台風域内の気圧と風速及び過度について

ON THE PRESSURE, WIND VELOCITY AND VORTICITY IN A TYPHOON

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1. Introduction

The studies of the structure of typhoons have been developed by many authors from both the theoretical and the analytical point of view, and some models of hurricane and typhoon have been presented. In the present paper, typhoon Sara (5914) which attacked Miyako-jima in 1959 and was named Miyàko-jima Typhoon for her severity is taken up as an example of typhoons, and the structure of her is examined by the changes of surface weather elements such as pressure and wind velocity observed at Miyako-jima station without inferring the mechanism of their developments.

The most favourable features of typhoon Sara as an example of typhoons are that she kept her central pressure constant and moved at the same speed along almost a straight line for some periods.

Since it is evident that even in the same typhoon the situation changes with different stages, these studies are concerned only with the above mentioned periods.

2. Model of Typhoon

As a preliminary note to treat the pressure and wind velocity in a typhoon, we shall describe briefly some of the well-known models of typhoon. In these models, both the pressure and wind velocity are assumed to be a function of the distance from the center of a typhoon.

(a) Rankine's combined vortex

This Model is based on the assumption that positive vorticity of the gradient wind distributes uniformly within the limited circular area and the vorticity in the outer region of the area is zero. If we admit the above mentioned assumption as the model of a tropical cyclone, we can compute the pressure and wind velocity in the following type;

$$(2.1)\quad \begin{cases} p=\dfrac{\rho\zeta_0^2}{8}\cdot r^2+p_c & r\leq a \\ p=-\dfrac{\rho a^4\zeta_0^2}{8}\cdot\dfrac{1}{r^2}+p_\infty & r>a \end{cases}$$

$$(2.2)\quad \begin{cases} v=\dfrac{\zeta_0}{2}\cdot r & r\leq a \\ v=\dfrac{\zeta_0 a^2}{2}\cdot\dfrac{1}{r} & r>a \end{cases}$$

where p_c and p_∞ are the pressure at the center and the surrounding area, r is the distance from the center, a the radius of the inner core distributed positive vorticity

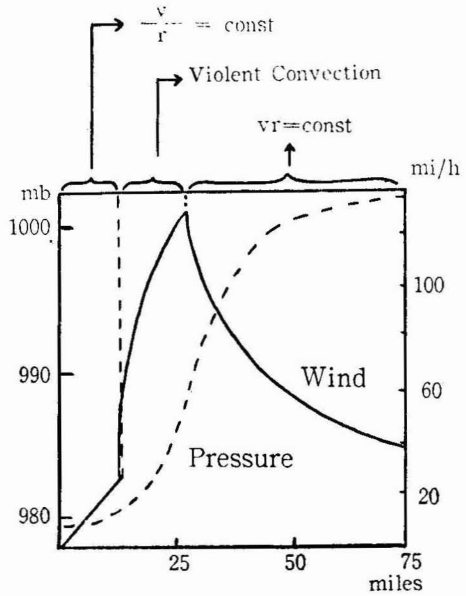

Fig. 1 Deppermann's model

ζ_0 and ρ is the density of the air.

(b) Deppermann's model

He noticed the existence of violent convection area between the inner and outer region and presented such a model of typhoon as shown in Fig. 1

(c) Syono's Model

He introduced theoretically the existence of negative vorticity around the area of positive vorticity, and proposed a model of typhoon.

