Transient Stability of the Power System with the Effect of Flux Decay

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Summary

In this paper, the transient stability of the single-machine system with the effect of flux decay is studied. The Lagrange - Charpit method is used for the construction of the Lyapunov function for stability study of the power system. A product - type nonlinearity depending on flux decay is well - defined. The positive definiteness of the obtained Lyapunov function and the negative semi - definiteness of its time derivative are easily verified in the region which satisfy the sector - condition of this nonlinearity. The stability boundaries given by the proposed Lyapunov function are compared with those obtained by conventional Lyapunov function.

Key words : Power system, Stability, Lyapunov method.

List of principal symbols

M = machine inertia in p.u. power second^2 per electrical radian.

D = machine damping coefficient in p.u. power - second per electrical radian.

P_I = mechanical power input, p.u..

P_E = electrical power output, p.u..

E_{ex} = exciter voltage referred to the armature circuit, p.u..

E_B = voltage of infinite bus, p.u..

E'_q = voltage proportional to field flux linkage, p.u..

x_d = direct axis synchronous reactance, p.u..

x'_d = direct axis transient reactance, p.u..

x_{12} = total reactance between the generator terminal and the infinite bus, p.u..

T'_o = open circuit transient time constant, s.

e = value of E'_q at the prefault stable equilibrium point.

δ = angle in radians between E_B and E'_q.

δ_o = value of δ at the prefault stable equilibrium point.

I = generator current.

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I_d = d - component of I.

1. Introduction

The effect of flux decay on the transient stability of a synchronous machine has been investigated by many authors, using the direct method of Lyapunov. Thus, various methods have been used to construct Lyapunov functions. Siddiqee developed a Lyapunov function by trials based on physical consideration[1]. Afterwards some techniques were used in attempts to improve Siddiqee's function. Among them are (a) variable gradient technique[2], (b) Initiating function technique[3] and (c) Kalman's procedure under Desoer-Wu's condition[4]. However, all of the resulting Lyapunov functions by these techniques are equivalent to the energy function developed by Siddiqee. Those techniques could not deal with the product-type nonlinearity well. This difficulty is due to violation of the sector condition of such nonlinearity which exists in generator output.

In a recent paper, the author has presented the Lagrange-Charpit method[5] of constructing Lyapunov functions for power systems. This method determines the arbitrary non-negative function ϕ which allows the Lyapunov function to be determined.

This method is applied to a synchronous machine with the effect of flux decay in this paper. Aforementioned difficulty is overcome through both the Lagrange-Charpit approach and some arrangement of the nonlinearity. Hence, a new Lyapunov function can be derived. Furthermore, the positive definiteness of the obtained Lyapunov function and the negative semi-definiteness of its time derivative can be easily verified in the region which satisfy the sector-condition of this nonlinearity.

A summary of the Lagrange-Charpit method is given in the following section.

2. Lagrange-Charpit method

Consider a nonlinear autonomous system represented by

$$\dot{\boldsymbol{x}} = f(\boldsymbol{x}), \qquad f(\boldsymbol{o}) = \boldsymbol{o} \tag{1}$$

where $\boldsymbol{x}$ is the n-vector, $f(\boldsymbol{x})$ is the n-vector function and $\boldsymbol{x}=\boldsymbol{o}$ is the equilibrium state which is asymptotically stable. A candidate Lyapunov function is obtained by solving

$$F(\boldsymbol{x}, V, \boldsymbol{P}) = \boldsymbol{P}^T f(\boldsymbol{x}) + \phi(\boldsymbol{x}) = 0 \tag{2}$$

where $\boldsymbol{P} = \partial V / \partial \boldsymbol{x}$ and $\phi(\boldsymbol{x})$ is an arbitrary non-negative function whose opposite sign becomes the time derivative of the obtained Lyapunov function.

