蒸溜塔の段数計算

Calculation of the Number of Equilibrium Stages

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Abstract

In order to calculate the number of plates for distillation, the McCABE-THIELE diagram is widely applied in this field.

This approach is used to give the number of equilibrium stages required for the change in composition from x_0 to x_n directly, calculating reflux and the relationship between constant A and constant B, without the diagram method of McCABE-THIELE.

The number of equilibrium stages are calculated, according to the following equation:

$$n = \frac{\log\left[\left(\frac{1+b}{B}\right)\frac{\log\frac{[x_n/(1-x_n)]}{[x_0/(1-x_0)]}}{\log\left[(a_0/A)\left(\frac{x_0}{1-x_0}\right)^{\frac{1+b}{B}}\right]}+1\right]}{\log\frac{1+b}{B}}$$

Constant A and constant B are shown in the following:

$$A = 1/\left(\frac{x_n}{1-x_n}\right)^{B-1}$$

and

$$B = \frac{\log(1/A)}{\log[x_n/(1-x_n)}-1$$

Also the reflux may be calculated from the following equations:

$$y_{n+1} = \left(\frac{r}{r+1}\right)x_n + \left(\frac{1}{r+1}\right)x_D$$

and

$$r = R/D$$

From these equations one can calculate the number of equilibrium stages and the number of plates for distillation.

1. Introduction

This is a study of distillation, especially the calculation of the number of equilibrium stages and the number of plates for the column.

Distillation is the separation of the constituents of a liquid mixture by partial vaporization of the mixture and separate recovery of vapor and residue. The more volatile components of the original mixture are obtained in the vapor; the less volatile in the residue. The extent of the separation depends upon the number of plates.

After FENSKE[1] attempted to make a theory for the number of theoretical plates required at total reflux, G. L. MATHESON[2] and W. K. LEWIS[3] tried to calculate the number of plates for the column. Also A. J. UNDERWOOD[4] developed a equation for the minimum reflux ratio.

Even though there are numerous studies of distillation and calculation of the number of equilibrium stages, the McCABE–THIELE diagram is widely applied in this field.

In this study, the number of equilibrium stages and the number of plates for distillation were obtained without the diagram method of McCABE–THIELE.

2. Principles of Fractionating Columns

Assume a column in continuous, constant molal overflow as in Fig. 1. A solution which consists of a binary mixture of volatile components, at its boiling point is fed at F, and is introduced on one of the plates in the mid-section of the column.

The liquid is fed together with reflux from the upper portion of the column, down over the plates below the point of intake.

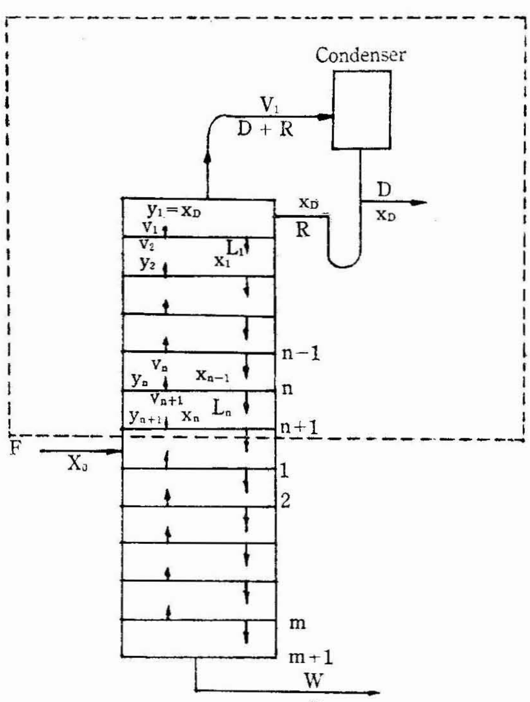

Fig. 1. A column in continuous and constant molal overflow.

In this illustration, heat to vaporize a portion of the binary mixture and the reflux is supplied by means of a steam condenser in a closed coil in the base of the column.

The residue W is withdrawn from the base of the column. The vaporized solution and reflux pass up the column undergoing rectification on each plate, and finally to the total condenser. A portion of the total condensate is returned to the column as reflux R, while the rest is withdrawn as product or distillate D.