(d) Others

Concerning pressure and wind velocity distribution in a typhoon, we have several empirical formulas as follows;

$$(2.3)\quad \begin{cases} p = p_\infty - \dfrac{\Delta p_c}{1+\dfrac{r}{r_0}} & \text{(K. Takahashi)} \\ p = p_\infty - \dfrac{\Delta p_c}{1+\left(\dfrac{r}{r_0}\right)^2} & \text{(V. Bjerknes)} \\ p = p_\infty - \dfrac{\Delta p_c}{\sqrt{1+\left(\dfrac{r}{r_0}\right)^2}} & \text{(T. Fujita)} \end{cases}$$

$$(2.4)\quad \begin{cases} v = Ar & : \text{Internal} \\ v = B/\sqrt{r} & : \text{External} \\ v = C/r & : \text{Outside} \end{cases} \qquad \text{(Y. Horiguchi)}$$

3. Case of typhoon Sara

In this paper, we take up the case of typhoon Sara which attacked Miyako-jima in September 15, 1959 and caused great damages. She travelled northwestward over the tropical North Pacific far south of the Ryukyu Islands, and deepened her central pressure from 964 mb at 5 h, Sept. 13, to 920 mb at 17 h, Sept. 14 keeping her course unchanged. After that time, she changed her course gradually to the north while passing over Miyako-jima with her central pressure as low as 908.4 mb at 18 h 56 m, Sept. 15. During the periods from 3 h, 15 to 12h, 16, we can suppose she conserved her state unchanged keeping her central pressure around 910 mb and moved at almost constant speed 20 km/hr. The conspicuous features of the typhoon is that her circular isobar was maintained and no front appeared during this periods. These studies are concerned only with the mean structure of the typhoon under the allowable assumption that she has almost invariable structure during the periods shown in Fig. 2.

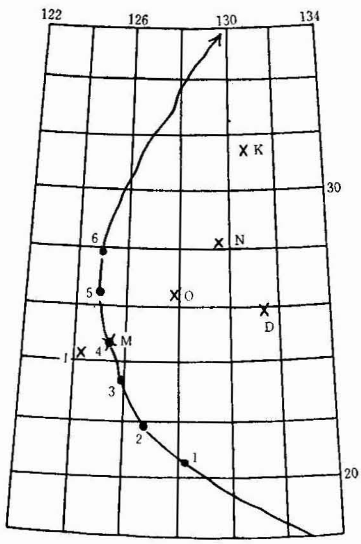

Fig. 2 Path of typhoon Sara.
I : Ishigaki-jima D : Minami-Daito
M: Miyako-jima N : Naze
O : Okinawa-jima K : Kagoshima

Supplementary Table to Fig. 2.

Number	Time	Number	Time
1	17 h 00 m, 14	4	18 h 45 m, 15
2	05 h 00 m, 15	5	04 h 00 m, 16
3	11 h 30 m, 15	6	12 h 00 m, 16

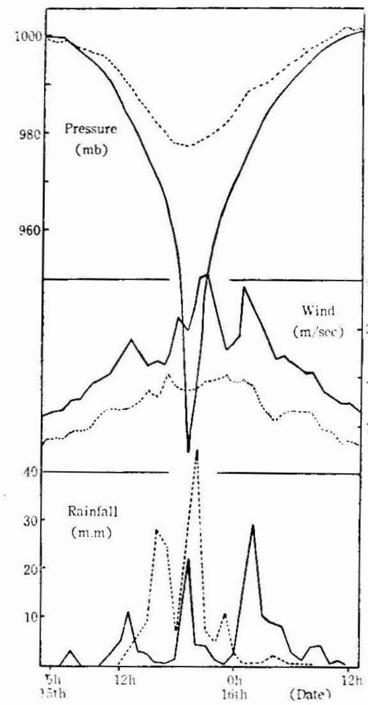

Fig. 3 Changes of the surface weather elements.
—— at Miyako-jima
------- at Ishigaki-jima

In these studies we assumed that all observations made at the equal distance from the center of the typhoon are equivalent, disregarding the time of observation and the position of the typhoon. Based on the idea, space cross sections ahead of and behind the center of the typhoon are made by means of rearranging the observations.

