Relationship of Parallel Model and Series Model for Permanent Magnet Synchronous Motors Taking Iron Loss Into Account

Naomitsu Urasaki, *Member, IEEE*, Tomonobu Senjyu, *Member, IEEE*, and Katsumi Uezato

***Abstract*—This paper investigates the relationship of parallel model and series model for permanent magnet synchronous motor taking iron loss into account. The expressions of flux linkage, terminal voltage, and electromagnetic torque are compared. It follows from the investigation that the parallel and series models are mathematically the same. In addition, the properties of the models are exhibited. The parallel model is superior in understanding the physical meaning to the series model. The series model is superior in low order of the state variables to the parallel model.**

***Index Terms*—Iron loss resistance, parallel model, permanent magnet synchronous motor, series model.**

NOMENCLATURE

v_{dq}	$d-q$ axes terminal voltages.
i_{dq}	$d-q$ axes line currents.
i_{dqm}	$d-q$ axes magnetizing currents.
i_{dqi}	$d-q$ axes iron loss currents.
Ψ_{dq}	$d-q$ axes flux linkages.
Ψ_{dqs}	$d-q$ axes self-flux linkages for series model.
Ψ_{dqm}	$d-q$ axes mutual flux linkages for series model.
p	Differential operator $(= d/dt)$.
N_r	Rotor speed.
ω_m	Mechanical angular velocity.
ω_e	Electrical angular velocity $(= P\omega_m)$.
P	Number of pole pairs.
τ	Electromagnetic torque.
R	Armature resistance.
L	Armature inductance.
K_e	emf constant.
P_i	Iron loss.
R_i	Iron loss resistance.
R_m	Equivalent iron loss resistance.
L_m	Equivalent armature inductance.
K_{em}	Equivalent emf constant.
i_{2d}	Equivalent exciting current.
i_{3dq}	Equivalent eddy currents.
Ψ_{3dq}	$d-q$ axes flux linkages for eddy current circuit.
R_3	Resistance of eddy current circuit.
L_1	Self inductance of armature circuit.
L_3	Self inductance of eddy current circuit.
M_{ij}	Mutual inductance between i and j windings.

Manuscript received February 9, 2003.

The authors are with the Department of Electrical and Electronics Engineering, University of the Ryukyus, Okinawa 903-0213, Japan (e-mail: urasaki@tec.u-ryukyu.ac.jp).

Digital Object Identifier 10.1109/TEC.2004.827291

I. INTRODUCTION

As the employment of vector-controlled ac motors, especially induction motor and permanent magnet synchronous motor (PMSM), has become standard in industrial drives, the improvement of ac motor drives has been an important issue. Traditionally, vector control strategies have been performed under the assumption that there is no iron loss in motors. However, since the iron loss influences the flux linkage and electromagnetic torque in ac motors, it has been necessary to compensate the influence of the iron loss in vector control strategies. For this reason, several authors have made an attempt to consider the iron loss in vector-controlled ac motor drives.

In those studies, equivalent circuits with an iron loss resistance are utilized. From a modeling point of view, they can be classified as either parallel or series type. The parallel model inserts an iron loss resistance in the equivalent circuit with the parallel fashion and it is employed in [1]–[5]. By contrast, the series model inserts an equivalent iron loss resistance in the equivalent circuit with the series fashion and it is employed in [6]–[9]. In this situation, however, the relationship between the parallel and series models has not been clarified.

It is the purpose of this paper to reveal the relationship of the two types of model for PMSM. In the first phase, the formulations of both the parallel and series models are illustrated. In the second phase, the expressions of flux linkage, terminal voltage, and electromagnetic torque are compared. In addition, the properties of both parallel and series models are exhibited.

II. MATHEMATICAL FORMULATION OF PMSM TAKING IRON LOSS INTO ACCOUNT

In the synchronous reference frame $(d-q)$, the voltage equation for PMSM is expressed as [10]

$$\left.\begin{aligned} v_d &= Ri_d + p\Psi_d - \omega_e\Psi_q \\ v_q &= Ri_q + p\Psi_q + \omega_e\Psi_d \end{aligned}\right\} \tag{1}$$

where the first term in the right-hand side represents the voltage drop due to the armature resistance R, and the second and third terms represent the transformer electromotive-force (emf) and speed emf, respectively.

From an expression of the $d-q$ axes flux linkages (Ψ_d, Ψ_q), it is possible to classify mathematical models for PMSM taking iron loss into account into two main categories (i.e., parallel or series type). In this section, expressions of flux linkage for the two types of model are explained.

