台風域内の気圧と風速及び過度について

ON THE PRESSURE, WIND VELOCITY AND VORTICITY IN A TYPHOON (II)

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1. Introduction

Concerning the surface structure of typhoons, many investigations have been developed by many authors from both the theoretical and analytical point of view.

In the present paper, as one of the examples of moderate typhoon, we will take up the case of Typhoon Ema (5920) which passed near Miyako-Jima in Nov. 1959, and the structure of which was examined by the changes of surface weather elements such as pressure, wind velocity, vorticity and rainfall intensity observed at Miyako-Jima station.

In order to examine the surface structure of the typhoon, we adopted, as a first step, an exponential function of the distance from the center of the typhoon as the expression of the pressure distribution. Based on this expression, we derived the gradient wind velocity and the vorticity of the typhoon from the equation of gradient wind. Using the results thus obtained, the surface wind velocity is calculated from the equations of motion in the frictional layer. Thus, we tried to present patterns of surface weather elements of Typhoon Ema.

Since the situation of the typhoon changes with different stages, this study is concerned only with the mean structure of the typhoon under the assumption that she has almost invariable structure during the periods shown in Fig. 1.

2. Case of Typhoon Ema (5920)

The path of Typhoon Ema is shown in Fig. 1. During the periods from 09:00, Nov. 12 to 09:00, Nov. 13, 1959, she passed near Miyako-Jima taking her course almost parallel to the line of the Ryukyu islands after travelling over the tropical North Pacific Ocean far south of the islands.

The conspicuous features of the typhoon is that she has been accompanied by a front during the periods while keeping her central pressure around 960 mb. The changes of surface weather elements with time is shown in Fig. 2.

In order to make the space cross sections of the typhoon, it was assumed that the observations made at the equal distance from the center of the typhoon are equivalent, disregarding the time of observation and the position of the typhoon. Based on this idea, space cross sections ahead of and behind the center of the typhoon are made by means of rearranging the observations at Miyako-Jima station.

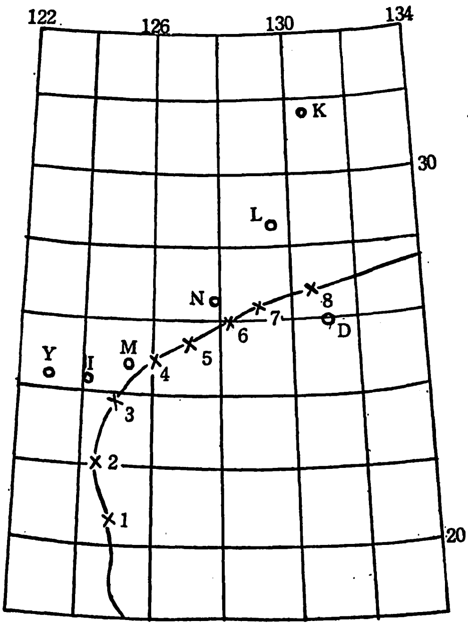

Fig. 1 Path of typhoon Ema.
I : Ishigaki-jima D : Minami-Daito
M : Miyako-jima L : Naze
N : Naha K : Kagoshima

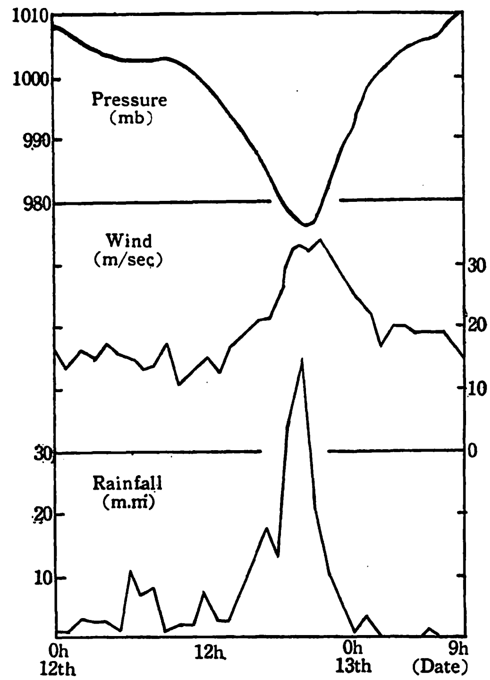

Fig. 2 Changes of the surface weather elements. (Miyako-jima)

Supplementary Table to Fig. 1

Number	Time	Number	Time
1	23 h 15 m, 11	5	24h 00 m, 12
2	09 h 00 m, 12	6	03 h 00 m, 13
3	15 h 00 m, 12	7	06 h 00 m, 13
4	21 h 00 m, 12	8	09 h 00 m, 13

Pressure at the center.

