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	作成者: Kato, Mitsuo, 加藤, 満生
	メールアドレス:
	所属:
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The b-function of μ-constant deformation of x^9+y^4

Mitsuo KATO*

Introduction

The Milner number μ of the singularity $x^9+y^4=0$ is 24, and the μ-constant deformation is given by

(0.1) $F=-(x^9/9+y^4/4)+t_1x^7y+t_2x^5y^2+t_6x^6y^2+t_{10}x^7y^2=0.$

For fixed t, the b-function of $F=0$ is of the following form :

$b_F(s)=(s+1)\tilde{b}_F(s),$

$\tilde{b}_F(s)=\prod_{0\leq p\leq 7,\ 0\leq q\leq 2}(s+(p+1)/9+(q+1)/4-e_{p,q}),$

where $e_{p,q}$ are functions of t and the values are 0 and 1.

The parameter space is stratified into 7 strata, by the condition that on each stratum $e_{p,q}$ are constant.

The stratification is as follows.

I. $t_1\neq 0,\ t_2+3t_1^2=0,$

$$e_{p,q}=\begin{cases}1 & \text{if } (p,q)=(7,1),(6,2),(7,2),\\ 0 & \text{otherwise.}\end{cases}$$

II. $t_1\neq 0,\ 2t_2+7t_1^2=0,\ 48t_6+2429t_1^6=0$

$$e_{p,q}=\begin{cases}1 & \text{if } (p,q)=(7,1),(5,2),(7,2)\\ 0 & \text{otherwise}\end{cases}$$

III. $t_1\neq 0$, *not either* I *or* II

$$e_{p,q}=\begin{cases}1 & \text{if } (p,q)=(7,1),(5,2),(6,2),(7,2)\\ 0 & \text{otherwise}\end{cases}$$

IV. $t_1=0,\ t_2\neq 0$

$$e_{p,q}=\begin{cases}1 & \text{if } (p,q)=(5,2),(6,2),(7,2)\\ 0 & \text{otherwise}\end{cases}$$

V. $t_1=t_2=0,\ t_6\neq 0$

$$e_{p,q}=\begin{cases}1 & \text{if } (p,q)=(6,2),(7,2)\\ 0 & \text{otherwise}\end{cases}$$

VI. $t_1=t_2=t_6=0,\ t_{10}\neq 0$

$$e_{p,q}=\begin{cases}1 & \text{if } (p,q)=(7,2)\\ 0 & \text{otherwise}\end{cases}$$

VII. $t_1=t_2=t_6=t_{10}=0$

$e_{p,q}=0$ *for all* (p,q).

III is divided into two strata :

III_1 $t_1\neq 0,\ t_2+3t_1^2\neq 0,\ 2t_2+7t_1^2\neq 0$ *then* $L(F)=3,$

III_2 $t_1\neq 0,\ 2t_2+7t_1^2=0,\ 48t_6+2429t_1^6\neq 0$ *then* $L(F)=4.$

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* Dept. of Math., Univ. of the Ryukyus.

The notations and the idea of the proofs are the same as those in the previous paper [1] .

§1.

1.1. If $t_1=0$ then by 1.5 of [1] , IV~VII follows.

From now on we assume $t_1 \neq 0$.

1.2. $A_m=\{f \in O: d(f) \geqq m/36\}$, $d(x)=1/9$, $d(y)=1/4$.

$A_{36}=Cx^5y^2+Cx^6y^2+(F, F_x, F_y)O \cap A_{36}$.

$A_{72}=Cx^{14}y^2+(F_x, F_y)(F, F_x, F_y)O \cap A_{72}$.

$A_{108}=(F_x, F_y)^2(F, F_x, F_y)O \cap A_{108}$.

Put

(1.2.1) $X_0=xD_x/9+yD_y/4$, $P_1(s)=36(X_0-s)$.

Then we have

(1.2.2) $36x(X_0F-F)+t_1yF_x+5t_1t_2x^4F_y+5t_1^2t_2x^3F_x$
$+(10t_1t_2^2+35t_1^3t_2)xyF_x+...$
$=(2t_2+7t_1^2)x^6y^2$.

Hence if $2t_2+7t_1^2 \neq 0$ then

$A_{36}=Cx^5y^2+(F, F_x, F_y)O \cap A_{36}$,

$A_{72}=(F_x, F_y)(F, F_x, F_y)O \cap A_{72}$.