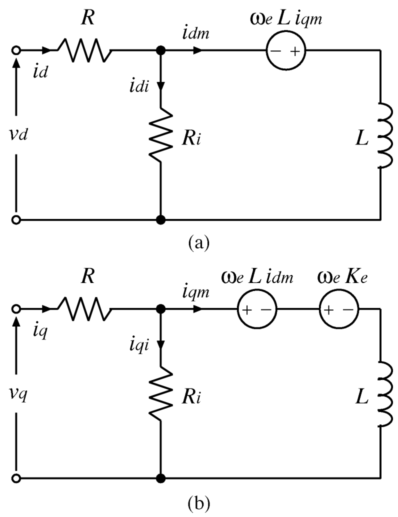

Fig. 1. $d-q$-axes equivalent circuits for parallel model. (a) d-axis. (b) q-axis.

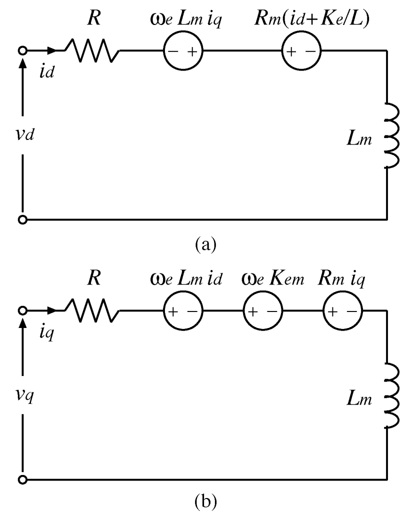

Fig. 2. $d-q$-axes equivalent circuits for series model. (a) d-axis. (b) q-axis.

A. Parallel Model

Fig. 1 shows the $d-q$ axes equivalent circuits for PMSM which are traditionally applied to consider iron loss [2]. In this circuit, an iron loss resistance R_i is inserted in parallel with the magnetizing branch. Thus, the $d-q$ axes line currents (i_d, i_q) are divided into the iron loss currents (i_{di}, i_{qi}) and magnetizing currents (i_{dm}, i_{qm}).

The iron loss P_i arises from the iron loss resistance R_i and is expressed as

$$P_i = R_i \left(i_{di}^2 + i_{qi}^2\right). \tag{2}$$

In steady-state condition, the iron loss currents (i_{di}, i_{qi}), illustrated in Fig. 1, are expressed as

$$\left.\begin{aligned} i_{di} &= -\tfrac{\omega_e \Psi_q}{R_i} \\ i_{qi} &= \tfrac{\omega_e \Psi_d}{R_i} \end{aligned}\right\}. \tag{3}$$

It is noted that since the $d-q$ axes magnetizing currents (i_{dm}, i_{qm}) are dc quantities in steady-state condition, the transformer emfs (Lpi_{dm}, Lpi_{qm}) become zero. Thus, these components do not appear in (3). As a result, the iron loss can be rewritten as

$$P_i = \frac{\omega_e^2 \left(\Psi_d^2 + \Psi_q^2\right)}{R_i}. \tag{4}$$

Supposing the iron loss resistance R_i is constant, (4) corresponds to only an eddy current loss, because P_i is proportional to the product of the square of electrical angular velocity ω_e^2 and the square of flux linkages ($\Psi_d^2 + \Psi_q^2$). For practical purposes, in order to include a hysteresis loss into P_i, the iron loss resistance R_i is usually treated as a function of the electrical angular velocity $R_i(\omega_e)$ [3].

The flux linkage equation for the parallel model is given as

$$\left.\begin{aligned} \Psi_d &= Li_{dm} + K_e \\ \Psi_q &= Li_{qm} \end{aligned}\right\} \tag{5}$$

where K_e corresponds to a permanent magnet flux linkage [2].

B. Series Model

A series type mathematical model for PMSM taking iron loss into account is derived from a magnetic coupling between an armature circuit and eddy current circuit [8]. In this model, the iron loss P_i arises from the resistance R_3 of the eddy current circuit (see the Appendix). In steady-state condition, from (A.2) in the Appendix ($p\Psi_{3d} = 0$, $p\Psi_{3q} = 0$), the iron loss P_i is expressed as

$$\begin{aligned} P_i &= R_3 \left(i_{3d}^2 + i_{3q}^2\right) \\ &= \frac{\omega_e^2 \left(\Psi_{3d}^2 + \Psi_{3q}^2\right)}{R_3} \\ &= \frac{\omega_e^2 \left(\Psi_d^2 + \Psi_q^2\right)}{R_3}. \end{aligned} \tag{6}$$

It is noted that the $d-q$ axes flux linkages (Ψ_{3d}, Ψ_{3q}) for the eddy current circuit are equal to the flux linkages (Ψ_d, Ψ_q) for the armature circuit when (A.6) in the Appendix is presumed.

