Fluctuations in spacetime and localization of the wave function

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Abstract. We discuss an uncertainty in spacetime of the order of the Planck length. It can be shown that the effect of this uncertainty is negligible for microscopic particles moving in spacetime. However, the situation changes drastically for macroscopic objects, and for which this uncertainty causes localization of the wave function.

1. Introduction

The subject of the title has attracted some attention recently [1-4]. Nevertheless several points remain to be clarified. In this article we approach the subject from a simple viewpoint, and develop this in a detailed investigation.

We consider a flat Minkowskian spacetime, and take the attitude that the coordinate systems of our spacetime cannot be independent of the theory of matter. The coordinate systems of our spacetime must be defined in terms relevant to the physical objects which inhabit the universe. Our first objective is to show that the coordinate axes of such a system must be seen to fluctuate when viewed from another coordinate system.

We assume that a light clock is used in order to construct the x^0-axis of an inertial coordinate system (x^μ). That is to say, a photon runs between two particles A and B, and the x^0-axis is composed of the events (say, A_n, $n = 0, 1, 2, \cdots$) defined by the intersection of

the light ray and the world line of the particle A (Figure). Here we need the help of an ideal Lorentz frame, (X^μ), in which the spacetime structure is represented by the Minkowski metric tensor $\eta_{\mu\nu}$, where $\eta_{\mu\nu}$ is defined by $-\eta_{00} = \eta_{11} = \eta_{22} = \eta_{33} = 1$ and $\eta_{\mu\nu} = 0$ $(\mu \neq \nu)$. We assume that the particles A and B seem macroscopically to be at rest from the frame (X^μ) and that for simplicity the photon and particles move in the X^0-X^1 plane. Let $\alpha/2$ be the spatial distance between these two particles, measured by means of the coordinate (X^μ). Let λ be the wavelength of the light and M be the mass of the particles. We have to assume

$$\lambda \ll \alpha, \quad \text{and} \quad \frac{\hbar}{Mc} \ll \alpha \tag{1.1}$$

in order to guarantee that the setup of the photon and particles plays the role of a clock, where $\hbar/Mc$ is the Compton wavelength of the particles.

When we observe the event A_{n-1} from the frame (X^μ), the coordinate $X^1(A_{n-1})$ is derived with an uncertainty $\approx \lambda$, and therefore the momentum of the particle A must have an uncertainty $\approx \hbar/\lambda$. This uncertainty creates an uncontrollable uncertainty $\Delta X^1(A_n)$ in the value of X^1 for the next event A_n :

$$\Delta X^1(A_n) \approx \frac{\hbar}{M\lambda}\frac{\alpha}{c}. \tag{1.2}$$

From this we can deduce that the value of $X^0(A_n)$ must have an uncertainty

$$\Delta X^0(A_n) \approx \frac{\hbar}{M\lambda}\frac{\alpha}{c}. \tag{1.3}$$

On the other hand, the value of $X^0(A_n)$ can be derived only with an error of the order of the wavelength λ of the light. Because these two uncertainties in $X^0(A_n)$ are from different origins, they should be added :

$$\Delta X^0(A_n) \approx \frac{\hbar}{M\lambda}\frac{\alpha}{c} + \lambda \geq 2\sqrt{\frac{\alpha\hbar}{Mc}}. \tag{1.4}$$

Furthermore this line of reasoning leads to the conclusion that there is no correlation between the fluctuations in $X^0(A_n)$ and those in $X^0(A_{n'})$ if $n \neq n'$.

Now, the Schwarzschild's radius GM/c^2 of the particles A and B (where G is the universal gravitational constant) should be assumed to be less than the size α of the clock, otherwise we cannot observe the internal events A_n $(n = 0, 1, 2, \cdots)$ from outside the radius. This assumption is reasonable because this clock is used in order to establish the coordinate system (x^μ) in an area beyond the radius. Combining this assumption

$$\alpha > \frac{GM}{c^2} \tag{1.5}$$

and eq. (1.4), we have

$$\Delta X^0(A_n) > \sqrt{\frac{G\hbar}{c^3}} \equiv l_g, \tag{1.6}$$

where $l_g = 1.6 \times 10^{-33}$cm is the so-called Planck length.

