BC} from Fig.13. γ_{AB} and γ_{BC} are also calculated from $\gamma_{AB} = \tan\phi_{AB}$ and $\gamma_{BC} = \tan\phi_{BC}$.

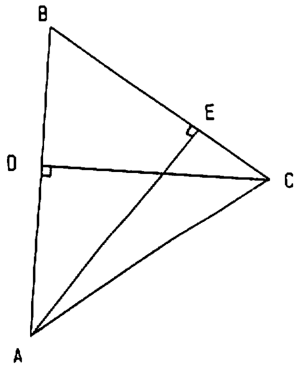

Fig. 12, Triangle before strain.

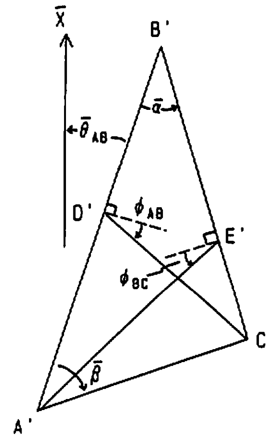

Fig. 13, Triangle after strain.

(4) λ_{AB} and λ_{BC} are obtained from the definition, that is,

$$\lambda_{AB} = \left(\frac{A'B'}{AB}\right)^2 \quad \text{and} \quad \lambda_{BC} = \left(\frac{B'C'}{BC}\right)^2$$

(5) Make an arbitrary γ'/λ' coordinate on Fig.14.

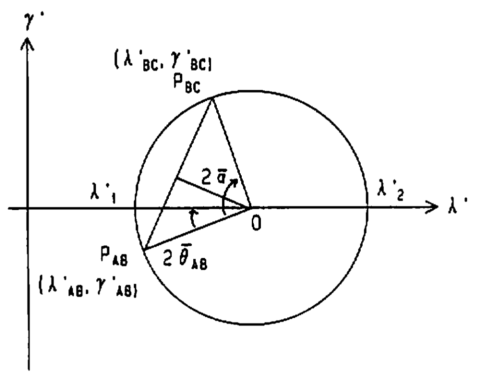

Fig. 14, Mohr's circle for deformation of strain grid.

(6) Calculate the coordinates of P_{AB} $(\lambda'_{AB}, \gamma'_{AB})$ and P_{BC} $(\lambda'_{BC}, \gamma'_{BC})$.

(7) Draw the points P_{AB} and P_{BC} in Fig.14.

(8) Draw a circle which connects two points P_{AB} and P_{BC}, center of which lies on λ' axis.

(9) Confirm the relation, $angle(P_{AB},O,P_{BC}) = 2\bar{\alpha}$.

(10) Then, we obtain $2\bar{\theta}_{AB}$. $\bar{\theta}_{AB}$ is the principal direction.

(11) Calculate λ_1 and λ_2 from λ'_1 and λ'_2.

Deformation of Glass Shard

Glass shard is a set of three joints that intersect 120° together. We thus know two angles α and β before deformation and we get two changed angles $\bar{\alpha}$ and $\bar{\beta}$ after deformation. The technique how to get strain is described here.

(1) Draw a line AB on Fig.15.
(2) Make a triangle $\triangle ABC$ by using α and β.
(3) Draw a line CD as $AB \perp CD$.
(4) Make a triangle $\triangle ABC$ by using $\bar{\alpha}$ and $\bar{\beta}$.
(5) Draw C D.
(6) Then we obtain ϕ_{AB}, which is the angle of shear for the side AB.
(7) Do similarly in Fig.16 to obtain the angle of shear ϕ_{AC} for the side AC. We know so far the angle of shear ϕ_{AB} for the side AB, ϕ_{AC} for AC and the angle $\bar{\alpha}$ between AB and AC.

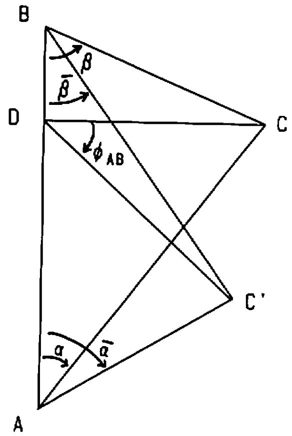

Fig. 15, Triangles of before and after strain with respect to the side AB.