The flux linkage equation for the series model is given as follows [8]:

$$\left.\begin{aligned} \Psi_d &= \Psi_{ds} + \Psi_{dm} \\ \Psi_q &= \Psi_{qs} + \Psi_{qm} \end{aligned}\right\} \tag{7}$$

$$\left.\begin{aligned} \Psi_{ds} &= L_m i_d + K_{em} \\ \Psi_{qs} &= L_m i_q \end{aligned}\right\} \tag{8}$$

$$\left.\begin{aligned} \Psi_{dm} &= \tfrac{R_m}{\omega_e} i_q \\ \Psi_{qm} &= -\tfrac{R_m}{\omega_e}\left(i_d + \tfrac{K_e}{L}\right) \end{aligned}\right\}. \tag{9}$$

The equivalent iron loss resistance R_m, equivalent armature inductance L_m, and equivalent emf constant K_{em} are defined as follows:

$$R_m = \frac{\omega_e^2 L^2}{R_3^2 + \omega_e^2 L^2} R_3 \tag{10}$$

$$L_m = \frac{R_3^2}{R_3^2 + \omega_e^2 L^2} L \tag{11}$$

$$K_{em} = \frac{R_3^2}{R_3^2 + \omega_e^2 L^2} K_e. \tag{12}$$

The details of the above expression of the flux linkage for the series model are explained in the Appendix. Since the expression of the flux linkage for the series model is described in terms of the line currents (i_d, i_q), the $d-q$ axes equivalent circuits become series fashion as shown in Fig. 2. As can be seen from Fig. 2, added speed emfs appear in the equivalent circuit. The effects of the iron loss on the armature circuit are reflected by these speed emfs. It is noted that the series model shown in Fig. 2 is expressive of only the armature circuit of PMSM. Thus, the iron loss cannot be obtained directly from this equivalent circuit, because the iron loss in the series model occurs in the eddy current circuit (see the Appendix). Nevertheless, since the series model includes the effect of the iron loss, it suffices for the vector-control strategies taking iron loss into account. As proved later, the iron loss in the eddy current circuit can be represented by using the quantities of the armature circuit.

III. Relationship of Flux Linkage for Parallel and Series Models

In this section, the expressions of the flux linkage are mathematically developed and the relationship of the two types of flux linkage is investigated. In these mathematical developments, the following assumption is introduced. Both the resistances R_i and R_3 are much greater than the armature reactance $(\omega_e L)$. In other words, the following relations are satisfied:

$$\left(\frac{\omega_e L}{R_i}\right)^2 \ll 1 \quad \text{and} \quad \left(\frac{\omega_e L}{R_3}\right)^2 \ll 1. \tag{13}$$

A. Flux Linkage Equation for Parallel Model

From (3) and (5), the $d-q$ axes magnetizing currents (i_{dm}, i_{qm}) are expressed as follows:

$$i_{dm} = i_d - i_{di} = i_d + \frac{\omega_e L}{R_i} i_{qm} \tag{14}$$

$$i_{qm} = i_q - i_{qi} = i_q - \frac{\omega_e L}{R_i}\left(i_{dm} + \frac{K_e}{L}\right). \tag{15}$$

Substituting (15) into (14) eliminates the q-axis magnetizing current i_{qm} in (14) as

$$i_{dm} = i_d + \frac{\omega_e L}{R_i}\left\{i_q - \frac{\omega_e L}{R_i}\left(i_{dm} + \frac{K_e}{L}\right)\right\}. \tag{16}$$

Transforming the term $(-(\omega_e L/R_i)^2 i_{dm})$ in the right-hand side into the left-hand side gives

$$\left\{1 + \left(\frac{\omega_e L}{R_i}\right)^2\right\} i_{dm} = i_d + \frac{\omega_e L}{R_i}\left(i_q - \frac{\omega_e K_e}{R_i}\right). \tag{17}$$

Finally, applying the relation indicated in (13) to (17) gives the d-axis magnetizing current expressed in terms of the line current (i_d, i_q) as

$$i_{dm} \simeq i_d + \frac{\omega_e L}{R_i}\left(i_q - \frac{\omega_e K_e}{R_i}\right). \tag{18}$$

In a similar way, the q-axis magnetizing current can be expressed as

$$i_{qm} \simeq i_q - \frac{\omega_e L}{R_i}\left(i_d + \frac{K_e}{L}\right). \tag{19}$$

Substituting (18) and (19) into (5) yields the flux linkage equation described in the form of

$$\left.\begin{aligned} \Psi_d &\simeq L i_d + K_e + \tfrac{\omega_e L}{R_i} L i_q \\ \Psi_q &\simeq L i_q - \tfrac{\omega_e L}{R_i}(L i_d + K_e) \end{aligned}\right\}. \tag{20}$$