As stated above, each of the events A_n is regarded as a scale mark on the x^0-axis. For example, we have $x^0(A_n) = n\alpha$ if we regard A_0 as the origin of the inertial coordinate system (x^μ). Hence eq. (1.6) means that the scale marks on the x^0-axis are observed to be fluctuating to the extent of l_g when they are seen from the coordinate system (X^μ). We therefore cannot definitely determine the coordinate transformation between the systems (X^μ) and (x^μ). (We have constructed only the x^0-axis. We do not complete the construction of the coordinate system (x^μ) here, because we need only eq. (1.6) in our following arguments. For the details of such a construction, see references [5] and [6].)

Thus we have arrived at the following conclusions (I) and (II) : (I) The coordinate transformation functions $X^\mu(x)$ have an uncertainty; in particular we have

$$\Delta X^0(x) \approx l_g \tag{1.7}$$

for each scale mark x on the x^0-axis. (II) There is a separation α between points x and x' on the x^0-axis above which there is no correlation between the fluctuations in $X^0(x)$ and in $X^0(x')$.

There is of course the possibility that eq. (1.7) is not universal and that some other formula, for example $\Delta X^0(x) = 0$, may be derived if we use a different clock to construct the coordinate system (x^μ). Here, however, we make a jump in the logic and assume that the above statements (I) and (II) are universally satisfied insofar as the coordinate (x^μ) has been defined intrinsically from the theory of matter. We will discuss some consequences of this standpoint in the following sections.

The above statement (II) does not mean that there must be some correlation between the fluctuations in $X^0(x)$ and those in $X^0(x')$ if the distance between x and x' is less than α. It should only be interpreted to mean that we do not know how the relation between $X^0(x)$ and $X^0(x')$ should be described for such x and x'.

In order to clarify our point of view, we should consider the order of magnitude of α: Combining the conditions (1.1) and (1.5), we have

$$\alpha \gg \sqrt{\frac{G\hbar}{c^3}} = l_g. \tag{1.8}$$

Another estimate of the order of α may be made by assuming our clock has the same order of precision as an atomic clock. The above (I) and (II) mean that the precision of our clock is approximately equal to l_g/α. On the other hand, the precision of atomic clocks is known to be

$$\Delta t/t \approx 10^{-9} \ \text{ to } \ 10^{-14}, \tag{1.9}$$

(see Ref. [7], p.393). Equating the two gives a range for α of

$$\alpha \approx 10^{-24} \ \text{ to } \ 10^{-19} \text{cm}, \tag{1.10}$$

which is considered to be consistent with the condition (1.8).

2. Fluctuations in metric tensor

We have assumed that the spacetime structure is represented by the Minkowski metric tensor $\eta_{\mu\nu}$ in the frame (X^μ). Hence, in the coordinate (x^μ), the structure is represented by the metric coefficients $g_{\mu\nu}(x)$;

$$g_{\mu\nu}(x) = \frac{\partial X^\alpha}{\partial x^\mu} \frac{\partial X^\beta}{\partial x^\nu} \eta_{\alpha\beta}. \tag{2.1}$$

Now, we know that we cannot definitely determine the functions $X^\mu(x)$, therefore we have to accept the conclusion that the values of the coefficients $g_{\mu\nu}(x)$ must have some uncertainties. The statements (I) and (II) in the previous section mean that the uncertainties in $g_{\mu\nu}(x)$ should be such that

$$\Delta \left[\int_{(n-1)\alpha/c}^{n\alpha/c} \sqrt{-g_{\mu\nu}(x)\frac{dx^\mu}{dt}\frac{dx^\nu}{dt}}dt \right] \approx l_g, \tag{2.2}$$

where $x^\mu(t)$ is the world line segment between A_{n-1} and A_n along the x^0-axis.