Time	Pressure	Time	Pressure
23 h 15 m, 11	965 mb	24 h 00 m, 12	960 mb
09 h 00 m, 12	960	03 h 00 m, 13	960
15 h 00 m, 12	960	06 h 00 m, 13	960
21 h 00 m, 12	960	09 h 00 m, 13	970

3. Equations of motion in the frictional layer

As a preliminary note to treat the pressure and wind velocity in the region of a typhoon, we shall sum up the fundamental equations in the frictional layer on which we have based the methods used in this paper.

Now let it be assumed the isobars in the region of a typhoon are of axial symmetry and the motion is stationary, then the equations of motion in the frictional layer are expressed in cylindrical coordinates as follows;

$$(3.1)\quad \begin{cases} v_r \dfrac{\partial v_r}{\partial r} - \dfrac{v_\theta^2}{r} - f v_\theta = -\dfrac{1}{\rho}\dfrac{\partial p}{\partial r} + \nu \dfrac{\partial^2 v_r}{\partial z^2} \\ v_r \dfrac{\partial v_\theta}{\partial r} + \dfrac{v_r v_\theta}{r} + f v_r = \nu \dfrac{\partial^2 v_\theta}{\partial z^2} \end{cases}$$

or, using the relations,

$$
(3.2)\quad \begin{cases} \zeta = \dfrac{\partial v_\theta}{\partial r} + \dfrac{v_\theta}{r} \\ P = p + \dfrac{1}{2}\rho\,(v_r^2 + v_\theta^2) \end{cases}
$$

we obtain the expressions

$$
(3.3)\quad \begin{cases} -(f+\zeta)v_\theta = -\dfrac{1}{\rho}\dfrac{\partial P}{\partial r} + \nu\dfrac{\partial^2 v_r}{\partial z^2} \\ (f+\zeta)v_r = \nu\dfrac{\partial^2 v_\theta}{\partial z^2} \end{cases}
$$

where v_r is the radial velocity, v_θ the tangential velocity, f the Coriolis parameter, ζ the relative vorticity, ρ the density of air and ν the eddy frictional coefficient.

In the first place, if we neglect the frictional terms on the right-hand side of the equations (3.3), since the gradient wind is unaffected by the friction, the first equation of these reduces to

$$
(3.4)\quad (f+\zeta)v_g = \frac{1}{\rho}\frac{\partial P}{\partial r}
$$

where v_g denotes the gradient wind velocity.
Combined with (3.2), this gives the equation of gradient wind

$$
(3.5)\quad fv_g + \frac{v_g^2}{r} = \frac{1}{\rho}\frac{\partial p}{\partial r}.
$$

The gradient wind velocity v_g is accordingly given by

$$
(3.6)\quad v_g = -\frac{1}{2}fr + \{(\frac{1}{2}fr)^2 + \frac{r}{\rho}(\frac{\partial p}{\partial r})\}^{\frac{1}{2}}.
$$

The vertical component of vorticity of the gradient wind ζ_g is defined by

$$
(3.7)\quad \zeta_g = \frac{\partial v_g}{\partial r} + \frac{v_g}{r}.
$$

Therefore, when the pressure distribution in the region of a typhoon is given as a function of r, the distance from the center, the gradient wind velocity and its vorticity may be computed from the equations (3.6) and (3.7).

We next reconsider the equations (3.1).
An approximate solution of these equations takes the form

$$
(3.8)\quad \begin{cases} v_r = -v_g C \exp(-\pi z/D) \cos(-\Upsilon + \pi z/D) \\ v_\theta = v_g\{1 + C \exp(-\pi z/D) \sin(-\Upsilon + \pi z/D)\} \end{cases}
$$

where

$$
\Upsilon = \frac{\pi}{4} + \psi, \quad C = \sqrt{2}\,\sin\psi,
$$

$$D=\pi\sqrt{\frac{2\nu}{f+K\zeta_g}},\quad K=1-0.173\,C\ (\sin\Upsilon-\cos\Upsilon),$$

and ψ is the angle between the isobar and the surface wind, D the frictional height and ν the eddy frictional coefficient.

Utilizing the results, the radial velocity v_r and tangential velocity v_θ corresponding to the height z can be computed.