4. Pressure and wind velocity

Assuming a homogeneous and imcompressible atmosphere, the flow is defined by the following equations:

$$(4.1)\begin{cases} \dfrac{\partial v}{\partial t}+u\dfrac{\partial v}{\partial x}+v\dfrac{\partial v}{\partial y}+fu=-\dfrac{1}{\rho}\dfrac{\partial p}{\partial y} \\ \dfrac{\partial u}{\partial t}+u\dfrac{\partial u}{\partial x}+v\dfrac{\partial u}{\partial y}-fv=-\dfrac{1}{\rho}\dfrac{\partial p}{\partial x} \\ \dfrac{\partial u}{\partial x}+\dfrac{\partial v}{\partial y}=0 \end{cases}$$

where, u and v are the components along X-and Y-axis, respectively of the horizontal wind, the X-axis points to the east and the Y-axis to the north, f is the Coriolis parameter.

In order to discuss exactly the pressure and wind velocity in a typhoon, it is necessary to solve the equations under the given corditions. Since it is not easy to solve the equations, we are to confine ourselves to presenting some empirical formulas of pressure and wind velocity in the case of typhoon Sara.

The pressure distribution in the space cross section to the axis of the distance plotted on Fig. 4 shows that the pressure holds its symmetrical nature as it is in the time cross section shown in Fig. 3. In this case, we can adopt the empirical formula proposed by Fujita as the pressure distributions.

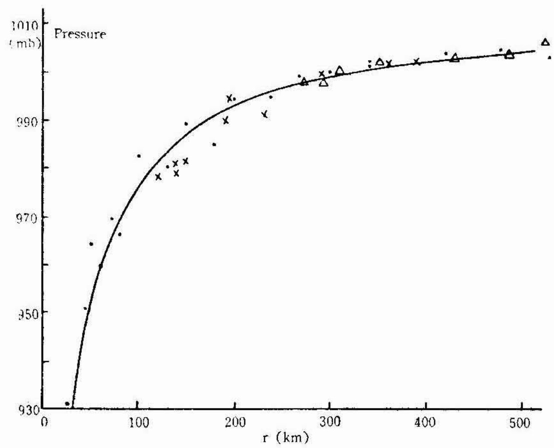

Fig. 4 Pressure distribution.

• : Miyako-jima
× : Ishigaki-jima
△ : Naha

That is

$$(4.2)\quad p=p_{\infty}-\frac{\Delta p_c}{\sqrt{1+\left(\frac{r}{r_0}\right)^2}}$$

As the parameter involved in the formula we can choose

$$p_{\infty}=1010\ \text{mb},\qquad p_c=910\ \text{mb}$$
$$\Delta p_c=100\ \text{mb},\qquad r_0=35\ \text{km}$$

Then the pressure pattern in typhoon Sara is given by the following expression:

$$(4.3)\quad p=1010-\frac{100}{\sqrt{1+\left(\frac{r}{35}\right)^2}}$$

The curve shown in Fig. 4 represents this formula, and showns good agreement with the actual pressures obtained by the observation.

On the other hand, the wind velocity shown in Fig. 5 is not of uniform convergence, but is of a somewhat irregular nature the same as the wind velocity shown in Fig. 3.

It is assumed that the wind velocity is empirically, in many

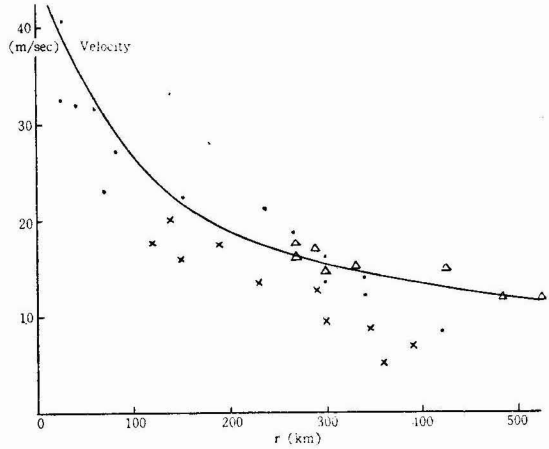

Fig. 5 Velocity distribution.