The characteristic equation for eqn. 2 is given by

$$\frac{dx_1}{\partial F/\partial P_1} = \frac{dx_2}{\partial F/\partial P_2} = \cdots\cdots = \frac{dx_n}{\partial F/\partial P_n}$$

$$= \frac{dv}{P_1 \frac{\partial F}{\partial P_1} + P_2 \frac{\partial F}{\partial P_2} + \dots\dots + P_n \frac{\partial F}{\partial P_n}}$$

$$= \frac{-dP_1}{\partial_1 F + P_1 \partial_V F} = \frac{-dP_2}{\partial_2 F + P_2 \partial_V F} = \dots = \frac{-dP_n}{\partial_n F + P_n \partial_V F} \tag{3}$$

where $\partial_n F = \partial F / \partial x_n$ and $\partial_V F = \partial F / \partial V$ and $\partial_1 F, \partial_2 F, \dots, \partial_n F$ include $\partial_1 \phi, \partial_2 \phi, \dots, \partial_n \phi$, respectively.

If we can obtain equations containing at least one component of $\boldsymbol{P}$ from eqn. 3, such that

$$\left.\begin{array}{l} G_1 \ (\boldsymbol{x}, V, \boldsymbol{P}, \partial \phi / \partial \boldsymbol{x}) = 0 \\ G_2 \ (\boldsymbol{x}, V, \boldsymbol{P}, \partial \phi / \partial \boldsymbol{x}) = 0 \\ \dots\dots\dots\dots\dots\dots\dots\dots\dots\dots \\ G_{n-1} \ (\boldsymbol{x}, V, \boldsymbol{P}, \partial \phi / \partial \boldsymbol{x}) = 0 \end{array}\right\} \tag{4}$$

then, in order for F and $\boldsymbol{G}$ to have a common solution, these functions must satisfy the conditions

$$[G_i, F] = \sum_{k=1}^{n} \left(\frac{dG_i}{dx_k} \frac{\partial F}{\partial P_k} - \frac{dF}{dx_k} \frac{\partial G_i}{\partial P_k} \right) = 0 \tag{5}$$

where $i = 1, 2, \dots, n-1$.

If $[G_i, F]$ still give partial differential equations including $\boldsymbol{P}$, they must also have a common solution with F. Hence, we let $[G_i, F] = G$ once again. These manipulations are continued until the 2nd-order partial differential equations for ϕ are given, because we would like ϕ to be in a quadratic form so that its positive semi-definiteness can be easily verified. Thus the unknown $\partial \phi / \partial \boldsymbol{x}$ and ϕ are determined from the conditions

$$\left.\begin{array}{l} [G_r, G_s] = \sum_{k=1}^{n} \left(\frac{dG_r}{dx_k} \frac{\partial G_s}{\partial P_k} - \frac{dG_s}{dx_k} \frac{\partial G_r}{\partial P_k} \right) = 0 \\ [G_t, F] = \sum_{k=1}^{n} \left(\frac{dG_t}{dx_k} \frac{\partial F}{\partial P_k} - \frac{dF}{dx_k} \frac{\partial G_t}{\partial P_k} \right) = 0 \end{array}\right\} \tag{6}$$

where $r, s = 1, 2, \dots, \max [t]$, $t > n-1$, $r \neq s$ and

$$\frac{dG_r}{dx_k} = \frac{\partial G_r}{\partial x_k} + P_k \frac{\partial G_r}{\partial V} \tag{7}$$

From the conditions 6, $\partial \phi / \partial \boldsymbol{x}$ and ϕ can be expressed by using $\boldsymbol{x}$.

If eqns. 2 and 4 can be solved so as to give $\boldsymbol{P}$ as the function of $\boldsymbol{x}$ and V as

$$\boldsymbol{P} = \boldsymbol{P} \ (\boldsymbol{x}, V) \tag{8}$$

the Pfaffian differential equation

$$\boldsymbol{P}^T dx = (\nabla V)^T d\boldsymbol{x} \tag{9}$$

is integrable. Hence, a possible Lyapunov function and its time derivative are

$$\left.\begin{aligned} V(\boldsymbol{x}) &= \int_O^x \boldsymbol{P}^T d\boldsymbol{x} \\ -\dot{V}(\boldsymbol{x}) &= \phi(\boldsymbol{x}) \end{aligned}\right\} \tag{10}$$