Furthermore let $x^\mu(t)$ be the world line segment between the events A_0 and A_n along the x^0-axis, and let s be the length of this segment. Then we have

$$s = \sum_{i=1}^{n} s_i, \tag{2.3}$$

where s_i is the length of the segment between A_{i-1} and A_i. Our statements (I) and (II) claim that each s_i distributes around the average value α with the standard deviation $\Delta s_i \approx l_g$, and that $s_i - \alpha$ and $s_j - \alpha$ distribute independently from each other if $i \neq j$. Hence the standard deviation Δs of the length s is estimated to be

$$\begin{aligned} (\Delta s)^2 &= \langle (s - n\alpha)^2 \rangle \\ &= \langle (\sum_i (s_i - \alpha))^2 \rangle = \sum (\Delta s_i)^2 \approx n \times l_g{}^2, \end{aligned} \tag{2.4}$$

where the parentheses $\langle \cdots \rangle$ denote the mean value of the quantity enclosed. Thus we have

$$\Delta s = \Delta \left[\int_0^{n\alpha/c} \sqrt{-g_{\mu\nu}(x) \frac{dx^\mu}{dt} \frac{dx^\nu}{dt}} dt \right] \approx \sqrt{n} l_g. \tag{2.5}$$

In order to express this uncertainty in $g_{\mu\nu}(x)$, we introduce a family of metric tensors

$$g_{\mu\nu}(x) = \eta_{\mu\nu} + h_{\mu\nu}(x), \tag{2.6}$$

where $h_{\mu\nu}(x)$ is assumed to fluctuate statistically around the average value 0. Furthermore we assume that each $h_{\mu\nu}$ fulfills the condition

$$|h_{\mu\nu}(x)| \ll 1 \tag{2.7}$$

and that the free Einstein equation

$$R^{\mu\nu} - \frac{1}{2} g^{\mu\nu} R = 0 \tag{2.8}$$

is satisfied for each $g_{\mu\nu}$.

Although there might be other methods to describe this uncertainty in $g_{\mu\nu}(x)$, we consider the above statistical description here.

Then, using eq. (2.7), we have

$$\int_0^{n\alpha/c} \sqrt{-g_{\mu\nu} \frac{dx^\mu}{dt} \frac{dx^\nu}{dt}} dt = n\alpha - \frac{c}{2} \int_0^{n\alpha/c} h_{00}(t, \mathbf{0}) dt \tag{2.9}$$

if $x^\mu(t)$ is the world line segment between the events A_0 and A_n along the x^0-axis. Hence eq. (2.5) leads to

$$\Delta s = \Delta \left[\frac{c}{2} \int_0^{n\alpha/c} h_{00}(t, \mathbf{0}) dt \right] \approx \sqrt{n} l_g. \tag{2.10}$$

(Similar resuls to (2.10) have been derived by various methods. See for example Ref. [7], p.1192.)

In the following, we abbreviate $h_{00}(x)$ to $h(x)$ for simplicity. We have assumed the condition (2.7), therefore the Einstein equation (2.8) gives the equation

$$\partial^{\mu}\partial_{\mu}h(x) = 0 \qquad (2.11)$$

for certain conditions on $h_{\mu\nu}(x)$.

Here we can use the following method introduced by Károlyházy et al. [1, 2]: Let

$$h(x) = \frac{1}{\sqrt{V}} \sum_{\mathbf{k}} \left[c_{\mathbf{k}} e^{i(\mathbf{k}\cdot\mathbf{x}-kx^0)} + c_{\mathbf{k}}{}^* e^{-i(\mathbf{k}\cdot\mathbf{x}-kx^0)} \right] \qquad (2.12)$$

be the Fourier expansion of $h(x)$ in a large box of volume V in three dimensional space with $k \equiv |\mathbf{k}|$. Each complex coefficient $c_{\mathbf{k}}$ is supposed to vary around the average value zero, and each set of specific values for every $c_{\mathbf{k}}$ determines an $h(x)$.