• : Miyako-jima
× : Ishigaki-jima
△ : Naha

cases, composed of two parts, the one being proportional to Δp and the other proportional to $\sqrt{\Delta p}$, where $\Delta p = p_\infty - p$.

That is

$$(4.4) \quad v = A \cdot \Delta p + B \cdot \sqrt{\Delta p}$$

As the parameters involved in the formula we can choose, $A=0$, $B=4.5$. Then we get the empirical formula of the velocity in typhoon Sara as follows:

$$(4.5) \quad v = 4.5 \times \sqrt{\Delta p}$$

The wind velocity curve obtained by this formula is shown in Fig. 5.

5. Vorticity

The relation between the vertical component of vorticity of the gradient wind ζ_g and that of surface wind ζ_s is expressed by cylindrical coordinates as follows;

$$(5.1) \quad \zeta_g = \frac{K}{r} \frac{\partial}{\partial r}(r v_s) = K\left(\frac{\partial v_s}{\partial r} + \frac{v_s}{r}\right) = K\zeta_s$$

where

$$K = \frac{1}{\sqrt{2}} \cos\left(\varphi + \frac{\pi}{4}\right)$$

and r is the distance from the center, v_s the surface wind velocity and φ is the angle between the isobar and the surface wind.

This means that the surface wind velocity can be used instead of the gradient wind to discuss the vorticity of the gradient wind.

In the case of typhoon Sara, the distributions of the surface wind vorticity can be computed by using the observed wind velocity at Miyako-jima station. The result of the computation using this observations is shown in Fig. 6.

It is of interest to see that the positive vorticity ahead of and behind the typhoon is concentrated near the center and out-ward of the area positive and negative vorticity exist alternately at intervals of about 30–40 km, and distribute symmetrically

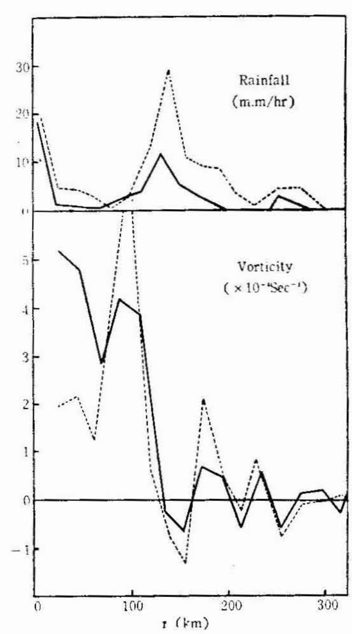

Fig. 6 Calculated vorticity by observed wind velocity and observed intensity of rainfall. (Miyako-jima)
—— ahead of typhoon
------- behind typhoon

about the center of the typhoon. This fact may be a very remarkable features concerning the structure of typhoon.

Suppose that we take a linear relation of distance as the vorticity distribution within the rain area, we may adopt the following relation

$$(5.2)\qquad \zeta=(-1.5\times10^{-6}r+4.5\times10^{-4})\ \mathrm{sec}^{-1}$$

showing that the value of the vorticity at the center is assumed to be $4.5\times10^{-4}\ \mathrm{sec}^{-1}$ and the boundary of the rain area to be 300 km.

Moreover, if we take the wind velocity field of the typhoon as the one defined by the expression already stated

$$(4.5)\qquad v=4.5\times\sqrt{\Delta p}$$

the distribution of vorticity which is shown in Fig. 7 can be computed. The result is about the same as the one obtained by Hughes (1952) for a mean typhoon.

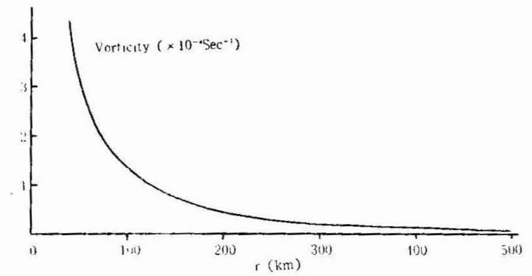

Fig. 7 Calculated vorticity by the formula (4.5).