The simplest case is that of two volatile components and constant L/V. If the composition of the saturated (equilibrium) vapor and liquid at the column pressure is determined for the alcohol-water system, the compositions of the vapor and liquid leaving any equilibrium stage are represented by a point on the equilibrium line.

A material balance for the more volatile component around the top of the column and the less volatile components in the lower section exists in the section of the apparatus of Fig. 1. bounded by the dotted line.

$$V_{n+1} = L_n + D \quad (1)$$

For the more volatile component,

$$V_{n+1}\ y_{n+1} = L_n x_n + D x_D \quad (2)$$

$$y_{n+1} = (L/V)\ x_n + (D/V)\ x_D \quad (3)$$

From equation (1), by substituting $L_n + D$ for V_{n+1}

$$y_{n+1} = \left(\frac{L_n}{L_n+D}\right) x_n + \left(\frac{D}{L_n+D}\right) x_D \quad (4)$$

If the molal overflow is constant, L/V is constant, then

$$V_{n+1} = V_n = V_{n-1} = V_{n-2} = \cdots\cdots = V_1 = R + D \quad (5)$$

Also,

$$L_1 = L_2 = L_3 = \cdots\cdots = L_n = R \quad (6)$$

From equation (2), (5) and (6)

$$V_{n+1} (y_{n+1}) = R (x_n) + D (x_n) \quad (7)$$

where

x = mole fraction of a component (the more-volatile component in a two-component system) in the liquid stream,
y = mole fraction of that component in the vapor stream,
V = moles of vapor flowing up past liquid in rectifying section,
L = moles of liquid flowing down in rectifying section,
D = moles of distillate product,
R = moles of reflux, per unit time,
W = moles of bottom, per unit time,
m = a plate or equilibrium stage in stripping section,
n = a plate or equilibrium stage in rectifying section.

3. Calculation of the Number of Equilibrium Stages

If the total condenser is used, the product has the composition of the vapor leaving the top plate; hence

$$x_D = x_0 = y_1 \quad (8)$$

Consider a section of the apparatus of Fig. 1 bounded by the dotted line which includes the portion of the tower above the nth plate.

A material balance on this section is

$$V_{n+1} = R + D \quad (9)$$

Then

$$(R + D)\ (y_{n+1}) = R\ (x_n) + D\ (x_0) \quad (10)$$

Therefore

$$y_{n+1} = \frac{R}{R+D}\ x_n + \frac{D}{R+D}\ x_0 \quad (11)$$

because

$$r = R/D \quad (12)$$

where r is reflux ratio.

From equation (11) and (12)

$$y_{n+1} = \left(\frac{r}{r+1}\right) x_n + \left(\frac{1}{r+1}\right) x_D \quad \cdots\cdots (13)$$

This equation shows the linear relationship between the vapor composition entering any plate and the liquid composition.

Equation (13) may be plotted, using the values y and x.

On the same plot, the y vs. x equilibrium curve for the given binary mixture is plotted. Such a diagram is shown in Fig. 2 to which a 45-degree line passing through the origin has been added for reference.

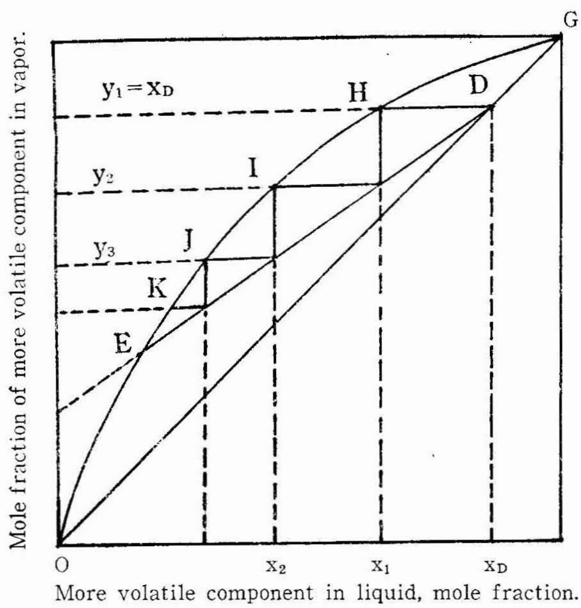

Fig. 2. McCabe-Thiele diagram.