Our objective in the remainder of this section is to find how each $c_{\mathbf{k}}$ distributes under the condition that eq. (2.10) must be satisfied. It would be natural to assume that

$$\langle c_{\mathbf{k}} \rangle = 0 \quad \text{and} \quad \langle c_{\mathbf{k}} c_{\mathbf{k}'} \rangle = \langle c_{\mathbf{k}} c_{\mathbf{k}'}{}^* \rangle = 0 \ \text{ for } \ \mathbf{k} \neq \mathbf{k}', \qquad (2.13)$$

and that $\langle |c_{\mathbf{k}}|^2 \rangle$ depends only on $|\mathbf{k}| = k$, i.e.

$$F(k) \equiv \langle |c_{\mathbf{k}}|^2 \rangle. \qquad (2.14)$$

Furthermore, we assume that the summation in (2.12) covers only the range $k \leq 1/\alpha$. This assumption is reasonable because, as said in our statement (II) in Section 1, we merely know that there is no correlation between the fluctuations of $X^0(x)$ and $X^0(x')$ for x and x' whose separation is α or more.

Now we have

$$\left\langle \left(\frac{c}{2} \int_0^{n\alpha/c} h(t,\mathbf{0})dt \right)^2 \right\rangle$$

$$= \frac{1}{4V} \left\langle \left[\sum_{\mathbf{k}} \{ i\frac{c\mathbf{k}}{k}(e^{-ikn\alpha} - 1) + c.c. \} \right]^2 \right\rangle$$

$$= \frac{1}{V} \sum_{\mathbf{k}} \frac{F(k)}{k^2}(1 - \cos kn\alpha) \tag{2.15}$$

$$= \frac{1}{(2\pi)^3} \int d^3\mathbf{k} \frac{F(k)}{k^2}(1 - \cos kn\alpha)$$

$$= \frac{1}{2\pi^2} \int_0^{1/\alpha} dk\, F(k)(1 - \cos kn\alpha)$$

$$= \frac{1}{2\pi^2}(n\alpha)^{-1} \int_0^n dk\, F\left(\frac{k}{n\alpha} \right)(1 - \cos k).$$

Hence if $F(k)$ is proportional to k^{-2},

$$F(k) = ak^{-2} \tag{2.16}$$

for some constant a, then we have

$$\left\langle \left(\frac{c}{2} \int_0^{n\alpha/c} h(t,\mathbf{0})dt \right)^2 \right\rangle = n\frac{a\alpha}{2\pi^2} \int_0^n dk\, k^{-2}(1 - \cos k), \tag{2.17}$$

and the condition (2.10) is satisfied. (Note that the value of the integral in (2.17) approaches a constant of order 1 as n increases). Here the constant a must fulfill the condition

$$a\alpha \approx l_g^2. \tag{2.18}$$

Before continuing, we should check whether the assumption (2.16)

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is consistent with (2.7) ;

$$\langle h(x)^2 \rangle = \frac{1}{V} \langle \left[\sum_{\mathbf{k}} \left(c_{\mathbf{k}} e^{i(\mathbf{kx}-\omega t)} + c.c. \right) \right]^2 \rangle$$

$$= \frac{4}{V} \sum_{k<1/\alpha} F(k) \tag{2.19}$$

$$= \frac{2}{\pi^2} \frac{a}{\alpha}$$

$$\approx \left(\frac{l_g}{\alpha} \right)^2 \ll 1,$$

where we have used (2.18) and (1.8). Thus far at least, there is no contradiction among our assumptions.