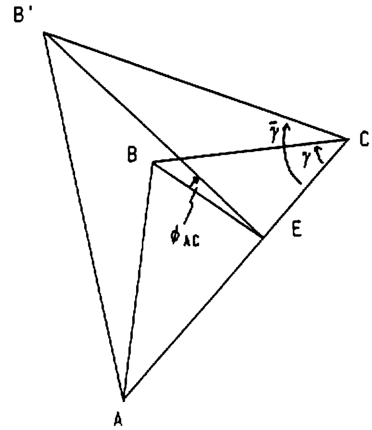

Fig. 16, Triangles of before and after strain with respect to the side AC.

(8) Obtain $\frac{\lambda'_1}{\lambda'_2}$ from Fig.17.

(9) Calculate $R^2 = \frac{\lambda_1}{\lambda_2}$ from $\frac{\lambda'_1}{\lambda'_2}$

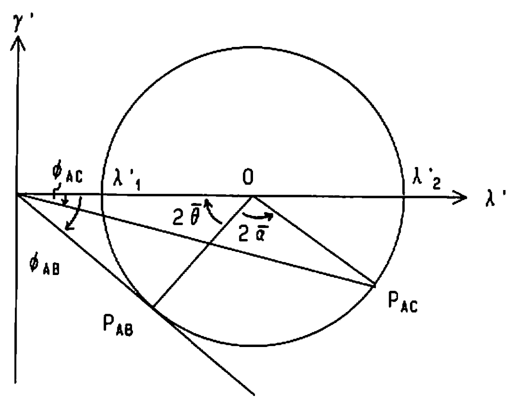

Fig. 17, Mohr's circle for deformation of glass shard.

This is the method where we use only graph to get strain. The other method where we need calculation to get strain is described from (8).

(8) Supposing the angle between AB and long axis to be $\overline{\theta}$, we have

$$\tan 2\overline{\theta} = \frac{\sin\phi_{AB}\ \sin\phi_{AC} - \sin\phi_{AB}\ \sin(\phi_{AC} + 2\overline{\alpha})}{\cos(\phi_{AC} + 2\overline{\alpha})\sin\phi_{AB} - \cos\phi_{AB}\ \sin\phi_{AC}}$$

from which we can calculate $\overline{\theta}$. It should be noted, however, that since the relation is introduced geometrically from Fig.17, we have to use absolute value for any angle.

(9) Draw a figure according to Fig.17 to get $\frac{\lambda'_1}{\lambda'_2}$

(10) Calculate R from $\frac{\lambda'_1}{\lambda'_2}$

Deformation of Fossil

Plotting length of hinge line h_0 or median line m_0 of brachiopod as abscissa and frequency as ordinate, points show normal distribution. Points on the frequency (ordinate) and ratio of $r_0 = \frac{h_0}{m_0}$ (abscissa) diagram show narrower normal distribution than those of the former.

It is sometimes difficult to distinguish whether a fossil is adult or immature, because shape of the adult and of immature is not always same. We sometimes distinguish them by plotting a *frequency/* r_0 diagram.

Deformation of 1 Fossil

Let us consider bilateral fossils such as brachiopod, trilobite etc. When we know the direction of long axis of strain ellipse, we get $\overline{\theta}$ and ϕ where $\overline{\theta}$ is the intersect angle between the long axis of strain ellipse and the symmetric axis of the bilateral fossil and ϕ is the shear angle for the symmetric axis.

(1) Draw γ'/λ' coordinate shown as Fig.18.

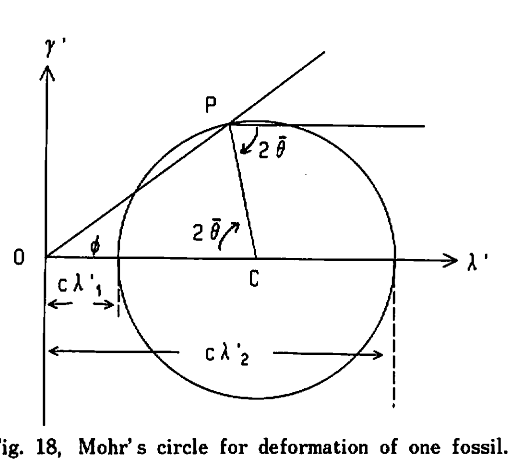

Fig. 18, Mohr's circle for deformation of one fossil.