Table 1. Results of the calculation. (Pressure, Wind velocity, Vorticity)

r (Km)	P (mb)	v_g (m. sec^{-1})	ζ_g ($\times10^{-4}$sec^{-1})	v_θ (m. sec^{-1})	v_r (m. sec^{-1})	v (m. sec^{-1})
20	966.2	22.5	15.9	22.2	1.0	22.2
40	971.7	29.2	9.8	28.2	2.6	28.4
60	976.5	33.1	7.0	30.6	4.2	30.7
80	980.6	35.3	5.5	29.2	5.7	31.0
100	984.2	36.7	4.0	26.3	6.8	29.8
150	991.6	36.4	2.2	23.3	9.6	28.0
200	996.8	34.1	1.1	17.4	8.5	24.8
300	1003.3	26.8	0.10	11.8	6.3	18.5
400	1006.5	19.1	−0.27	7.7	4.3	12.6
500	1008.2	12.5	−0.34	4.7	2.8	8.2
600	1009.1	7.5	−0.28	2.7	1.3	4.9

4. Pressure and wind velocity

The pressure distribution in the region of Typhoon Ema plotted in Fig. 3 shows its symmetrical nature about the center of the typhoon.

Based on the observation, we can adopt an exponential function of the distance r as the expression of the pressure distribution.
That is

$$(4.1)\quad p(r)=p(\infty)-\Delta p\ \exp\left(-\frac{r}{r_0}\right)$$

where $p(r)$ and $p(\infty)$ are the pressure at the distance r and the surrounding area, Δp the depth of the pressure at the center and r_0 is a constant.
As the parameters involved in the formula, we can choose

$$p(\infty)=1010\text{mb},\ \Delta p=50\text{mb},\ r_0=150\text{Km}.$$

Then the pressure pattern in Typhoon Ema is given by the following expression;

$$(4.2)\quad p(r)=1010-50\ \exp\left(-\frac{r}{150}\right)\ \text{mb}.$$

where r is the value in Km.

The results calculated by this formula are listed in Table 1, and the curve shown in Fig. 3 which represents this formula is in good agreement with the actual pressure obtained by the observation.

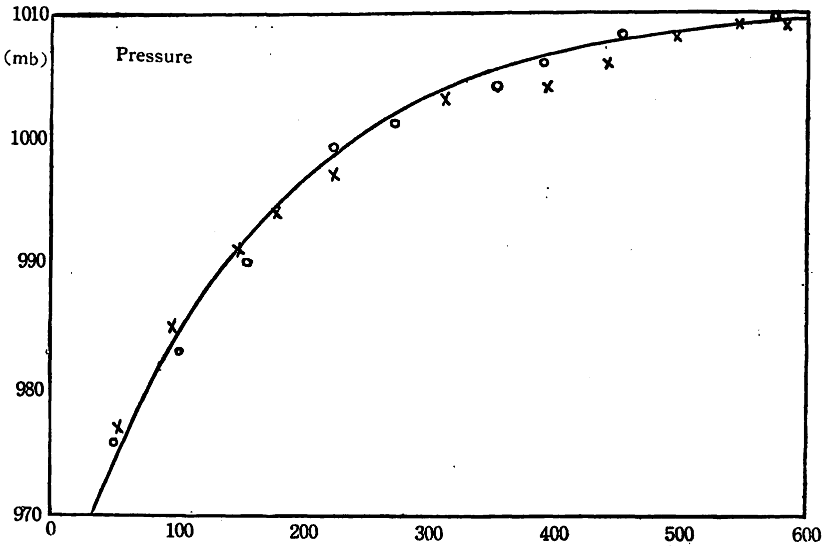

Fig. 3 Pressure distribution.
× : ahead of the center ○ : behind the center

Next we consider the wind velocity corresponding to this pressure distribution. By differentiating (4.1) with respect to r, we get

$$\text{(4.3)} \quad \frac{\partial p}{\partial r}=\Delta p\frac{1}{r_0}\exp\left(-\frac{r}{r_0}\right).$$

Substituting (4.3) in (3.6), we have

$$\text{(4.4)} \quad v_g=-\frac{1}{2}fr+\left\{\left(\frac{1}{2}fr\right)^2+\frac{\Delta p}{\rho}\frac{r}{r_0}\exp\left(-\frac{r}{r_0}\right)\right\}^{\frac{1}{2}}.$$

Then choosing the parameters involved in (4.4) as

$$f=0.63\times10^{-4}\text{sec}^{-1},\quad \rho=1.1\times10^{-3}g.\,\text{cm}^{-3},$$

we get the gradient wind pattern in Typhoon Ema as follows;

$$\text{(4.5)} \quad v_g=-(3.15r)+V^{\frac{1}{2}}\quad \text{cm. sec}^{-1}$$

where $$V=(3.15r)^2+3.03r\times\exp\left(-\frac{r}{150}\right)\times10^5$$

and r is the value in Km.