Equation (13) is represented by line DE and the equilibrium curve is given by GHIJKEO.

Line DE is the operating line in this case.

If the diagrams changed from McCABE-THIELE diagram to log-log diagram, the operation line will show the following,

$$Y = AX^B \quad \cdots\cdots (14)$$

where

$$X = \left(\frac{x}{1-x}\right) \quad \text{and}$$

$$Y = \left(\frac{y}{1-y}\right) \quad \cdots\cdots (15)$$

Therefore the operation line will be shown as the following equation:

$$\log Y = B \log X + \log A \quad \cdots\cdots (16)$$

In case of a binary system, relative volatility can be expressed as,

$$a = \left(\frac{y}{1-y}\right)\left(\frac{1-x}{x}\right)$$

Then,

$$\left(\frac{y}{1-y}\right) = a\left(\frac{x}{1-x}\right) \quad \cdots\cdots (17)$$

The equilibrium curve will be expressed by equation (18),

$$a = a_0 X^b \quad \cdots\cdots (18)$$

From equation (15) and (17)

$$Y = aX \quad \cdots\cdots (19)$$

Substituting for a, from equation (18)

$$Y = (a_0 X^b) X = a_0 X^{1+b}$$

Then $Y = a_0 X^{1+b}$ .... (20)

The equation (20) is the line of equilibrium on a log–log diagram.

The above relationship is illustrated in Fig. 3, where

$X_0\ X_1\ X_2$ .... is the operating line on the feed plate and

$Y_0\ Y_1\ Y_2$ .... is the equilibrium line.

This is shown by the following equations:

$$Y_0 = a_0 (X_0)^{1+b} \quad \cdots\cdots (21)$$

$$Y_0 = A (X_1)^B \quad \cdots\cdots (22)$$

Then

$$a_0 (X_0)^{1+b} = A (X_1)^B \quad \cdots\cdots (23)$$

$$(X_1)^B = (a_0/A) (X_0)^{1+b} \quad \cdots\cdots (24)$$

Equation (24) may be simplified to

$$X_1 = (a_0/A)^{\frac{1}{B}} (X_0)^{\frac{1+b}{B}} \quad \cdots\cdots (25)$$

For the next plate, the nth plate,

$$Y_1 = a_0 (X_1)^{1+b} \cdots (26)$$

$$Y_1 = A (X_2)^B \cdots (27)$$

Then

$$a_0 (X_1)^{1+b} = A (X_2)^b$$

and

$$X_2 = (a_0/A)^{\frac{1}{B}} (X_1)^{\frac{1+b}{B}} \quad \cdots\cdots (28)$$

Substituting for X_1 from equation (25),

$$X_2 = (a_0/A)^{\frac{1}{B}} \left[(a_0/A)^{\frac{1}{B}} (X_0)^{\frac{1+b}{B}}\right]^{\frac{1+b}{B}}$$

$$= (a_0/A)^{\frac{1}{B}} \left[(a_0/A)^{\frac{1+b}{B^2}} (X_0)^{\left(\frac{1+b}{B}\right)^2}\right]$$

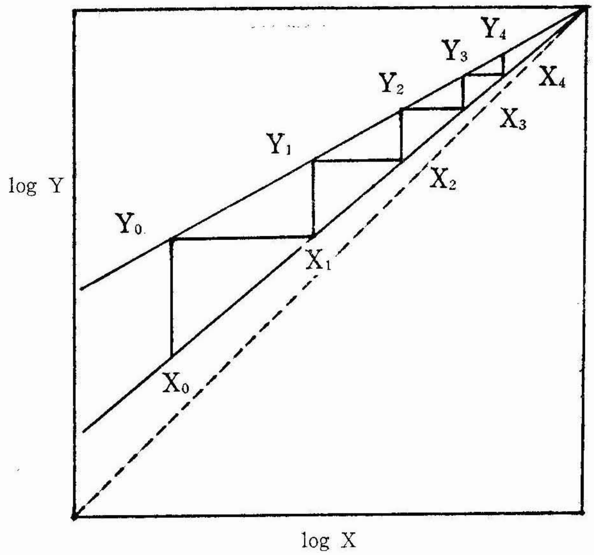

Fig. 3. The diagram of the equilibrium line and operation line.