B. Flux Linkage Equation for Series Model

Applying (13) to (10) simplifies the expression of the equivalent iron loss resistance R_m as

$$R_m = \frac{\left(\frac{\omega_e L}{R_3}\right)^2}{1 + \left(\frac{\omega_e L}{R_3}\right)^2} R_3 \simeq \frac{\omega_e^2 L^2}{R_3}. \tag{21}$$

Applying (13) to (11) simplifies the equivalent armature inductance L_m as

$$L_m = \frac{1}{1 + \left(\frac{\omega_e L}{R_3}\right)^2} L \simeq L. \tag{22}$$

In a similar way, the equivalent emf constant K_{em} indicated in (12) is simplified as

$$K_{em} \simeq K_e. \tag{23}$$

Thus, substituting (21) to (23) into (7) to (9) results in the flux linkage equation described in the form of

$$\left.\begin{aligned} \Psi_d &\simeq L i_d + K_e + \tfrac{\omega_e L}{R_3} L i_q \\ \Psi_q &\simeq L i_q - \tfrac{\omega_e L}{R_3}(L i_d + K_e) \end{aligned}\right\}. \tag{24}$$

C. Comparison of Flux Linkage for Two Types of Model

It can be concluded from the following two points of view that the iron loss resistance R_i and the resistance R_3 of the eddy current circuit are identical.

1) The iron loss P_i arises from an added resistance. The resistance corresponds to R_i in the parallel model and R_3 in the series model, respectively.
2) From (4) and (6), the forms of the iron loss P_i for two types of model are the same. Furthermore, the iron loss P_i corresponds to only eddy current loss as long as both the resistances R_i and R_3 are constant.

From the sameness of the resistances, (20) is identical to (24). Thus, the expressions of $d-q$ axes flux linkages (Ψ_d, Ψ_q) for the parallel and series models are mathematically the same. In other words, expressing the magnetizing currents (i_{dm}, i_{qm}) with the line current (i_d, i_q) as shown in (18) and (19) and replacing the term $(\omega_e^2 L^2/R_i)$ with the symbol R_m, where $R_i = R_3$, convert the expression of the flux linkage for the parallel model into that for the series model.

The validity of the relations indicated in (13) will be confirmed. Fig. 3(a) shows the iron loss resistance R_i versus rotor speed N_r for a 160-W PMSM [11]. The specifications of the PMSM are listed in Table I. As can be seen from Fig. 3(a), the iron loss resistance R_i is almost proportional to the rotor speed N_r. The linear characteristic of the iron loss resistance qualitatively agrees with the results obtained in the literature [4], [5]. Fig. 3(b) shows the square of impedance ratio $(\omega_e L/R_i)^2$ versus rotor speed N_r. The ratio is calculated with using the electrical angular velocity ω_e $[\mathrm{rad/s}]$ $(= (2\pi/60)PN_r\ [\mathrm{rpm}])$, armature inductance $L[\mathrm{H}]$ listed in Table I, and iron loss resistance $R_i[\Omega]$ shown in Fig. 3(a). As can be seen from Fig. 3(b), the ratio is much less than 1 (order of the ratio is 10^{-4}.) over the wide speed range. Furthermore, similar results can be obtained from other

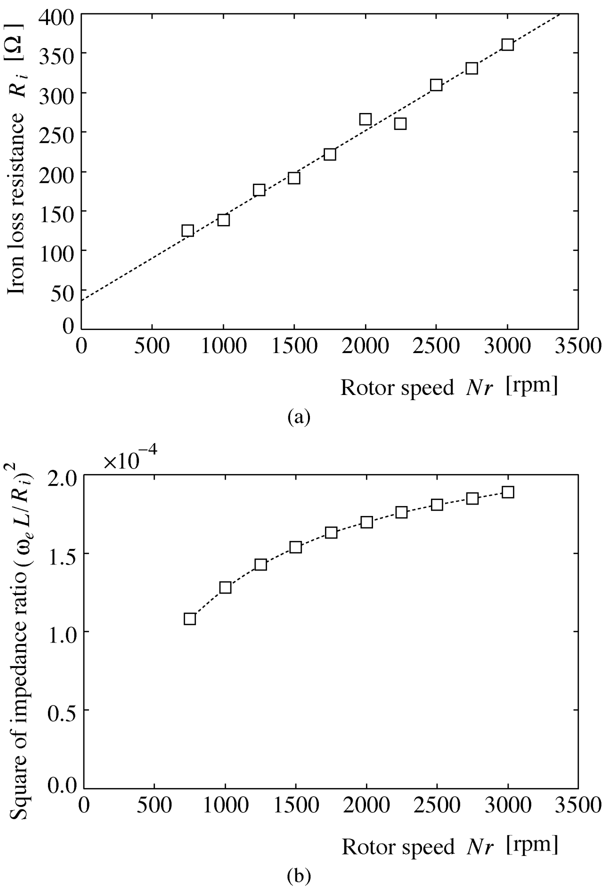

Fig. 3. Characteristic of iron loss versus rotor speed at 160-W PMSM. (a) Iron loss resistance. (b) Square of impedance ratio.