Now substituting (4.5) in (3.7) the following is found

$$\text{(4.6)} \quad \zeta_g=-0.63\times10^{-4}+V^{-\frac{1}{2}}\,(0.1985r\times10^{-3}+Z)\ \text{sec}^{-1}$$

where $$Z=3.03\times\left(\frac{3}{2}-\frac{r}{300}\right)\exp\left(-\frac{r}{150}\right).$$

The results calculated from these formulas are listed in Table 1.

We shall calculate next the radial velocity v_r from the second equation of the formulas (3.8). In this case it was assumed that ψ, the angle between the isobar and the surface wind, takes the constant value 20° for $r\geqq 150$ Km and then varies with r for $r<150$ Km so as to decrease linearly the value of $\sin\psi$ to zero at the center of the typhoon.

Thus, Γ, C and K in the equations (3.8) become 65°, 0.4836 and 0.96 respectively for $r\geqq 150$ Km.
The eddy frictional coefficient ν is estimated to be the constant value 0.7 m^{2}sec^{-1}.
We listed the values of the wind velocity for $z=20$ m in Table 1.

Fig. 4 shows the distribution of surface wind velocity observed together with that of calculation. It is seen that, over the area as far as 250 Km from the center, the calculated wind velocity for $z=20$ m does almost coincide with the observed wind velocity behind the center of the typhoon. On the contraly, in the area beyond 250 Km from the center of the typhoon, the actual wind velocity is large compared to the calculated wind velocity. This difference may be explained by the fact that there existed a front accompanied by the typhoon during this period. From this point, it is found that the boundary of the typhoon undisturbed by the front is about 250 Km from the center of the typhoon.

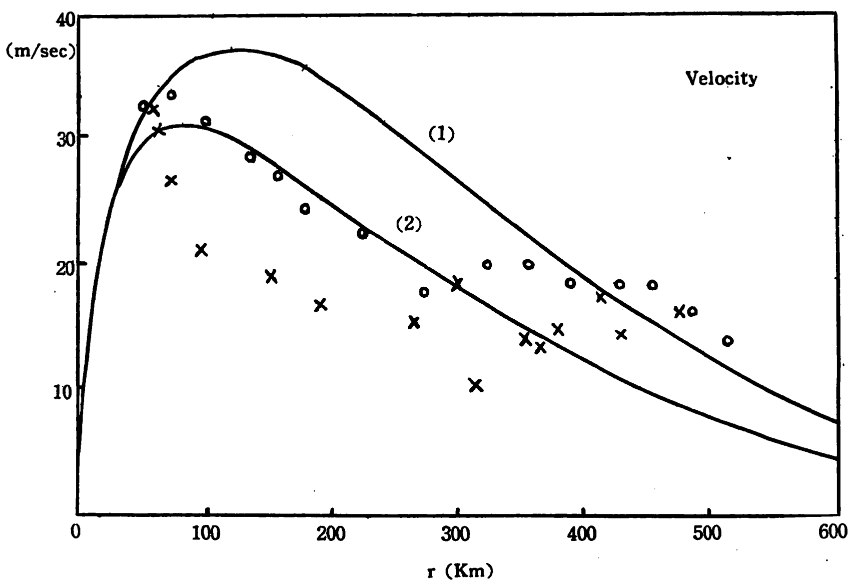

Fig. 4 Velocity distribution.

× : ahead of the center. ○ : behind the center.
(1) : gradient wind (2) : surface wind at $z=20$ m

5. Vorticity and Rainfall intensity

In Fig. 5, the solid curve shows the vorticity of the gradient wind computed by the formula (4.6), while the other two lines show the vorticities of the surface wind behind and ahead of the center of the typhoon.

The gradient wind vorticity gradually decreases its value and becomes negative after taking zero at $r \fallingdotseq 320$ Km. It is to be noted that there exists the region of negative vorticity in the pattern of vorticity.

On the other hand, no symmetry can be seen among the changes of vorticities of the surface wind behind and ahead of the center. But, generally speaking, the vorticities in both side within the distance about 250 Km from the center decrease their values almost parallel to that of the gradient wind. In the area beyond 250 Km from the center, the changes of the vorticities in both side of the center are quite irregular as those of the surface wind.

The fact the difference arises mainly at the distance beyond 250 Km seems to account for the existence of the disturbance by the front.