1.3. By 1.2, the following five elements of $J_F(s)$ are easily found :

(1.3.1) $P_{11}(s)=x^2P_1(s)+t_1xyD_x-t_1^{-1}(2t_2+7t_1^2)yP_1(s)$
$+(5t_1t_2-2t_1^{-1}t_2(2t_2+7t_1^2))(x^5D_y+t_1x^4D_x$
$+(2t_2+7t_1^2)x^2yD_x)+...$,

(1.3.2) $P_{12}(s)=xyP_1(s)+t_1y^2D_x+(2t_2+7t_1^2)x^6D_y$
$+t_1(2t_2+7t_1^2)x^5D_x+5t_1t_2x^4yD_y+(5t_1^2t_2+(2t_2+7t_1^2)^2)x^3yD_x$
$+t_1(2t_2+7t_1^2)(24t_2+49t_1^2)xy^2D_x+...$,

(1.3.3) $P_{13}(s)=y^2P_1(s)+t_1x^7D_y+...$,

(1.3.4) $P_{21}(s)=y(P_1(s)-1)P_1(s)-t_1^2x^6D_xD_y+...$,

(1.3.5) $P_4(s)=(P_1(s)-6)(P_1(s)-2)(P_1(s)-1)P_1(s) -t_1^4x^4yD_x^3D_y+...$.

1.4. Let α be a root of $\tilde{b}_F(s)$, and $Q\delta(Q \in C[D_x, D_y])$ be the corresponding element of B_{pt} with the properties (1.3.1), (1.3.2) of [1] .

By $F_xQ\delta=F_yQ\delta=0$ we have

(1.4.1) $Q^*=cD_x^pD_y^q$, $0 \leqq p \leqq 7$, $0 \leqq q \leqq 2$

where c is a nonzero constant and Q^* denotes the homogeneous part of Q of the lowest degree.

1.5. $(p, q) \neq (7, 1), (7, 2)$.

Proof. Use the equality $(X_0F-F)Q\delta=0$.

1.6. If $(p, q)=(5, 2)$ then $t_2+3t_1^2=0$.

Proof. Put $Q=D_x^5D_y^2+at_1D_x^7D_y+...$.

$P_{11}(\alpha)Q\delta=0$ implies $\alpha=-(6/9+3/4)$, $a=-2/7$, and $(X_0F-F)Q\delta=0$ implies $t_2+3t_1^2=0$.

1.7. If $(p, q)=(6, 2)$ then $2t_2+7t_1^2=0$, $48t_6+2429t_1^6=0$.

Proof. Put $Q=D_x^6D_y^2+a_1D_x^8D_y+a_2D_xD_y^4+b_2D_x^{10}$
$+a_3D_x^3D_y^3+a_4D_x^5D_y^2+a_5D_x^7D_y+\ldots$.
$F_xQ\delta=P_{11}(\alpha)Q\delta=0$ determines α, a_1, a_2, b_2, a_3, a_4, a_5, and $(X_0F-F)Q\delta=0$ implies the desired equalities.

§2. $2t_2+7t_1^2\neq 0$.

2.1. By 1.2, there exists an equation

(2.1.1) $((P_1(s)-1)P_1(s)+t_1^2(2t_2+7t_1^2)^{-1}(xP_1(s-1)D_x$
$+t_1yD_x^2+P_1(s)-32xD_x)+\ldots)F^s$
$=2(t_2+3t_1^2)x^5y^2sF^{s-1}$.

2.2. $t_2+3t_1^2=0$.

By (2.1.1), there exists an element of $J_F(s)$:

(2.2.1) $P_2(s)=(P_1(s)-1)P_1(s)+t_1^2(2t_2+7t_1^2)^{-1}(xP_1(s)D_x$
$+P_1(s)+4xD_x)+$ *higher terms.*

Let α be a root of $\tilde{b}_F(s)$, and $Q\delta$, $Q^*=D_x^pD_y^q$ be as in 1.4.

By 1.5, 1.7, $(p, q)\neq(7, 1), (6,2), (7,2)$.

For (p, q) with $0\leqq p+q\leqq 1$, by $P_2(\alpha)Q\delta=0$, α has at most two possibilities, that is

for $(p, q)=(0, 0), (0, 1)$, $\alpha=-((p+1)/9+(q+1)/4+k/36)$, $k=0, 1$,
for $(p, q)=(1, 0)$, $\alpha=-(2/9+1/4+k/36)$, $k=0, 2$.

For (p, q) with $p+q\geqq 2$, by $P_{11}(\alpha)Q\delta=P_{12}(\alpha)Q\delta=P_{13}(\alpha)Q\delta=0$, α has only one possibility : $\alpha=-((p+1)/9+(q+1)/4)$.

Since $\tilde{b}_F(s)$ has distinct 24 roots, the above possibilities of α really give all the roots.

This proves the case I of Introduction.

2.3. $2t_2+7t_1^2\neq 0$, $t_2+3t_1^2\neq 0$.

Let α, Q, $Q^*=D_x^pD_y^q$ be as above.