TABLE I
MOTOR SPECIFICATIONS

rated power	P_n	160 W
rated torque	τ_n	0.5 N·m
rated speed	N_n	3,000 rpm
armature resistance	R	2.14 Ω
armature inductance	L	0.0079 H
emf constant	K_e	0.0658 V·s/rad
number of pole pairs	P	2

motor parameters [2], [3], [9]. Accordingly, the relations indicated in (13) are valid.

IV. Relationship of Terminal Voltage for Parallel and Series Models

It has been clarified that the expressions of the flux linkage for the parallel and series models are mathematically the same. As can be seen from (1), the voltage equations for two types of model are also the same. In this section, the essential properties of them are exhibited.

A. Voltage Equation for Parallel Model

Substituting (5) into (1) yields the voltage equation described in the form of

$$\left.\begin{aligned} v_d &= Ri_d + p(Li_{dm}) - \omega_e L i_{qm} \\ v_q &= Ri_q + p(Li_{qm}) + \omega_e (L i_{dm} + K_e) \end{aligned}\right\}. \tag{25}$$

The voltage equation for the parallel model is intuitively understandable (i.e., it is seen that the first, second, and third terms correspond to the voltage drop due to the armature resistance R), transformer emf, and speed emf, respectively. However, the parallel model has the disadvantage of increase of state variables [i.e., the magnetizing currents $(i_{dm}, i_{qm}]$. In addition, the magnetizing currents should be estimated because they cannot be obtained directly.

B. Voltage Equation for Series Model

Substituting (7) to (9) into (1) yields the voltage equation described in the form of

$$\left.\begin{aligned} v_d &= (R+R_m)i_d + p\left(L_m i_d + \tfrac{R_m}{\omega_e} i_q\right) \\ &\quad -\omega_e\left(L_m i_q - \tfrac{R_m}{\omega_e}\tfrac{K_e}{L}\right) \\ v_q &= (R+R_m)i_q + p\left(L_m i_q - \tfrac{R_m}{\omega_e} i_d\right) \\ &\quad +\omega_e(L_m i_d + K_{em}) \end{aligned}\right\} \tag{26}$$

where $L_m \simeq L$ and $K_{em} \simeq K_e$. The series model has the advantage that there is no need to estimate the magnetizing currents (i_{dm}, i_{qm}). However, it is difficult to intuitively understand physical meanings of the formulation. For instance, although the terms $(R_m i_d)$ and $(R_m i_q)$ in (26) are the voltage drops due to the equivalent iron loss resistance R_m, they are physically elements of speed emfs $((\omega_e \Psi_d), (\omega_e \Psi_q))$.

V. Relationship of Electromagnetic Torque for Parallel and Series Models

A. Torque Equation for Parallel Model

The electromagnetic torque τ for the parallel model is derived from the interaction between the $d-q$ axes line currents (i_d, i_q) and flux linkages (Ψ_d, Ψ_q) as

$$\begin{aligned} \tau &= P(\Psi_d i_q - \Psi_q i_d) \\ &= P(\Psi_d i_{qm} - \Psi_q i_{dm}) + P(\Psi_d i_{qi} - \Psi_q i_{di}) \\ &= PK_e i_{qm} + \frac{1}{\omega_m}\frac{\omega_e^2\left(\Psi_d^2 + \Psi_q^2\right)}{R_i} \end{aligned} \tag{27}$$

where the first term in the right-hand side corresponds to the output torque and the second term corresponds to the loss torque due to the iron loss. It is noted that multiplying the second term in (27) by the mechanical angular velocity ω_m gives the iron loss P_i defined in (4).