3. Formulation of the problem

Neglecting resistance and transient saliency, the dynamic equations of a single-machine connected to an infinite-bus, taking into consideration of the flux decay are given by

$$\left.\begin{aligned} M\frac{d^2\delta}{dt^2} + D\frac{d\delta}{dt} &= P_1 - P_E \\ \frac{dE'_q}{dt} &= \frac{1}{T'_o}[E_{ex} - (x_d - x'_d) I_d - E'_q] \end{aligned}\right\} \tag{11}$$

where

$$P_E = \frac{E'_q E_B}{x_{12} + x'_d} \sin\delta, \qquad I_d = \frac{E'_q - E_B \cos\delta}{x_{12} + x'_d}$$

Defining state variable as

$$\left.\begin{aligned} x_1 &= \delta - \delta_0 \\ x_2 &= \dot{\delta} \\ x_3 &= E'_q - e \end{aligned}\right\} \tag{12}$$

we can represent the system as follows:

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \end{bmatrix} = \begin{bmatrix} f_1(\boldsymbol{x}) \\ f_2(\boldsymbol{x}) \\ f_3(\boldsymbol{x}) \end{bmatrix} = \begin{bmatrix} x_2 \\ -[Dx_2 + K_1(x_3 + e)\sin(x_1 + \delta_o) - P_I]/M \\ -\eta_1 x_3 - \eta_2 g(x_1) \end{bmatrix} \tag{13}$$

where

$$g(x_1) = \cos\delta_o - \cos(x_1 + \delta_o)$$

$$\eta_1 = \frac{x_{12} + x_d}{(x_{12} + x'_d)T'_o}, \qquad \eta_2 = \frac{(x_d - x'_d)E_B}{(x_{12} + x'_d)T'_o}$$

$$K_1 = \frac{E_B}{x_{12} + x'_d}$$

4. Construction of Lyapunov function

A scalar function V is obtained as a function which satisfies the linear partial differential equation given by eqn. 2. The characteristic equation then becomes

$$\frac{dx_1}{f_1(\boldsymbol{x})}=\frac{dx_2}{f_2(\boldsymbol{x})}=\frac{dx_3}{f_3(\boldsymbol{x})}=\frac{dV}{P_1f_1(\boldsymbol{x})+P_2f_2(\boldsymbol{x})+P_3f_3(\boldsymbol{x})}$$
$$=\frac{-dP_1}{\partial_1 F}=\frac{-dP_2}{\partial_2 F}=\frac{-dP_3}{\partial_3 F} \tag{14}$$

From the above equation, we obtain

$$\left.\begin{aligned}
&\frac{\alpha dx_1+\beta dx_2}{\alpha x_2-\beta[Dx_2+K_1(x_3+e)\ sin(x_1+\delta_o)-P_l]/M}\\
&=\frac{dP_2}{-P_1+(D/M)P_2-\partial\phi/\partial x_2}\\
&\frac{r(x_1)dx_1+\xi dx_3}{r(x_1)x_2-\xi(\eta_1x_3+\eta_2g(x_1))}\\
&=\frac{dP_3}{P_2(K_1/M)\ sin\ (x_1+\delta_o)+P_3\eta_1-\partial\phi/\partial x_3}
\end{aligned}\right\} \tag{15}$$

where α, β, ξ are arbitrary constants and $\gamma(x_1)$ is an arbitrary function. Integrating the numerators, we obtain

$$\left.\begin{aligned}
G_1&=\alpha x_1+\beta x_2-P_2=0\\
G_2&=R(x_1)+\xi x_3-P_3=0
\end{aligned}\right\} \tag{16}$$

where

$$R(x_1)=\int_O^{x_1}\gamma(x_1)dx_1$$

and then the following two equations must be satisfied;

$$\left.\begin{aligned}
G_3&=\alpha x_2-\beta[Dx_2+K_1(x_3+e)\ sin\ (x_1+\delta_o)-P_l]/M\\
&\quad+P_1-(D/M)P_2+\partial\phi/\partial x_2=0\\
G_4&=r(x_1)x_2-\xi[\eta_1x_3+\eta_2g(x_1)]\\
&\quad-P_2(K_1/M)\ sin(x_1+\delta_o)-P_3\eta_1+\partial\phi/\partial x_3=0
\end{aligned}\right\} \tag{17}$$

where G_3 and G_4 coincide with $[G_1, F]$ and $[G_2, F]$, respectively.