(2) Draw OP by using ϕ
(3) Draw PC by using $2\overline{\theta}$
(4) Draw a circle of which radius is CP
(5) Calculate the value of R from $R^2 = \dfrac{\lambda_1}{\lambda_2} = \dfrac{c\lambda'_2}{c\lambda'_1}$

The other method to get R is the one using the Breddin graph ($\phi / \overline{\theta}$ diagram). When θ is changed to $\overline{\theta}$, $90° - \theta$ is changed to $90° - \overline{\theta} - \phi$. As we know the relation $\tan\theta = R \tan\overline{\theta}$, we have

$$R' = \frac{\tan\overline{\theta}}{\tan\theta}$$

$$R' = \frac{\tan(90° - \overline{\theta} - \phi)}{\tan(90° - \theta)}$$

Multiply both sides separately, we have

$$R'^2 = \frac{\tan\overline{\theta}\tan(90° - \overline{\theta} - \phi)}{\tan\theta\tan(90° - \theta)}$$

Since $\tan(90° - \theta) = \cot\theta$, the denominator equals 1, thus the equation is

$$R'^2 = \tan\overline{\theta}\tan(90° - \overline{\theta} - \phi) = \frac{\tan\overline{\theta}}{\tan(\overline{\theta} + \phi)}$$

When we draw the curve defined by the above equation, a graph of the curves is called as the Breddin graph with abscissa $\overline{\theta}$ and ordinate ϕ. We get easily the value of R by using the graph, though the smaller the angle ϕ, the obscure the R' value. When $\phi = 0$, $R' = 1$, that is, $R = 1$. We get also the R' value by calculating the last equation $R'^2 = \dfrac{\tan\overline{\theta}}{\tan(\overline{\theta} + \phi)}$ without using the graph. For example, from deformation where $\overline{\theta} = -6.5°$ and $\phi = -15°$, we get $R'^2 = 0.289$, thus $R = 1.859$. Since we don't know the principal direction of strain in field, this method is not the practical one.

Deformation of 2 Fossils

If two fossils are different species each other but are bilateral together, we can getthe R value. There are three methods to get the R value.

(1) Calculation

$$R'^2 = \frac{\tan\overline{\theta}}{\tan(\overline{\theta} + \phi_A)}$$

$$= \frac{1 - \tan\overline{\theta}\tan\phi_A}{\tan\overline{\theta} + \tan\phi_A} \tan\overline{\theta}$$

$$R'^2 = \frac{\tan(\overline{\theta} + \overline{\alpha})}{\tan(\overline{\theta} + \overline{\alpha} + \phi_B)}$$

$$= \frac{1 - \tan\overline{\theta}\tan(\overline{\alpha} + \phi_B)}{\tan\overline{\theta} + \tan(\overline{\alpha} + \phi_B)} \frac{(\tan\overline{\theta} + \tan\overline{\alpha})}{(1 - \tan\overline{\theta}\tan\overline{\alpha})}$$

We get the R value by solving the equations numerically with R' and $\overline{\theta}$ as unknowns.

The equations are changed to a 4th order equation, thus the method is not the practical one.

(2) Mohr's diagram

When we know two shear angles ϕ_A and ϕ_B for each side A and B, and the intersect angle $\overline{\alpha}$ between the side A and B, we will get the long axis direction $\overline{\theta}$ of strain ellipse by applying the values ϕ_A, ϕ_B and $\overline{\alpha}$ to the Mohr's diagram as illustrated in Fig.19.

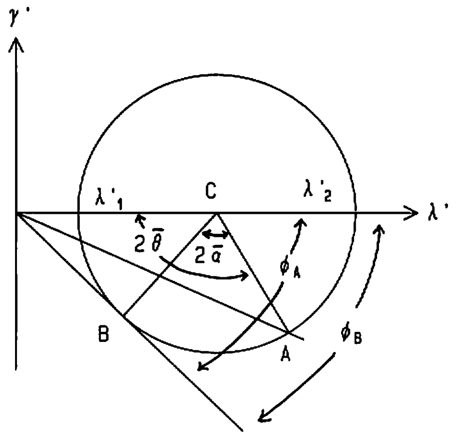

Fig. 19. Mohr's circle for deformation of two fossils.