B. Torque Equation for Series Model

The electromagnetic torque τ for the series model is derived from the interaction between the $d-q$ axes line currents and flux linkages, which is calculated from (7) to (9) together with the relations $R_m \simeq (\omega_e L)^2/R_3$, $L_m \simeq L$, and $K_{em} \simeq K_e$, as

$$\begin{aligned} \tau &= P(\Psi_d i_q - \Psi_q i_d) \\ &= PK_e\left\{i_q - \frac{R_m}{\omega_e L}\left(i_d + \frac{K_e}{L}\right)\right\} \\ &\quad + P\frac{R_m}{\omega_e}\left\{\left(i_d + \frac{K_e}{L}\right)^2 + i_q^2\right\}. \end{aligned} \tag{28}$$

From (19), the first term in the right-hand side in (28) is identical to the output torque for the parallel model as follows:

$$\begin{aligned}&\text{First term}\\&= PK_e\left\{i_q - \frac{\omega_e L}{R_i}\left(i_d + \frac{K_e}{L}\right)\right\}\\&= PK_e i_{qm}.\end{aligned} \tag{29}$$

Multiplying the second term in the right-hand side in (28) by the mechanical angular velocity ω_m gives

$$\begin{aligned}&\text{Second term} \times \omega_m\\&= \omega_m P\frac{R_m}{\omega_e}\left\{\left(i_d + \frac{K_e}{L}\right)^2 + i_q^2\right\}\\&= R_m\left\{\left(i_d + \frac{K_e}{L}\right)^2 + i_q^2\right\}.\end{aligned} \tag{30}$$

It is noted that (30) is identical to the iron loss P_i. Actually, substituting (24) into (6) and replacing the term $(\omega_e^2 L^2/R_3)$ with R_m yields (30).

It follows form above mathematical developments that although the torque equations for two types of model differ on appearances, they are mathematically the same.

VI. Summary of Relationship Between Parallel and Series Models

The relationship of the parallel and series models for PMSM is summarized as follows.

1) Although the flux linkage equations and voltage equations for two types of model differ on appearances, they are mathematically the same. The mathematical equivalence of them can be interpreted as the equivalent transformation between parallel and series electrical circuits. In addition, the torque equations for two types model are mathematically the same although they also differ on appearances.
2) In the parallel model, the iron loss P_i arises from the iron loss resistance R_i. Since the iron loss resistance is inserted in the armature circuit, the iron loss can be obtained directly from the $d-q$ axes equivalent circuit of PMSM shown in Fig. 1. In the series model, the iron loss P_i arises from the resistance R_3 of the eddy current circuit. Since the armature circuit includes only added emfs by the magnetic coupling of the eddy current circuit, the iron loss cannot be obtained directly from the equivalent circuit of PMSM shown in Fig. 2. Alternatively, the iron loss is obtained directly from the eddy current circuit indicated in (6). Fortunately, as indicated in (30), the iron loss can also be calculated by using the quantities of the armature circuit.
3) The parallel model is superior in understanding the physical meaning to the series model. The parallel model is capable of expressing physical phenomena evidently, while the series model cannot be understood intuitively. By contrast, the series model is superior in low order of the state variables to the parallel model. The number of the state variable does not change even if the iron loss is considered. For the parallel model, the number of state variables increases due to the magnetizing currents. In addition, the magnetizing currents should be estimated because they cannot be obtained directly.

VII. Conclusion

This paper has investigated the relationship for the parallel and series models for PMSM taking iron loss into account. The expressions of the flux linkage for two types of model are mathematically developed and compared. The investigation has revealed the mathematical equivalence of the parallel and series models. In addition, the properties of the models are exhibited. The parallel model is superior in understanding the physical meaning to the series model. The series model is superior in low order of the state variables to the parallel model.

Appendix

Fig. A1 shows the $d-q$ axes equivalent circuits of PMSM taking iron loss into account. The d-axis equivalent circuit consists of the armature circuit, field circuit, and added eddy current circuit. The q-axis equivalent circuit consists of the armature circuit and the eddy current circuit. The iron loss arises from the resistance R_3 of the eddy current circuit.

From Fig. A1, the $d-q$ axes flux linkages for the armature circuit are given as

$$\left.\begin{aligned}\Psi_d &= L_1 i_d + M_{13} i_{3d} + M_{12} i_{2d}\\ \Psi_q &= L_1 i_q + M_{13} i_{3q}\end{aligned}\right\}. \tag{A.1}$$

The eddy current circuit is short circuit and its voltage equations is expressed as

$$\left.\begin{aligned}0 &= R_3 i_{3d} + p\Psi_{3d} - \omega_e \Psi_{3q}\\ 0 &= R_3 i_{3q} + p\Psi_{3q} + \omega_e \Psi_{3d}\end{aligned}\right\} \tag{A.2}$$

where the flux linkages are given as

$$\left.\begin{aligned}\Psi_{3d} &= L_3 i_{3d} + M_{13} i_d + M_{32} i_{2d}\\ \Psi_{3q} &= L_3 i_{3q} + M_{13} i_q\end{aligned}\right\}. \tag{A.3}$$