Then the application of eqn. 6 to $\{G_i,\ i=1-4\}$ and F gives

$$\begin{aligned}
[G_1, G_2]&=0\\
[G_1, G_3]&=2\{\alpha-\beta(D/M)\}+\partial^2\phi/\partial x_2^2=0\\
[G_1, G_4]&=\gamma(x_1)-\beta(K_1/M)sin(x_1+\delta_0)+\partial^2\phi/\partial x_3\partial x_2=0
\end{aligned}$$

$$\left.\begin{aligned}
&[G_2, G_3]=\gamma(x_1)-\beta(K_1/M)\ sin(x_1+\delta_o)+\partial^2\phi/\partial x_2\partial x_3=0\\
&[G_2, G_4]=2\xi\eta_1-\partial^2\phi/\partial x_3^2=0\\
&[G_3, C_4]=\xi\eta_2\ sin(x_1+\delta_0)-(d\gamma(x_1)/dx_1)x_2\\
&\qquad +\alpha(K_1/M)\ sin(x_1+\delta_o)\\
&\qquad +\alpha(K_1/M)x_1\ cos(x_1+\delta_o)\\
&\qquad +\beta(K_1/M)x_2\ cos(x_1+\delta_o)+\eta_1\gamma(x_1)\\
&\qquad -\partial^2\phi/\partial x_3\partial x_1=0\\
&\quad =\alpha(K_1/M)x_1(x_3+e)cos(x_1+\delta_o)\\
&\qquad +\eta_2 R(x_1)\ sin(x_1+\delta_o)\\
&\qquad +\xi\eta_2 x_3\ sin(x_1+\delta_o)\\
&\qquad +\alpha\{K_1(x_3+e)\ sin(x_1+\delta_o)-P_1\}/M\\
&\qquad +(\eta_1 x_3+\eta_2 g(x_1))\gamma(x_1)\\
&\qquad +(\partial^2\phi/\partial x_2\partial x_1)x_2-\partial\phi/\partial x_1=0\\
&[G_4, F]=0
\end{aligned}\right\}$$

Using the above conditions, we have

$$\begin{aligned}
\phi=&\ \xi\eta_1 x_3^2+x_3[\{\beta(K_1/M)\ sin(x_1+\delta_o)-\gamma(x_1)\}x_2\\
&+\xi\eta_2 g(x_1)+\eta_1 R(x_1)+\alpha(K_1/M)x_1\ sin(x_1+\delta_o)]\\
&+\{\beta(D/M)-\alpha\}x_2^2+\Phi(x_1)x_2+\eta_2 R(x_1)g(x_1)\\
&+\alpha x_1\{K_1 e\ sin(x_1+\delta_o)-P_l\}/M
\end{aligned}\tag{19}$$

where $\Phi(x_1)$ is an arbitrary function.

In order to establish the positive semi-definiteness of ϕ, we choose

$$\left.\begin{aligned}
&\xi=(\beta K_1\eta_1)/M\eta_2)\\
&\gamma(x_1)=\beta(K_1/M)\ sin(x_1+\delta_o)\\
&\Phi(x_1)=-2\sqrt{\alpha\{\beta(D/M)-\alpha\}\ x_1 L(x_1)}
\end{aligned}\right\}\tag{20}$$

where

$$0\le\alpha\le(D/M)$$

$$\begin{aligned}
L(x_1)=&\frac{1}{M}[K_1\{e-\frac{\eta_2}{\eta_1}g(x_1)-\frac{\alpha\eta_2}{4\beta\eta_1^2}x_1\ sin(x_1+\delta_o)\}*\\
&*sin(x_1+\delta_o)-P_l]
\end{aligned}$$

Let α and β be replaced by $\alpha'D$ and 1, respectively, then ϕ becomes

$$\begin{aligned}
\phi=&\frac{K_1}{M}[\sqrt{\frac{1}{\eta_2}}\{\eta_1 x_3+\eta_2 g(x_1)\}+\frac{\alpha' D}{2\eta_1 M}\sqrt{\eta_2}\ x_1\ sin(x_1+\delta_0)]^2\\
&+\frac{D}{M}[\sqrt{1-\alpha'}\,x_2-\sqrt{\alpha'\ x_1 L(x_1)}]^2
\end{aligned}\tag{21}$$