(3) Breddin graph ($\phi/\overline{\theta}$ diagram)

When we know the same values as the method of Mohr's diagram, we get $\overline{\theta}$ by applying the values to the Breddin graph where we should take angle as absolute value.

Deformation of more than 2 Fossils

In this case we use the former methods mentioned in the sections "Deformation of 1 fossil" and "Deformation of 2 fossils" to get R value. The other methods, which use only for deformation of more than 2 fossils, are described as follows.

(1) $L/\overline{\alpha}$ diagram ($h/\overline{\alpha}$ or $m/\overline{\alpha}$ diagram)

The condition of the method is that the fossils are the same species and h_0 is constant, but their bilaterality is not necessary. Let an arbitrary line PQ be a reference line. Measure angles intersecting the reference line and hinge lines and call the angles as $\overline{\alpha}$. h_0 is length of hinge line before deformation. Replacing $\overline{\theta}$ by $\overline{\alpha}-\overline{\theta}_1$ in the equation $\lambda' = \lambda'_1 \cos^2\overline{\theta} + \lambda'_2 \sin^2\overline{\theta}$, we have

$$\lambda' = \left(\frac{h_0}{h}\right)^2 = \frac{\cos^2(\overline{\alpha}-\overline{\theta}_1)}{\lambda_1} + \frac{\sin^2(\overline{\alpha}-\overline{\theta}_1)}{\lambda_2}$$

where $\overline{\theta}_1$ is the angle between PQ and the long axis $\overline{X}$ of strain ellipse. Since

$R^2 = \dfrac{\lambda_1}{\lambda_2}$, we have

$$h = h_0\sqrt{\frac{\lambda_1}{1+(R^2-1)\sin^2(\overline{\alpha}-\overline{\theta}_1)}}$$

When we plot the curve with $\overline{\alpha}$ as abscissa and h as ordinate, we get a diagram, where the highest point of the curve is $h_0\sqrt{\lambda_1}$ and the lowest is $h_0\sqrt{\lambda_2}$ We get the value of R by

$$R=\frac{h_0\sqrt{\lambda_1}}{h_0\sqrt{\lambda_2}}$$

The direction of the highest point is the long axis $\overline{X}$ of strain ellipse. Though we use here hinge lines to get R, we can use median lines instead.

(2) $\phi/\overline{\alpha}$ diagram

The condition of the method is that fossils are bilateral, but the size of the fossils is not necessarily equal together and fossils of different species are possible to use in the method.

$$\tan\phi=\frac{\gamma'}{\lambda'}=\frac{(\lambda'_2-\lambda'_1)\sin(\overline{\theta}-\overline{\alpha})\cos(\overline{\theta}-\overline{\alpha})}{\lambda'_1\cos^2(\overline{\theta}-\overline{\alpha})+\lambda'_2\sin^2(\overline{\theta}-\overline{\alpha})}$$

Dividing the denominator and numerator by $\lambda'_1\sin(\overline{\theta}-\overline{\alpha})\ \cos(\overline{\theta}-\overline{\alpha})$, and considering the relation $R^2=\frac{\lambda'_2}{\lambda'_1}$ we have

$$\tan\phi=\frac{(R^2-1)\tan(\overline{\theta}-\overline{\alpha})}{1+R^2\tan(\overline{\theta}-\overline{\alpha})}$$

We get the direction $\overline{\theta}_1$ of long axis $\overline{X}$ of strain ellipse and the angle $\overline{\theta}_2$ from long axis to the direction ϕ_{max} from the curve as its intersect point to $\overline{\alpha}$ axis and its maximum point, respectively. R is calculated from the equation $R=\cot\overline{\theta}_2$, using $\overline{\theta}_2$, or from the next equation.

$$R^2=1+2(\tan\phi_{max})^2+2\tan\phi_{max}\sec\phi_{max}$$

It is described in the later section "Deformation associate with radial spherulitic texture" how to get the equation.

(3) r/ | ϕ | diagram

The condition of the method is that fossils are the same species and r_0 $(=\frac{h_0}{m_0})$ is constant. m_0 is the length of median line before deformation. The diagram is shown with $|\phi|$ as abscissa and r $(=\frac{h}{m})$ as ordinate. Since the largest value of r is r_0R and the smallest is r_0R', we get the value of R from $R^2=\frac{r_0R}{r_0R'}$.