By 1.5, 1.6, 1.7, $(p, q)\neq(7, 1), (5, 2), (6, 2), (7, 2)$.

By 1.2, there exists an element of $J_F(s)$:

(2.3.1) $P_3(s)=(P_1(s)-2)(P_1(s)-1)P_1(s)+$ *higher terms.*

By (2.1.1), there exists an element of $J_F(s)$:

(2.3.2) $P_{22}(s)=x((P_1(s)-1)P_1(s)+t_1^2(2t_2+7t_1^2)^{-1}(xP_1(s)D_x$
$+P_1(s)+4xD_x))-2(t_2+3t_1^2)(2t_2+7t_1^2)^{-1}xP_1(s)$
$+$ *higher terms.*

If $(p, q)=(0, 0)$ then, by $P_3(\alpha)Q\delta=0$, α has at most three possibilities :
$\alpha=-(1/9+1/4+k/36)$, $k=0, 1, 2$.

If $(p, q)=(1, 0)$ then, by $P_{22}(\alpha)Q\delta=0$, α has at most two possibilities :
$\alpha=-(2/9+1/4+k/36)$, $k=0, 2$.

If $(p, q)=(0, 1)$ then, by $P_{21}(\alpha)Q\delta=0$, α has at most two possibilities :
$\alpha=-(1/9+2/4+k/36)$, $k=0, 1$.

If $p+q\geqq 2$ then, by $P_{11}(\alpha)Q\delta=P_{12}(\alpha)Q\delta=P_{13}(\alpha)Q\delta=0$, α has only one possibility : $\alpha=-((p+1)/9+(q+1)/4)$.

All of these possibilities give all the roots of $\tilde{b}_F(s)$.

This proves the case III_1 of Introduction.

§3. $2t_2+7t_1^2=0$.

3.1.

By (1.2.2), we have an element of $J_F(s)$:

(3.1.1) $P_{14}(s)=xP_1(s)+t_1yD_x+\textit{higher terms}$.

By a routine computation we get an equation :

(3.1.2) $((P_1(s)-2)(P_1(s)-1)P_1(s)+\textit{higher terms})F^s$
$$=(5/2)(48t_6+2429t_1^6)x^6y^2sF^{s-1}.$$

3.2. $2t_2+7t_1^2=0,\ 48t_6+2429t_1^6=0$.

Let α, Q, $Q^*=D_x^pD_y^q$ be as in 1.4.

By 1.5, 1.6, $(p, q)\neq(7, 1), (5, 2), (7, 2)$.

By (3.1.2), we have an element of $J_F(s)$:

(3.2.1) $P_{31}(s)=(P_1(s)-2)(P_1(s)-1)P_1(s)+\textit{higher terms}$.

If $(p, q)=(0, 0)$ then, by $P_{31}(\alpha)Q\delta=0$, α has at most three possibilities :
$$\alpha=-(1/9+1/4+k/36),\ k=0,\ 1,\ 2.$$

If $(p, q)=(0, 1)$ then, by $P_{21}(\alpha)Q\delta=0$, α has at most two possibilities :
$$\alpha=-(1/9+2/4+k/36),\ k=0,\ 1.$$

If $p\geqq 1$ or $q\geqq 2$ then, by $P_{13}(\alpha)Q\delta=P_{14}(\alpha)Q\delta=0$, α has only one possibility :
$$\alpha=-((p+1)/9+(q+1)/4).$$

All these possibilities of α give all the roots of $\tilde{b}_F(s)$.

This proves the case II of Introduction.

3.2. $2t_2+7t_1^2=0,\ 48t_6+2429t_1^6\neq 0$.

By 1.5, 1.6, 1.7, $(p, q)\neq(7, 1), (5, 2), (6, 2), (7, 2)$.

If $(p, q)=(0, 0)$ then, by $P_4(\alpha)Q\delta=0$, α has at most four possibilities :
$$\alpha=-(1/9+1/4+k/36),\ k=0,\ 1,\ 2,\ 6.$$

If $(p, q)=(0, 1)$ then, by $P_{21}(\alpha)Q\delta=0$, α has at most two possibilities :
$$\alpha=-(1/9+2/4+k/36),\ k=0,\ 1.$$

If $p\geqq 1$ or $q\geqq 2$ then, by $P_{13}(\alpha)Q\delta=P_{14}(\alpha)Q\delta=0$, α has only one possibility :
$$\alpha=-((p+1)/9+(q+1)/4).$$

All these possibilities of α give all the roots of $\tilde{b}_F(s)$, and this proves the case III_2 of Introduction.

REFERRENCES

[1] M. Kato : The b function of a μ-constant deformation of x^7+y^5, Bulletin of the College of Science University of the Ryukyus, No. 32 (1981).