Solving eqns. 16, 17 for P_1, P_2 and P_3, we obtain

$$\left.\begin{aligned} P_1 &= \alpha'(D/M)x_2+\alpha'(D/M)^2+\{K_1(x_3+e)\sin(x_1+\delta_o)-P_l\}/M \\ &\quad +2(D/M)\sqrt{\alpha'(1-\alpha')x_1L(x_1)} \\ P_2 &= \alpha'(D/M)x_1+x_2 \\ P_3 &= (K_1/M)g(x_1)\{(K_1\eta_1)/(M\eta_2)\}x_3 \end{aligned}\right\} \quad (22)$$

Hence, the scalar function V is given by eqn. 10 as

$$\begin{aligned} V(x) &= \frac{1}{2}(x_2+\alpha'\frac{D}{M}x_1)^2+\frac{K_1}{2M}\left\{\sqrt{\frac{\eta_2}{\eta_1}}g(x_1)+\sqrt{\frac{\eta_1}{\eta_2}}x_3\right\}^2 \\ &\quad +\frac{1}{2}\alpha'(1-\alpha')(\frac{D}{M})^2x_1^2+2\frac{D}{M}\sqrt{\alpha'(1-\alpha')}\int_o^{x_1}\sqrt{x_1L(x_1)dx_1} \\ &\quad +\frac{1}{M}[K_1\{e-\frac{\eta_2}{2\eta_1}g(x_1)\}g(x_1)-P_lx_1] \end{aligned} \quad (23)$$

To inspect the definiteness of V, we rewrite the right-hand side of eqn. 23, except the first and second terms, such that

$$V_1=\int_o^x H(x_1)\,dx_1 \quad (24)$$

where

$$\begin{aligned} H(x_1) &= (D/M)^2\alpha'(1-\alpha')x_1+L(x_1) \\ &\quad +2(D/M)\sqrt{\alpha'(1-\alpha')}\sqrt{x_1L(x_1)} \\ &\quad +\alpha'(K_1D\eta_2/4M^2\eta_1^2)x_1\sin^2(x_1+\delta_o) \end{aligned}$$

As we can write

$$H(x_1)=\begin{cases} [(D/M)\sqrt{\alpha'(1-\alpha')}\sqrt{x_1}+\sqrt{L(x_1)}]^2 \\ \qquad +\dfrac{K_1\alpha'D\eta_2}{4M^2\eta_1^2}x_1\sin^2(x_1+\delta_o) \\ \qquad\qquad (x_1\geq 0) \\ -[(D/M)\sqrt{\alpha'(1-\alpha')}\sqrt{-x_1}-\sqrt{-L(x_1)}]^2 \\ \qquad +\dfrac{K_1\alpha'D\eta_2}{4M^2\eta_1^2}x_1\sin^2(x_1+\delta_o) \\ \qquad\qquad (x_1<0) \end{cases} \quad (25)$$

the sector-condition $x_1 H(x_1)\geq 0$ can be verified in the neighbourhood of the origin. Then V_1 *is* a non-negative function around the origin. Hence, V given by eqn. 23 is a positive definite function.

If we choose $\alpha'=0$ in eqn. 23, the Lyapunov function becomes

$$V(x) = \frac{1}{2}x_2^2 + \frac{K}{2M}\left\{\sqrt{\frac{\eta_2}{\eta_1}}\, g(x_1) + \sqrt{\frac{\eta_1}{\eta_2}}\, x_3\right\}^2$$
$$+ \frac{1}{M}\left[K_1\left\{e - \frac{\eta_2}{2\eta_1}\, g(x_1)\right\} g(x_1) - P_f x_1\right] \tag{26}$$

which is the equivalent function given by references 1, 3, 4 and 5.