Then

$$X_2 = (a_0/A)^{\frac{1}{B}} (a_0/A)^{\frac{1+b}{B^2}} (X_0)^{\left(\frac{1+b}{B}\right)^2} \quad \cdots\cdots (29)$$

For the (n−1) th plate,

$$Y_2 = a_0 (X_2)^{1+b} \quad \cdots\cdots (30)$$

$$Y_2 = A (X_3)^B \quad \cdots\cdots (31)$$

$$(X_3)^B = (a_0/A) (X_2)^{1+b}$$

Then

$$X_3 = (a_0/A)^{\frac{1}{B}} (X_2)^{\frac{1+b}{B}} \quad \cdots\cdots (32)$$

Substituting for X_2 from equation (29)

$$X_3 = (a_0/A)^{\frac{1}{B}}\left[(a_0/A)^{\frac{1}{B}}(a_0/A)^{\frac{1+b}{B^2}}(X_0)^{\left(\frac{1+b}{B}\right)^2}\right]^{\frac{1+b}{B}}$$

$$= (a_0/A)^{\frac{1}{B}}(a_0/A)^{\frac{1+b}{B^2}}(a_0/A)^{\frac{(1+b)2}{B^3}}(X_0)^{\left(\frac{1+b}{B}\right)^3} \qquad \cdots\cdots (33)$$

Then,

$$X_3 = (a_0/A)^{\frac{1}{B}}(a_0/A)^{\frac{1+b}{B^2}}(a_0/A)^{\frac{(1+b)2}{B}}(X_0)^{\left(\frac{1+b}{B^3}\right)^3} \qquad \cdots\cdots (34)$$

In the general case,

$$X_n = (a_0/A)^{\frac{1}{B}}(a_0/A)^{\frac{1+b}{B^2}}(a_0/A)^{\frac{(1+b)}{B^3}}(a_0/A)^{\frac{(1+b)}{B^4}} \cdots\cdots$$

$$\cdots\cdots (a_0/A)^{\frac{(1+b)^{n-1}}{B^n}}(X_0)^{\left(\frac{1+b}{B}\right)^n} \qquad \cdots\cdots (35)$$

$$X_n = (a_0/A)^{\frac{1}{B}\left[1+\left(\frac{1+b}{B}\right)+\left(\frac{1+b}{B}\right)^2+\left(\frac{1+b}{B}\right)^3+\cdots\cdots+\left(\frac{1+b}{B}\right)\right]}(X_0)^{\left(\frac{1+b}{B}\right)^n}$$

Then

$$X_n = (a_0/A)^{\frac{(1+b)^n-B^n}{(1+b-B)B^n}}(X_0)^{\left(\frac{1+b}{B}\right)^n} \qquad \cdots\cdots (36)$$

Dividing equation (36) by X_0

$$X_n/X_0 = \left[(a_0/A)(X_0)^{\frac{1+b}{B}}\right]^{\frac{(1+b)^n-B^n}{(1+b-B)B^n}} \qquad \cdots\cdots (37)$$

Defining

$$X_n = \frac{x_n}{1-x_n} \qquad \text{and} \qquad X_0 = \frac{x_0}{1-x_0}$$

$$\left(\frac{x_n}{1-x_n}\right)\Big/\left(\frac{x_0}{1-x_0}\right) = \left[(a_0/A)\left(\frac{x_0}{1-x_0}\right)^{\frac{1+b}{B}}\right]^{\frac{\left(\frac{1+b}{B}\right)^n}{\left(\frac{1+b}{B}\right)B^n}} \qquad \cdots\cdots (38)$$