In steady-state condition, the equivalent eddy currents are derived from (A.2) and (A.3) as

$$\left.\begin{aligned}i_{3d} &= -\tfrac{\omega_e^2 L_3 M_{13}}{R_3^2+\omega_e^2 L_3^2} i_d + \tfrac{R_3 \omega_e M_{13}}{R_3^2+\omega_e^2 L_3^2} i_q - \tfrac{\omega_e^2 L_3 M_{32}}{R_3^2+\omega_e^2 L_3^2} i_{2d}\\ i_{3q} &= -\tfrac{\omega_e^2 L_3 M_{13}}{R_3^2+\omega_e^2 L_3^2} i_q - \tfrac{R_3 \omega_e M_{13}}{R_3^2+\omega_e^2 L_3^2} i_d - \tfrac{R_3 \omega_e M_{32}}{R_3^2+\omega_e^2 L_3^2} i_{2d}\end{aligned}\right\}. \tag{A.4}$$

Substituting (A.4) into (A.1) results in the flux linkages expressed in the form of

$$\left.\begin{aligned}\Psi_d &= \left(L_1 - \tfrac{\omega_e^2 L_3 M_{13}^2}{R_3^2+\omega_e^2 L_3^2}\right) i_d + \tfrac{R_3 \omega_e M_{13}^2}{R_3^2+\omega_e^2 L_3^2} i_q\\ &\quad + \left(M_{12} - \tfrac{\omega_e^2 L_3 M_{32} M_{13}}{R_3^2+\omega_e^2 L_3^2}\right) i_{2d}\\ \Psi_q &= \left(L_1 - \tfrac{\omega_e^2 L_3 M_{13}^2}{R_3^2+\omega_e^2 L_3^2}\right) i_q - \tfrac{R_3 \omega_e M_{13}^2}{R_3^2+\omega_e^2 L_3^2} i_d\\ &\quad - \tfrac{R_3 \omega_e M_{32} M_{13}}{R_3^2+\omega_e^2 L_3^2} i_{2d}\end{aligned}\right\}. \tag{A.5}$$

Neglecting leakage inductances in both the armature and eddy current circuits gives the following relations:

$$\left.\begin{aligned}&L_1 \simeq L_3 \simeq M_{13} \equiv L\\ &M_{12} \simeq M_{32}\end{aligned}\right\}. \tag{A.6}$$

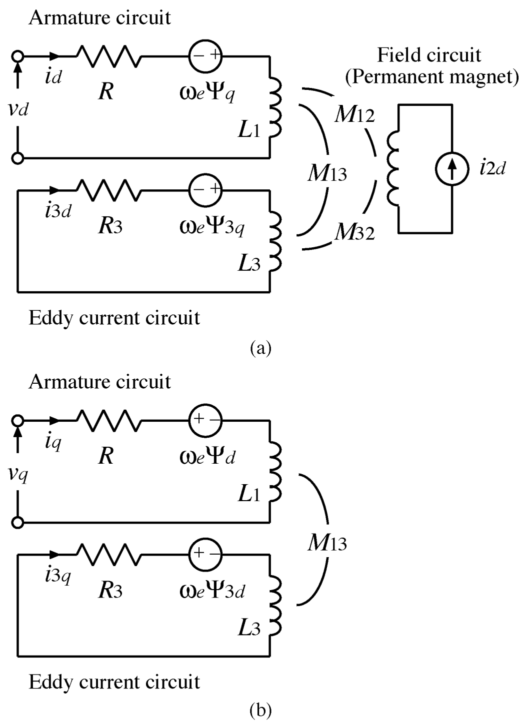

Fig. A1. $d-q$ axes equivalent circuits of PMSM taking iron loss into account. (a) d-axis. (b) q-axis.

It is noted that (A.3) is equal to (A.1) when the relations are true. Applying the relations to (A.5) results in the flux linkages as

$$\left.\begin{aligned}\Psi_d &= \frac{R_3^2}{R_3^2+\omega_e^2L^2}Li_d+\frac{\omega_eL^2}{R_3^2+\omega_e^2L^2}R_3i_q+\frac{R_3^2}{R_3^2+\omega_e^2L^2}K_e\\ \Psi_q &= \frac{R_3^2}{R_3^2+\omega_e^2L^2}Li_q-\frac{\omega_eL^2}{R_3^2+\omega_e^2L^2}R_3i_d-\frac{\omega_eL}{R_3^2+\omega_e^2L^2}R_3K_e\end{aligned}\right\} \tag{A.7}$$

where $K_e(= M_{12}i_{2d})$ corresponds to the permanent magnet flux. Defining the equivalent iron loss resistance R_m, equivalent armature inductance L_m, and equivalent emf constant K_{em} as (10) to (12), respectively, it is seen that (A.7) becomes (7).