Comparison of Fig. 6 and 7 shows that it seems natural to take formula (4.5) as the wind pattern of the typhoon. As was already shown in Fig. 6 the actual distribution of vorticity in a typhoon is, in many cases, not so simple as can be seen in Fig. 7. This is of course due to the fact that in our case we adopted the formula (4.5) as wind pattern, and that in the case of Hughes he obtained his result for mean typhoon.

Concerning the relation between the vorticity and rainfall, it seems reasonable to think that rainfall in a typhoon is closely connected with its vorticity. As was pointed out by Syono, the horizontal mass convergence in the frictional layer is proportional to the vorticity of gradient wind, and the intensity of vortical rainfall proportional to that of the vorticity in a typhoon. Thus, vorticity itself is considered as one of the most important necessary conditions for rainfall. If we consider rainfall in a typhoon only from the view point of vorticity, it seems that the relation between rainfall intensity and the changes of wind velocity observed at a station may be expressed as follows;

If a station is situated ahead (behind) of a typhoon, in another words, a typhoon is approaching to (is going away from) a station, it is natural to increase (decrease) gradually its wind velocity. But, on the contrary, if its wind velocity does not increase (decrease) as it is expected, the vorticity may increase its value for this periods. Consequently, we can expect to have rainfall within 1 or 2 hours after the beginning of these periods.

In the case of typhoon Sara, as is shown in Fig. 3, it seems apparently that the changes of wind velocity correspond to that of rainfall intensity, but it may be understood from the above mentioned considerations.

6. Summary

In this paper, the features of pressure and wind velocity, including its vorticity, are investigated associated with the typhoon Sara. Pressure and wind patterns of the typhoon are presented by empirical formulas on the basis of actual observations. Vorticity is computed by using both actual observations and empirical formula representing wind velocity and the results are compared. Some relation between the changes of wind velocity and that of rainfall intensity at a station is suggested. On the whole, typhoon can not be represented by a simple model due to its complicated structure and it is imperfect dynamically in our case since Coriolis force and friction are neglected. However, it seems that this kind of simplification may still offer an explanation for the structure of typhoon.

References

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Table 12 Analytical data of brine water