In the following, we assume for simplicity that the distribution of $c_{\mathbf{k}}$ is Gaussian for each $\mathbf{k}$. Then the equation (2.16) means that the mean value of a quantity $Q(c_{\mathbf{k}}, c_{\mathbf{k}}^*)$ is given by the integral

$$\frac{k^2}{\pi a} \int Q(c_{\mathbf{k}}, c_{\mathbf{k}}^*) e^{-k^2 |c_{\mathbf{k}}|^2 / a} dc_{\mathbf{k}} dc_{\mathbf{k}}^*. \tag{2.20}$$

3. Localization of the wave function

Throughout this argument we have relied on the Lorentz frame (X^μ) in order to ascertain the uncertainty (2.5) in the metric coefficients $g_{\mu\nu}(x)$. The frame (X^μ) is ideal in the sense that the metric tensor is assumed to have no uncertainty when it is described in this frame. Here, we make a second jump in the logic, and postulate the following : Although there is no such ideal frame as (X^μ), nevertheless the existence of the uncertainty (2.5) and of the size α (1.8) is guaranteed for every inertial coordinate system (x^μ).

In this section, we argue from this postulate that the uncertainty in our spacetime causes the localization of wave function of a particle.

We will use the statistical method which has been developed in the previous section in order to express the uncertainty in $g_{\mu\nu}(x)$.

We consider a scalar particle with mass m moving in our space-time. Assuming that the motion is sufficiently slow with reference to the coordinate (x^μ), we have

$$
\begin{aligned}
L &\equiv -mc\sqrt{-g_{\mu\nu}(x)\frac{dx^\mu}{dt}\frac{dx^\nu}{dt}} \\
&= -mc^2\sqrt{1 - h(x) - \left(\frac{\mathbf{v}}{c}\right)^2} \\
&= \frac{m}{2}\mathbf{v}^2 + \frac{mc^2}{2}h(x) - mc^2,
\end{aligned}
\tag{3.1}
$$

where $v^j \equiv dx^j/dt$ $(j = 1, 2, 3)$. Hence the Hamiltonian H is given by

$$
H = H_0 + V(x) \quad \text{with} \quad H_0 \equiv \frac{p^2}{2m} \quad \text{and} \quad V(x) \equiv -\frac{mc^2}{2}h(x). \tag{3.2}
$$

Let ψ be an initial state of this particle, and let $\psi(t)$ and $\rho(t)$ be denoted by

$$
i\hbar\frac{d}{dt}\psi(t) = (H_0 + V(x))\psi(t) \quad \text{and} \quad \rho(t) = |\psi(t)\rangle\langle\psi(t)|, \tag{3.3}
$$

where $\psi(0) = \psi$. We are taking the point of view that the final state of the particle is given by the statistical average of $\rho(t)$ over the distribution of $h(x)$.

Here, however, we meet with an obstacle : It is very difficult to write the solution $\psi(t)$ of (3.3) in a simple enough form that we can calculate the average of $\rho(t)$. In order to save the situation, we will begin with the equation

$$
i\hbar\frac{d}{dt}\tilde{\psi}(t) = V(x)\tilde{\psi}(t), \quad \tilde{\psi}(0) = \psi. \tag{3.4}
$$

This can be explicitly solved, giving

$$
\langle\mathbf{x}|\tilde{\psi}(t)\rangle = \langle\mathbf{x}|\psi\rangle e^{-\frac{i}{\hbar}\int_0^t V(\tau, \mathbf{x})d\tau}. \tag{3.5}
$$

First, we estimate the magnitude of the phase in (3.5) :

$$
\langle \left[\frac{1}{\hbar} \int_0^t V(\tau, \mathbf{x}) d\tau \right]^2 \rangle
$$
$$
= \left(\frac{mc}{2\hbar} \right)^2 \frac{1}{V} \langle \left[\sum_{\mathbf{k}} (\frac{ic_{\mathbf{k}}}{k} e^{i\mathbf{k}\mathbf{x}} \left(e^{-ickt} - 1 \right) + c.c.) \right]^2 \rangle
$$
$$
= \left(\frac{mc}{\hbar} \right)^2 \frac{1}{V} \sum_{\mathbf{k}} \frac{F(k)}{k^2} (1 - \cos ckt)
$$
$$
= a \left(\frac{mc}{\hbar} \right)^2 \frac{ct}{2\pi^2} \int_0^{ct/\alpha} \frac{1}{k^2} (1 - \cos k) dk,
$$