(4) Wellman method

This is a fully graphical method, background of which is described on the textbook after Wellman (1973).

Deformation from Circular to Elliptical Objects

General case

When we plot points (X_f, Y_f) in the X_f / Y_f diagram with X_f as ordinate and Y_f as abscissa, gradient of the line represents the mean axial ratio R. X_f and Y_f are length of short and long axis of elliptical object after deformation, respectively.

Since the method is simple, it is convenient to get a rough R value by the method. If the amount of deformation is great, however, it is difficult to measure the size of pebbles because edges of the pebbles are broken. If the amount of deformation is small, on the contrary, it is difficult to distinguish between long axis and short axis of the pebbles.

The method that is described as follows (method of 3 direction) is more precise than the simple X_f / Y_f diagram method.

(1) Mark central points of all the elliptical objects.

(2) Consider three arbitrary reference lines which are not parallel together. Draw three lines inside each elliptical object. The lines pass through a central point of the elliptical object.

(3) Add length of the lines which are parallel each other for all objects,
$a = \Sigma a_i$, $b = \Sigma b_i$, $c = \Sigma c_i$.

(4) Since the ellipse is deformed from a circle, the next relation holds assuming k as the radius of the circle. Consider the ellipse which contacts the three lines of which lengths are a, b and c.

$$\lambda_a : \lambda_b : \lambda_c = \left(\frac{a}{k}\right)^2 : \left(\frac{b}{k}\right)^2 : \left(\frac{c}{k}\right)^2 = a^2 : b^2 : c^2$$

(5) Thus the case is same as that described in the section"Deformation of strain grid".

(6) We get the principal direction and axial ratio R of strain from the method of the section "Deformation of strain grid".

Deformation Associated with Pressure Solution

Changes of length among the center points of spherical objects (ooid etc.) represent bulk deformation on the section plane. Even if the objects do not associate with pressure solution, we can apply the method to get strain, if the objects were homogeneously scattered on the plane or if we know heterogeneous distribution of the objects before deformation. When we plot the distance D as ordinate, which is the distance among center points of objects before deformation, and the angle α as abscissa, which is the angle measured from an arbitrary reference line, the points (α, D) are scattered along a line of mean distance m. When we draw $\overline{D} / \overline{\alpha}$ diagram after deformation by measuring the value of $\overline{D}$ and $\overline{\alpha}$, values $\overline{D}$ reflect the stretch directed to $\overline{\alpha}$. Thus we see that the direction of maximum elongation is $\overline{\alpha}_1$ and that of minimum is $\overline{\alpha}_2$, and obtain R from $R = \frac{m_x}{m_y}$.

Deformation Associated with Radial Spherulitic Texture

We recognize sometimes fan-like streak lines as chords that pass through the center points of elliptical objects. We thus measure angles $\bar{\alpha}$ that are the angle between an arbitrary reference line and the streak lines. A shear angle ϕ for the angle $\bar{\alpha}$ is obtained as the difference between the angle α before deformation and the angle $\bar{\alpha}$ after deformation. We draw $\phi / \bar{\alpha}$ diagram from the values of $\bar{\alpha}$ and shear angle ϕ for the direction $\bar{\alpha}$. The principal direction of strain is indicated by the two points that are the intersect points to the curve $(\phi / \bar{\alpha})$ and α axis $(\gamma = 0)$. If we obtain the angle $\bar{\theta}_2$ between principal axis of strain and direction of γ_{max}, we calculate R from $\tan\bar{\theta}_2 = R'$ where $R' = \frac{1}{R}$. The other method to calculate R value from using only the ϕ_{max} value is described as follows. From the relation $\gamma_{max} = \tan\phi_{max} = \frac{\lambda_1 - \lambda_2}{2\sqrt{\lambda_1 \lambda_2}}$ we have

$$4\tan^2\phi_{max} = \frac{\lambda_1}{\lambda_2} + \frac{\lambda_2}{\lambda_1} - 2$$

Since $R^2 = \frac{\lambda_1}{\lambda_2}$, we change the equation as

$$R^4 - 2R^2(2\tan^2\phi_{max} + 1) + 1 = 0$$

Solving the equation with regard to R^2, we have

$$R^2 = 1 + 2\tan^2\phi_{max} + 2\tan\phi_{max}\sec\phi_{max}$$

Deformation from Elliptical to Elliptical Objects

Owing to the data of 144 thin sections performed by Cloos (1947) we see that the maximum angle of long axes of deformed ooids from the long axis of strain (fluctuation= $2\phi_{max}$) decreases when the value of axial ratio R_s of strain ellipse increases. It means that after strong strain, the long axis's directions that are initially distributed randomly tend to converge to a certain direction, which is of the long axis of strain X_s. The axial ratio R_f and the direction of long axis after deformation of the objects that are elliptical objects before deformation depend on the next three factors.

