Derivation of the Equivalent Circuit of a Multi-Terminal Network Given by Generalization of Helmholtz-Thevenin's Theorem

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Abstract

The revision of generalized Helmholtz-Thevenin's theorem in the previous paper [M. Hosoya: Bull. Faculty of Science, Univ. of the Ryukyus, 83 (2007) 1-2] is unnecessary. The Helmholtz-Thevenin's equivalent circuit (HTC) shown in the original paper[M. Hosoya: Bull. Faculty of Science, Univ. of the Ryukyus, 71 (2001) 39-45] is valid. A new and general method to derive HTC is given.

Conclusion

Any n-terminal network can be equivalently transformed into the Helmholtz-Thevenin's circuit(HTC) which is shown in the former paper[1]. For example, a five-terminal network can be always transformed into the one in Fig. 1.

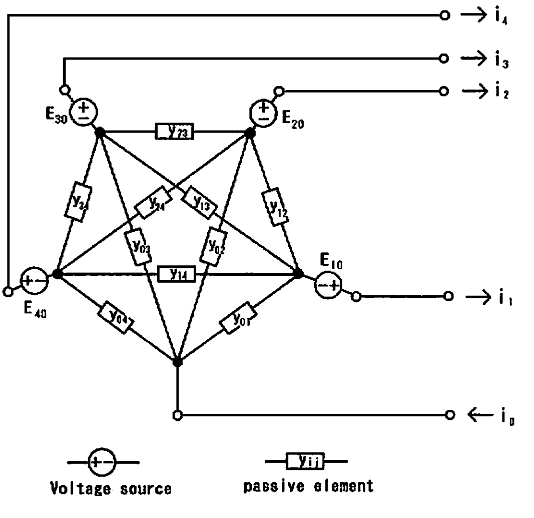

Fig.1 The Helmholtz-Thevenin's equivalent circuit of a five-terminal network.

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The transformation is possible because the following equation is generally obtained with ordinary procedure which is based on Kirchhoff's first and second laws.

$$\begin{bmatrix} A_{11} & A_{12} & A_{13} & A_{14} & A_{15} & A_{16} & A_{17} & A_{18} \\ A_{21} & A_{22} & A_{23} & A_{24} & A_{25} & A_{26} & A_{27} & A_{28} \\ A_{31} & A_{32} & A_{33} & A_{34} & A_{35} & A_{36} & A_{37} & A_{38} \\ A_{41} & A_{42} & A_{43} & A_{44} & A_{45} & A_{46} & A_{47} & A_{48} \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \\ V_3 \\ V_4 \\ I_1 \\ I_2 \\ I_3 \\ I_4 \end{bmatrix} = \begin{bmatrix} B_1 \\ B_2 \\ B_3 \\ B_4 \end{bmatrix} \tag{1}$$

Here Vectors $\{V_i\}$ and $\{I_i\}$ represent the terminal Voltage and current respectively, while matrix $\{A_{ij}\}$ and vector $\{B_i\}$ constitute of the constant values. We note that {Bi} has its origin in the sources of circuits. We can change Eq.1 into the following form using Gauss-Jordan elimination.

$$\begin{bmatrix} 1 & 0 & 0 & 0 & Z_{11} & Z_{12} & Z_{13} & Z_{14} \\ 0 & 1 & 0 & 0 & Z_{21} & Z_{22} & Z_{23} & Z_{24} \\ 0 & 0 & 1 & 0 & Z_{31} & Z_{32} & Z_{33} & Z_{34} \\ 0 & 0 & 0 & 1 & Z_{41} & Z_{42} & Z_{43} & Z_{44} \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \\ V_3 \\ V_4 \\ I_1 \\ I_2 \\ I_3 \\ I_4 \end{bmatrix} = \begin{bmatrix} V_{10} \\ V_{20} \\ V_{30} \\ V_{40} \end{bmatrix} \tag{2}$$

Here matrix $\{Z_{ij}\}$ and vector $\{V_{io}\}$ are composed of the constant values. The equation may be more comprehensible if we rewrite it as follows.

$$\begin{bmatrix} V_1 \\ V_2 \\ V_3 \\ V_4 \end{bmatrix} + \begin{bmatrix} Z_{11} & Z_{12} & Z_{13} & Z_{14} \\ Z_{21} & Z_{22} & Z_{23} & Z_{24} \\ Z_{31} & Z_{32} & Z_{33} & Z_{34} \\ Z_{41} & Z_{42} & Z_{43} & Z_{44} \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \\ I_3 \\ I_4 \end{bmatrix} = \begin{bmatrix} V_{10} \\ V_{20} \\ V_{30} \\ V_{40} \end{bmatrix} \tag{3}$$

Each component of vevtor $\{V_{io}\}$ corresponds to each Voltage-source in Fig. 1 one by one.

$$E_{io} = V_{io} \tag{4}$$

Matrix $\{Z_{ij}\}$ is regarded to be an impedance matrix. We can also get the following equivalent form of Eq.1 if we choose $\{I_i\}$ as pivots in Gauss-Jordan elimination.

$$\begin{bmatrix} I_1 \\ I_2 \\ I_3 \\ I_4 \end{bmatrix} + \begin{bmatrix} Y_{11} & Y_{12} & Y_{13} & Y_{14} \\ Y_{21} & Y_{22} & Y_{23} & Y_{24} \\ Y_{31} & Y_{32} & Y_{33} & Y_{34} \\ Y_{41} & Y_{42} & Y_{43} & Y_{44} \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \\ V_3 \\ V_4 \end{bmatrix} = \begin{bmatrix} I_{10} \\ I_{20} \\ I_{30} \\ I_{40} \end{bmatrix} \tag{5}$$

Here matrix $\{Y_{ij}\}$ is an admittance matrix and vector $\{I_{i0}\}$ corresponds to the current source in the equivalent circuit of Mayer-Norton's type. Components of the admittance matrix $\{Y_{ij}\}$ have almost one-to-one correspondence to the passive elements in Fig.1.

$$y_{ij} = -Y_{ij} \qquad (i \neq j)$$
$$y_{0i} = \sum_{j=1}^{4} Y_{ij} \tag{6}$$

Thus all the values which appear in Fig. 1 are given by Eq. (3) and (5). Such situation does not depend on the number of terminal. So we obtained the generalized result about the most convenient equivalent circuit of multi-terminal network.

Consequently the HTC in the original paper[1] is correct, so its revision in the previous paper[2] was unnecessary. However the HTC can not be gained from its MNC(Mayer-Norton's equivalent circuit) with Blakesley transformation[3]. Hence "proof" in the paper (2) based on Fig.5 is wrong.

Reference

1) M. Hosoya, "The Straightforward Expansion of Helmholtz-Thevenin Theorem to Multi-Terminal Networks", Bulletin of the Faculty of Science, University of the Ryukyus No. 71, pp. 39-45, 2001.

2) M. Hosoya, "Revised Helmholtz-Thevenin's Theorem to Multi-Terminal Networks", Bulletin of the Faculty of Science, University of the Ryukyus No. 83, pp.1-2, 2007.

3) T. H. Blakesley, A new Electrical Theorem, Phil. Mag., vol. 37, 448-450, 1894.