From equation (38)

$$\frac{\log\dfrac{\left(\frac{x_n}{1-x_n}\right)}{\left(\frac{x_0}{1-x_0}\right)}}{\log\left[(a_0/A)\left(\frac{x_0}{1-x_0}\right)^{\frac{1+b}{B}}\right]} = \frac{\left(\frac{1+b}{B}\right)^n - 1}{\left(\frac{1+b}{B}\right)} \qquad \cdots\cdots (39)$$

From equation (39) an equation (40) can be derived for calculating the number of equilibrium stages n.

$$\left(\frac{1+b}{B}\right)^n = \left(\frac{1+b}{B}\right)\frac{\log\dfrac{\frac{x_n}{1-x_n}}{\frac{x_0}{1-x_0}}}{\log\left[(a_0/A)\left(\frac{x_0}{1-x_0}\right)^{\frac{1+b}{B}}\right]} + 1 \qquad \cdots\cdots (40)$$

Then the equilibrium stages are shown as the follows:

$$n = \frac{\log\left[\left(\frac{1+b}{B}\right)\frac{\log\frac{\left(\frac{x_n}{1-x_n}\right)}{\left(\frac{x_0}{1-x_0}\right)}}{\log\left[(a_0/A)\left(\frac{x_0}{1-x_0}\right)^{\frac{1+b}{B}}\right]}+1\right]}{\log\left(\frac{1+b}{B}\right)} \quad \cdots\cdots (41)$$

In the case of total reflux,

$$Y = X \quad \cdots\cdots (42)$$

Then

$$A = B = 1 \quad \cdots\cdots (43)$$

From equations (43) and (38)

$$\frac{\left(\frac{x_n}{1-x_n}\right)}{\left(\frac{x_0}{1-x_0}\right)} = \left[a_0\left(\frac{x_0}{1-x_0}\right)\right]^{\frac{(1+b)^n-1}{B}} \quad \cdots\cdots (44)$$

In distillation

$$\left(\frac{y_{n-1}}{1-y_{n-1}}\right) = \left(\frac{x_n}{1-x_n}\right) \quad \cdots\cdots (45)$$

From equations (14) and (45)

$$\left(\frac{x_n}{1-x_n}\right) = \left(\frac{1}{A}\right)^{\frac{1}{B-1}} \quad \cdots\cdots (46)$$

Then

$$\log\left(\frac{x_n}{1-x_n}\right) = \frac{1}{B-1}\log\left(\frac{1}{A}\right) \quad \cdots\cdots (47)$$

The constant B will be calculated as follows:

$$B = \frac{\log\left(\frac{1}{A}\right)}{\log\left(\frac{x_n}{1-x_n}\right)} - 1 \quad \cdots\cdots (48)$$

and

$$A = \frac{1}{\left(\frac{x_n}{1-x_n}\right)^{B-1}} \quad \cdots\cdots (49)$$

From equation (37)

$$\log\left[\left(\frac{x_n}{1-x_n}\right)\Big/\left(\frac{x_0}{1-x_0}\right)\right] = \frac{(1+b)^n-B^n}{1+b-B}\log\left[(a_0/A)\left(\frac{x_0}{1-x_0}\right)^{\frac{1+b}{B}}\right] \quad \cdots\cdots (50)$$

If the equilibrium curve is given and a_0 and b are known, and $\left(\frac{x_n}{1-x_n}\right)$ and $\left(\frac{x_0}{1-x_0}\right)$ are constant, then from equation (50) and (48), the constants A and B can be obtained.

Then equation (14) can be calculated.

$$Y = AX^B \quad \cdots\cdots (14)$$

If x and y are obtained by equation (14), the operation line can be obtained.

$$y_{n+1} = \left(\frac{r}{r+1}\right) x_n + \left(\frac{1}{r+1}\right) x_D \qquad (9)$$

By the operation line, represented as (9), and equation (12) the reflux can be calculated:

$$r = R/D \qquad (12)$$

4. Conclusion

By use of the exponential series, the equilibrium curve and the operation line of the McCABE-THIELE diagram can be expressed as exponential functions.

The number of the ideal stages in a distillation column can be calculated without using the method of the McCABE-THIELE diagram.

5. References

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