(3.6)

giving

$$
\langle \left[\frac{1}{\hbar} \int_0^t V(\tau, \mathbf{x}) d\tau \right]^2 \rangle
$$
$$
\approx \begin{cases} (l_g \, mc/\hbar)^2 \, (ct/\alpha)^2 & \text{for } ct/\alpha \ll 1; \\ (l_g \, mc/\hbar)^2 \, ct/\alpha & \text{for } ct/\alpha \gg 1 , \end{cases}
$$

(3.7)

where we have made use of (2.18).

Note that the ratio $l_g mc/\hbar$ of the Planck length to the Compton wavelength of the particle is much less than 1 if the particle is microscopic, and that it is much greater than 1 for a macroscopic particle. Then from eq. (3.7) we can conclude the following (m) and (M):

(m) If the particle is microscopic, the magnitude of the phase in (3.5) reaches the order of unity only when the time t is quite large, i.e.

$$
t > \left(\frac{\hbar}{mc} / l_g \right)^2 \frac{\alpha}{c}.
$$

(3.8)

For the case of, for example, an electron, we have

$$
\frac{\hbar}{mc} \simeq 4 \times 10^{-11} \text{cm},
$$

(3.9)

and eq. (3.8) gives

$$
t > 10^{44} \times \frac{\alpha}{c}.
$$

(3.10)

(M) If the particle is macroscopic, the magnitude of the phase in (3.5) can reach the order of unity even if t is quite small, i.e.

$$t \approx \left(\frac{\hbar}{mc} / l_g \right) \frac{\alpha}{c}. \tag{3.11}$$

For the case $m = 1$g, for example, we have

$$\frac{\hbar}{mc} \simeq 10^{-37} \mathrm{cm}, \tag{3.12}$$

and eq. (3.11) gives

$$t \approx 10^{-4} \times \frac{\alpha}{c}. \tag{3.13}$$

Next, writing

$$\tilde{\rho}(t) = |\tilde{\psi}(t)\rangle\langle\tilde{\psi}(t)|, \tag{3.14}$$

we calculate the mean value of $\langle \mathbf{x} | \tilde{\rho}(t) | \mathbf{y} \rangle$:

The mean value of $\langle \mathbf{x} | \tilde{\rho}(t) | \mathbf{y} \rangle$

$$\begin{aligned}
&= \langle \mathbf{x} | \psi \rangle \langle \psi | \mathbf{y} \rangle \int e^{-\frac{i}{\hbar} \int_0^t [V(\tau, \mathbf{x}) - V(\tau, \mathbf{y})] d\tau} \\
&\qquad\qquad \times \prod_{\mathbf{k}} \frac{k^2}{\pi a} e^{-k^2 |c_{\mathbf{k}}|^2 / a} dc_{\mathbf{k}} dc_{\mathbf{k}}^* \\
&= \langle \mathbf{x} | \psi \rangle \langle \psi | \mathbf{y} \rangle \exp\left[-a \left(\frac{mc}{\hbar} \right)^2 \frac{|\mathbf{x} - \mathbf{y}|}{2\pi^2} \right. \\
&\qquad \left. \times \int_0^{|\mathbf{x}-\mathbf{y}|/\alpha} \frac{1}{k^2} \left(1 - \frac{\sin k}{k} \right) \left(1 - \cos \frac{ckt}{|\mathbf{x} - \mathbf{y}|} \right) \right],
\end{aligned} \tag{3.15}$$

where $|\mathbf{x} - \mathbf{y}|$ denotes the distance between the two points $\mathbf{x}$ and $\mathbf{y}$ in three dimensional space. Then similar considerations to before lead us to the following two conclusions (m') and (M'):

(m') In the case of a microscopic particle, the fluctuation in the potential $V(x)$ destroys the correlation between $\langle \mathbf{x} | \tilde{\psi}(t) \rangle$ and $\langle \mathbf{y} | \tilde{\psi}(t) \rangle$

when and only when the points **x** and **y** are distant from each other, i.e.