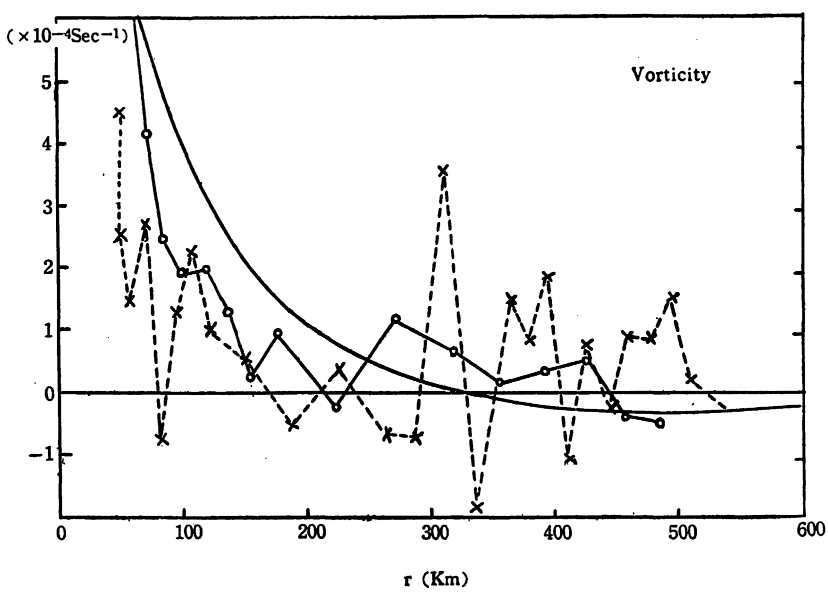

Fig. 5 Vorticity distribution.
×——×——× : ahead of the center
○——○——○ : behind the center
———— : vorticity calculated by eq. (4.6)

In Fig. 6, the dashed lines represent the observed rainfall intensity and the solid lines represent the results of the calculation from the formula (5.1). Since the vortical rain has a close connection with the vorticity, it was determined to what extent the relation may exist between the rainfall intensity and the vorticity.

In order to check the relation between them, we calculated the value of rainfall intensity from the formula (5.1) which has been known to express the intensity of vortical rainfall,

$$(5.1) \quad W=A\frac{\zeta_s}{\sqrt{f+\zeta_s}}$$

where ζ_s is the vorticity of surface wind and A asumed in our case, to be constant taking the value of 10×10^2 in c. g. s.

It would appear that the formula (5.1) gives a good approximation for values of the rainfall intensity within the area having the radius around 200 Km.

Thus, it can be estimated that the vortical rainfall is confined within the area.

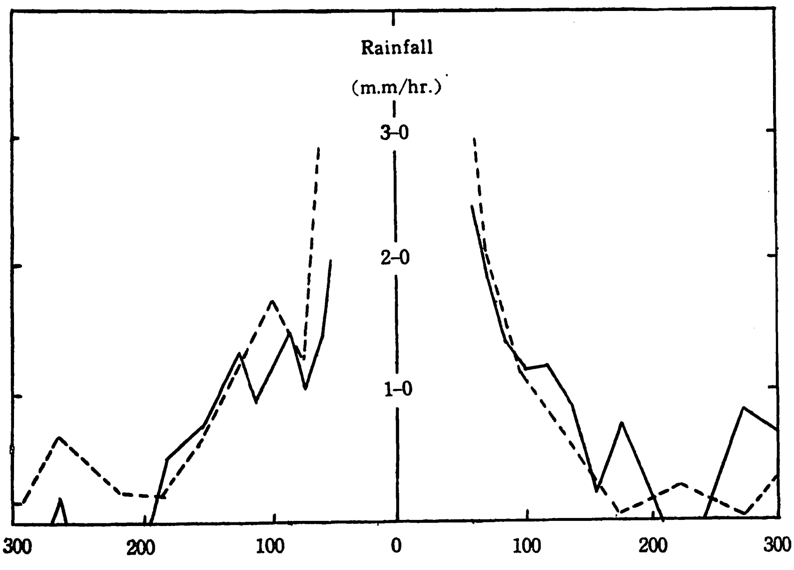

Fig. 6 Intensity of rainfall.
——— intensity of rainfall (calculated)
-------- intensity of rainfall (observed)

6. Summary

In this paper, the features of pressure, wind velocity, vorticity and rainfall intensity are investigated as associated with Typhoon Ema. The pressure pattern is given as an exponential function of the distance from the center. Wind velocity and vorticity corresponding to the given pressure pattern are computed.
The results of the calculation are compared with those of the observation.

As was already stated in the introduction the case chosen for this study is not independent from the disturbance of the front. Owing to the influence of the front on the typhoon, some disagreement can be seen among the results of the calculation and those of the observation.

As a whole, it seems that we can explain, to some extent, the surface structure of Typhoon Ema by the methods adopted in this paper.

References

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