(1) Axial ratio of initial object ellipse $R_i = \frac{X_i}{Y_i}$

(2) Axial ratio of strain ellipse $R_s = \frac{X_s}{Y_s}$

(3) Principal direction of initial object ellipse θ

Axial Ratio R_i of Initial Ellipse is Constant

The conglomerate pebbles, the axial ratio R_i of which is constant before deformation, direct their long axes randomly. When they are suffered strain $R_s=\frac{X_s}{Y_s}$ they deform their shape to be flat. Deformed more strongly, they change their form to be more flat than ever. We see that the stronger the strain the long axes of pebbles after deformation are rearranged more parallel direction. We make a R_f/ϕ diagram from the values R_i, R_s and θ by using (6.18) and (6.25). The maximum and minimum values of R_f are obtained from (6.18) and (6.25) as follows.

When we take the values $\theta=\phi=0°$, we have

$$R_{fmax}=R_s\cdot R_i.$$

When we take the values $\theta=\phi=90°$, we have

$$R_{fmin}=\frac{R_s}{R_i}\quad(R_s>R_i)$$

$$=\frac{R_i}{R_s}\quad(R_s<R_i)$$

Thus we can seek out the strain R_s and the axial ratio R_i of initial object ellipse by drawing the R_f/ϕ diagram of deformed conglomerate pebbles in field. If the R_f/ϕ diagram shows a closed curve, that is, the "onion curve", the relation $R_s>R_i$ holds.

The R_f/ϕ diagram method

(1) Take an arbitrary reference line

(2) Measure individual angles ϕ between the reference line and long axis of marker ellipse.

(3) Calculate $R_f=\frac{X_f}{Y_f}$

(4) Draw the R_f/ϕ diagram

(5) We get the direction of long axis of strain ellipse as a symmetry axis of the R_f/ϕ curve.

(6) Since the relations hold as

$$R_{fmax}=R_sR_i$$

$$R_{fmin}=\frac{R_s}{R_i}\quad(R_s>R_i)$$

$$R_{fmin}=\frac{R_i}{R_s}\quad(R_s<R_i)$$

,we have the value R_s and R_i from the equation

$$R_s=\sqrt{R_{fmax}\cdot R_{fmin}}\quad(R_s>R_i)$$

$$R_s=\sqrt{\frac{R_{fmax}}{R_{fmin}}}\quad(R_s<R_i)$$

Axial Ratio R_i of Initial Ellipse is not Constant

When R_i takes value ranging $0<R_{imin}<R_i<R_{imax}$, the next relations hold.

$$R_{fmax}=R_s \cdot R_{imax}$$

$$R_{fmin}=\frac{R_s}{R_{imax}} \quad (R_s>R_{imax})$$

$$=\frac{R_{imax}}{R_s} \quad (R_s<R_{imax})$$

Thus we can calculate the values R_s and R_{imax} by the above equation as well as the former deformation with constant R_i. We cannot get R_{imin} on the contrary to the fo rmercase of deformation.

References

Cloos,E.,1947. Oolite deformation in the southern mountain fold, Maryland. *Geol.Soc.Amer.Bull.*, 58 : 843-918.

Jaeger,J.C.,1956. Elasticity, fracture and flow.208pp.,Methuen,London.

Means,W.D.,1976. Stress and strain, Basic concepts of continuum mechanics for geologists. 339pp., Springer-Verlag.

Ragan,D.M.,1973. Structural geology, An introduction to geometrical techniques. 2nd ed., 208pp., John Wiley & Sons.

Ramsay,J.G.,1967. Folding and fracturing of rocks. 568pp., McGraw-Hill.

Ramsay,J.G. and Huber,M.I.,1983. The techniques of modern structural geology, Volume 1:Strain analysis. 307pp., Academic Press.