No.	Location	Date	Cl^- (g/L)	HCO_3^- (g/L)	I^- (mg/L)	Ca^{++} (mg/L)	Mg^{++} (mg/L)	Evaporate Residue (g/L)	SiO_2 (mg/L)	Zn^{++} (mg/L)	Mn^{++} (mg/L)	$KMnO_4$ cons (mg/L)	Gas produce (m^3/day)	HBO_2^- (mg/L)
3	Naha 5 Ku Kokusai OnsenYU	1958												
		May. 16	0.179	1.026	0.27			1.1855		0.20		81.36		17.53
		Jul. 21	0.189	1.019		3.31	0.75	1.2345				70.36		21.80
		Aug. 4	0.163	1.019										21.03
		Aug. 21	0.154	1.017				1.1772				89.51		
		Oct. 11	0.154	1.017	0.56	2.03	0.99	1.1770	7.99	0.04		89.05		
5	Tomigusuku Son Ryukyu Hirio Co.	1958												
		Aug. 5	0.708	0.577	2.07	7.04	2.92	2.2990	15.5	0.09		14.05		21.91
		Aug. 13	1.052	0.571		5.86	3.37							12.27
		Aug. 21	1.063	0.580						0.09				
7	Haebaru Son Ryukyu Sei to	1958												
		Aug. 27	0.966	0.603						0.30		47.47		
		Dec. 29	1.020	0.621		6.87	3.78	2.2030		0.07		44.06		
1	Naha 10 Ku Kokuei Kan	1958												
		May. 28	1.611	0.743	5.54	13.00	5.03	3.3872				108.48		26.29
		Jul. 21	1.549	0.747	5.17	10.77	4.58	3.2975	29.5	0.07	0.04	106.77		27.17
		Aug. 4	1.571	0.728		12.84	4.78	3.3495				105.06		28.49
		Aug. 21	1.659	0.715										
2	Nishihara Son Ryukyu Noren	1957												
		Nov. 15	1.295	0.915		10.35	4.44	2.9895	31.2					
		Dec. 2	1.438	0.894						0.87				
		May. 29. 1958	1.683	0.803	5.45	9.94	5.08	3.7500	10.5	0.06	0.01	118.65		26.29
		Dec. 2	1.101	0.939						0.04		107.08		
		Feb. 16. 1959	1.183	0.936										
6	Naha Miegusuku Ryukyu Suisan Co.	1958												
		Aug. 5	2.718	0.581	10.62	18.80	12.21	5.1655	4.5	0.10	0.06	120.68		35.06
		Nov. 8	2.798	0.581	11.51	21.47	12.83	5.1920				130.79		
8	Yonabaru Cho Tobu Haiden	1958												
		Aug. 27	1.756	0.713	5.20	14.11	5.62	3.6240		0.55		194.39		
		Nov. 6	3.491	0.546	10.64	46.38	21.69	6.4170	30.0	0.68	0.02	178.20	22.0	
		Nov. 20	3.940	0.496	10.90	53.26	26.18			0.49	0.07	171.76	22.7	
		Nov. 23	5.328	0.389	14.82	76.45	37.90			0.55		174.98	21.1	
		Dec. 3	3.079	0.584	8.37	36.08	15.12			0.22	0.04	197.61	18.8	
		Dec. 5	3.495	0.559		41.23	21.09			0.20		204.06	21.8	
		Dec. 18	3.858	0.508		48.45	24.68			0.20		213.73	20.5	
		Dec. 25	3.183	0.577		37.79	17.81			0.30			23.0	
		Dec. 29	3.460	0.565		46.38	20.50			0.31	0.05	187.88	16.0	
		Jan. 3. 1959	3.252	0.567		37.79	19.20			0.20	0.06	184.65	24.5	
		Jan. 5	2.837	0.599	7.98					0.31		210.51	15.7	
		Jan. 6	3.875	0.508						0.23	0.04		22.5	
		Jan. 7	4.169	0.492	11.03			7.0160		0.20		174.98	23.3	
		Jan. 8	4.256	0.479	8.28					0.12	0.03		24.0	
		Jan. 10	3.235	0.568						0.25		175.74	19.2	
		Jan. 12	4.065	0.495						0.17		168.53	20.4	
		Jan. 13	4.403									194.39	20.4	
		Jan. 14	4.385									178.20	22.0	
		Jan. 24	3.806									197.61		
4	Naha Miebashi Kokusai Shoji Co.	1958												
		Jun. 9	9.639	0.276	43.88	281.65	112.62	17.3485	15.5			174.98		52.15
		Jun. 16	10.679	0.226	50.81	324.70	110.40	19.1560	9.5	2.30	0.70	191.10		56.98
		Jun. 23	7.508	0.377	33.40	168.36	74.44					110.37		52.15
		Sep. 19	11.210	0.226	54.08	333.29	132.56	20.1700		0.35	0.01	86.79		
		Nov. 24	7.301	0.383	32.27	148.60	67.07			0.68	0.15	87.80		
		Dec. 5	7.162	0.402		147.70	67.17			0.86	0.26	152.41		
		Dec. 22	17.300	0.151	78.50	546.32	203.14			3.15	1.00			
		Dec. 23	7.162	0.395		141.73	63.84			0.62	0.24	81.36		
		Jan. 10. 1959	9.930	0.328	44.48					1.92	0.44	115.24		
		Jan. 13	15.951	0.191	69.34					2.05	0.67	149.19		
		Jan. 24	17.383	0.145	79.38					2.15	0.67	152.54		
		Jan. 24	6.885	0.403	30.53			11.7800		0.65	0.20	100.70		