$$|\mathbf{x} - \mathbf{y}| > \left(\frac{\hbar}{mc}/l_g\right)^2 \alpha. \tag{3.16}$$

For an electron, we have

$$|\mathbf{x} - \mathbf{y}| > 10^{44} \times \alpha. \tag{3.17}$$

(M') In the case of a macroscopic particle, the fluctuation in the potential $V(x)$ destroys the correlation between $\langle \mathbf{x}|\tilde{\psi}(t)\rangle$ and $\langle \mathbf{y}|\tilde{\psi}(t)\rangle$ even when the points **x** and **y** are very close together , i.e.

$$|\mathbf{x} - \mathbf{y}| \approx \left(\frac{\hbar}{mc}/l_g\right) \alpha, \tag{3.18}$$

where we have assumed that the time t is sufficiently large. For the case $m = 1$g, we have

$$|\mathbf{x} - \mathbf{y}| \approx 10^{-4} \times \alpha. \tag{3.19}$$

The above discussions of $\tilde{\psi}(t)$ can be summarized as follows : The fluctuating potential $V(x)$ alters the phase of the wave function of a particle moving in it. If the particle is microscopic, however, the change of the phase is only slight unless much time passes, and furthermore, the phases at two different points can be random only when they are well separated. If the particle is macroscopic, on the other hand, the phase is altered to a large extent in a very short time, and the phases at two different points can be random even when they are very close.

Now we wish to draw some conclusions about the time evolution $\psi(t)$ from the above discussions of $\tilde{\psi}(t)$. First, (m) and (m') can be interpreted as follows : If the particle is microscopic, it is not until a long period of time has passed that the effect of the fluctuating

$V(x)$ is reflected by a significant change in $\psi(t)$. Therefore throughout that period the time evolution of $\psi(t)$ will be governed by the free Hamiltonian H_0. That is to say, not only the magnitude of $h(x)$ but also of $V(x) = -mc^2 h(x)/2$ is sufficiently small for $V(x)$ to act as only a small perturbation on H_0.

On the other hand, the situation changes drastically for a macroscopic particle: Although $h(x)$ is very small as stated in (2.7), the potential $V(x) = -mc^2 h(x)/2$ can be rather large for a macroscopic mass m. It thus appears that the interaction is quite strong between such a particle and the potential $V(x)$, and that the $V(x)$ cannot be considered as only a small perturbation on H_0. Consequently, the phenomenon described in (M) and (M') will apply also in the case of $\psi(t)$. That is to say, the fluctuating $V(x)$ will disturb the phase of $\psi(t)$, and the statistical mean value of

$$\langle \mathbf{x}|\rho(t)|\mathbf{y}\rangle \tag{3.20}$$

will vanish even when the points $\mathbf{x}$ and $\mathbf{y}$ are very close, i.e.

$$|\mathbf{x} - \mathbf{y}| \approx \left(\frac{\hbar}{mc}/l_g\right)\alpha. \tag{3.21}$$

In this way the fluctuation in $V(x)$ causes localization of the wave function of a macroscopic particle. (See [8]).

4. Concluding remarks

We have taken the view that every coordinate system of our space-time should be defined in terms of the physical objects which inhabit the universe. From this approach, we have deduced that there must be some uncertainty in the metric tensor and that this uncertainty, if it is expressed by a certain statistical method, can be shown to cause the localization of the wave function of a macroscopic particle.

Eq. (1.8) is the only condition by which we can estimate the order of α. Therefore some ambiguities still remain in the equations (3.8), (11), (16) and (18). For example, we cannot say anything about the explicit size of localized wave packets.

However the author has confidence that the following possibility has been shown in this article: This uncertainty of the scale l_g in spacetime has some practical effects for a macroscopic particle, and it plays some role when we consider the quantum mechanical description of macroscopic objects.

Acknowledgements

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