The stability boundaries are drawn for the system constants given in Table 1. The asymptotic stability region C_{max} is determined according to Willem[6]. Thus we obtain

$$\left.\begin{array}{ll} C_{max} = 0.683 & (\alpha' = 0) \\ C_{max} = 1.003 & (\alpha' = 0.25) \\ C_{max} = 0.757 & (\alpha' = 1) \end{array}\right\} \tag{27}$$

Table 1

$x_d = 1.15$	$x'_d = 0.3$	$E_B = 1.0$
$e = 1.03$	$M = 1$	$D = 0.5$
$T_o' = 6.6$	$\delta_0 = 0.316$	$x_{12} = 0.1$

Figs. 1-3 show the cross-sections of the stability surfaces. The superiorities of the stability regions proposed in this paper (α' =0.25) are obvious from these figures. In the present paper, optimal value of α' was determined by inspection. However, it is difficult to determine the optimal values of α' by inspection for multi-machine system. This problem will be studied further in the future.

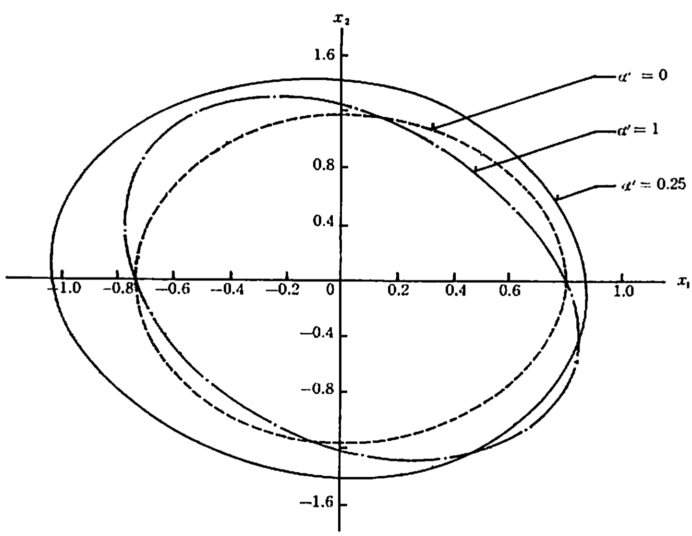

Fig. 1: Cross-sections of stability surfaces in the plane $x_3 = 0$

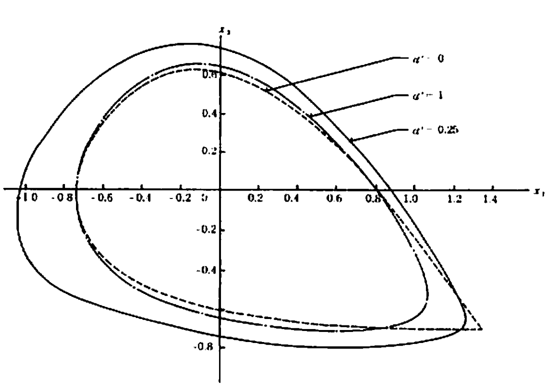

Fig. 2: Cross-sections of stability surfaces in the plane $x_2 = 0$

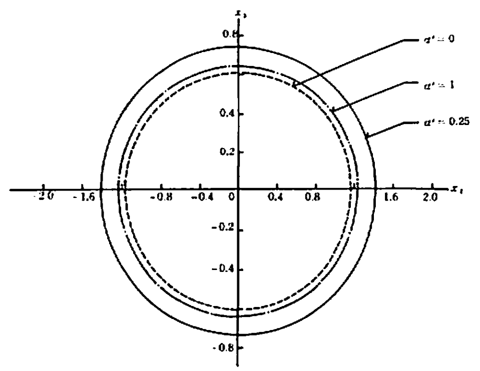

Fig. 3: Cross-sections of stability surfaces in the plane $x_1 = 0$

5. Conclusion

The transient stability of the single-machine system with the effect of flux decay has been studied. The Lagrange-Charpit method is applied to construct Lyapunov function. The conventional energy function is a special case of the Lyapunov function proposed in this paper.

Numerical examples show that the proposed Lyapunov function gives better estimation of stability boundary than those currently available.

The application to the system with effects of flux decay and AVR action will be discussed on another occasion.

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