REFERENCES

[1] L. Xu and J. Yao, "A compensated vector control scheme of a synchronous reluctance motor including saturation and iron losses," *IEEE Trans. Ind. Applicat.*, vol. 28, pp. 1330–1338, Nov./Dec. 1992.

[2] S. Morimoto, Y. Tong, Y. Takeda, and T. Hirasa, "Loss minimization control of permanent magnet synchronous motor drives," *IEEE Trans. Ind. Electron.*, vol. 41, pp. 511–517, Sept./Oct. 1994.

[3] E. Levi, "Impact of iron loss on behavior of vector controlled induction machines," *IEEE Trans. Ind. Applicat.*, vol. 31, pp. 1287–1296, Nov./Dec. 1995.

[4] H. Rasmussen, P. Vadstrup, and H. Borsting, "Rotor field oriented control with adaptive iron loss compensation," in *Proc. Ind. Applicat. Soc.*, Phoenix, AZ, 1999.

[5] T. Noguchi, P. Nakmahachalasint, and N. Watanakul, "Precise torque control of induction motor with on-line parameter identification in consideration of core loss," in *Proc. Power Conversion Conf.*, Nagaoka, Japan, 1997, pp. 113–118.

[6] T. Mizuno, J. Takayama, T. Ichioka, and M. Terashima, "Decoupling control method of induction motor taking stator core loss into consideration," in *Proc. Int. Power Electron. Conf.*, Tokyo, Japan, 1990, pp. 69–74.

[7] K. Uezato, T. Senjyu, and Y. Tomori, "Modeling and vector control of synchronous reluctance motors including stator iron loss," *IEEE Trans. Ind. Applicat.*, vol. 30, pp. 971–976, July/Aug. 1994.

[8] T. Senjyu, T. Shimabukuro, and K. Uezato, "Vector control of synchronous permanent magnet motors including stator iron loss," *Int. J. Electron.*, vol. 80, no. 2, pp. 181–190, 1996.

[9] J. Jung and K. Nam, "A vector control scheme for EV induction motors with a series iron loss model," *IEEE Trans. Ind. Electron.*, vol. 45, pp. 617–624, Aug. 1998.

[10] I. Boldea and S. A. Nasar, "Unified treatment of core losses and saturation in the orthogonal-axis model of electrical machines," *Proc. Inst. Elect. Eng., Elect. Power Applicat.*, pt. B, vol. 134, no. 6, pp. 355–363, Nov. 1987.

[11] N. Urasaki, T. Senjyu, and K. Uezato, "An accurate modeling for permanent magnet synchronous motor drives," in *Proc. Appl. Power Electron. Conf. Expo.*, New Orleans, LA, 2000, pp. 387–392.

Naomitsu Urasaki (M'98) was born in Okinawa Prefecture, Japan, in 1973. He received the B.S. and M.S. degrees in electrical engineering from the University of the Ryukyus, Okinawa, Japan, in 1996 and 1998, respectively.

Currently, he is a Research Associate with the Department of Electrical and Electronics Engineering, Faculty of Engineering at the University of the Ryukyus, where he has been since 1998. His research interests are in the areas of modeling and control of ac motors.

Mr. Urasaki is a member of the Institute of Electrical Engineers of Japan.

Tomonobu Senjyu (M'02) was born in Saga Prefecture, Japan, in 1963. He received the B.S. and M.S. degrees in electrical engineering from the University of the Ryukyus, Okinawa, Japan, in 1986 and 1988, respectively, and the Ph.D. degree in electrical engineering from Nagoya University, Nagoya, Japan, in 1994.

Currently, he is a Professor with the Department of Electrical and Electronics Engineering, Faculty of Engineering at the University of the Ryukyus, where he has been since 1988. His research interests are in the areas of stability of ac machines, advanced control of electrical machines, and power electronics.

Dr. Senjyu is a member of the Institute of Electrical Engineers of Japan.

Katsumi Uezato was born in Okinawa Prefecture, Japan, in 1940. He received the B.S. degree in electrical engineering from the University of the Ryukyus, Okinawa, Japan, in 1963, the M.S. degree in electrical engineering from Kagoshima University, Kagoshima, Japan, in 1972, and the Ph.D. degree in electrical engineering form Nagoya University, Nagoya, Japan, in 1983.

Currently, he is a Professor with the Department of Electrical and Electronics Engineering, Faculty of Engineering at the University of the Ryukyus, where he has been since 1972. He is engaged in research on stability and control of synchronous machines.

Dr. Uezato is a member of the Institute of Electrical Engineers of Japan.