The Wake Behind a Circular Cylinder at Low Reynolds Numbers

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§1. Introduction

Concerning the wake behind a circular cylinder and other two dimensional bluff bodies, many investigations have been developed since the time of Benard and various features of the wake have been released besides a summary of the known phenomena found in Goldstein.

In the present paper some observations developed by colour dye on the behavior of wake behind a circular cylinder at an intermediate range of Reynolds number, ($R=Ud/\nu$, where U is the cylinder speed, d the cylinder diameter and ν kinematic viscosity) $10<R<150$, are presented.

The aim of this paper is to observe the region of flow immediately behind the cylinder, to find the character of the flow in the wake and to obtain some details of the wake geometry.

§2. Experimental Method

The experiments were performed in a glass water tank of 75 cm in length, 50 cm in breadth and 35 cm in depth.

In order to move a circular cylinder uniformly, the carrier to which the cylinder is fixed is slided on the straight rails above the tank by means of a drawing screw.

It is possible to vary the speed of the carrier continuously in the range from 0.3 cm/sec to 1.5 cm/sec using an electric motor and a slide transformer.

Dyed water is put into the water around the cylinder producing the wake with a small injector to make it possible to observe visually the flow below the free surface.

The dye used is Fluorescein, concentrations being so small that the properties of the dyed water are not altered from those of the ambient water.

The horizontal layer below the free surface is illuminated from both sides of the tank normal to the flow direction through narrow slits.

The illuminated part of the wake behind a circular cylinder is photographed with a camera placed above the tank.

Various circular cylinders of diameter 2.37 mm, 3.02 mm, 4.21 mm, 4.71 mm, 5.33 mm, 6.65 mm, and 9.82 mm are used so that Reymolds number from about 10 to 150 can be obtained. The ratios of the diameter of the cylinder to the width of the tank range from 1/240 to 1/51.

It is necessary to allow the water to remain at rest for two hours or more after each run and before beginning each experiment, the experimental conditions are carefully controlled.

§3.0. Formation of the wake

It has long been recognized that for two-dimensional flow behind a circular cylinder at a Reynolds number of 3~6 a pair of fixed vortices is formed immediately behind the cylinder. The fixed vortex-pair is separated from the main flow by the vortex layers which are continuations of the laminar boundary layer of the cylinder after its separation.

As the Reynolds number is increased the vortices elongate in the direction of flow and after becoming asymmetry the vortices move away from the cylinder.

It is also commonly accepted that the wake below the vortex-pair becomes unstable at a Reynolds number of about 40, and the first appearance of a vortex street is a consequence of this instability. As the Reynolds number is increased the vortex street due to vortex shedding from the cylinder is generated.

While many investigations have been made by many authors concerning the origin of the vortex street, it seems likely that the mechanics of the formation of periodic wakes is still an interesting problem to be studied.

In this section, some deatailed observations about the relation between the vortex-pair and the wake below the vortex-pair are presented.

§3.1. Three stages of the wake

The process of the development of wake behind a circular cylinder with Reynolds numbers may be discussed from three different stages in its formation.

(a) Standing vortex-pair and straight line wake.

For Reynolds numbers below about 30, a pair of vortices formed immediatly behind the cylinder is quite symmetric and stable and the wake behind the vortex-pair is a straight line as shown in the figures from Fig. 1.1 to Fig. 1.4.

These two eddies grow and become more and more elongated in the flow direction with increasing the Reynolds number.

The straight line wake is not formed by the fluid flown out of the vortex-pair but is formed by the flow downstreem of the cylinder.

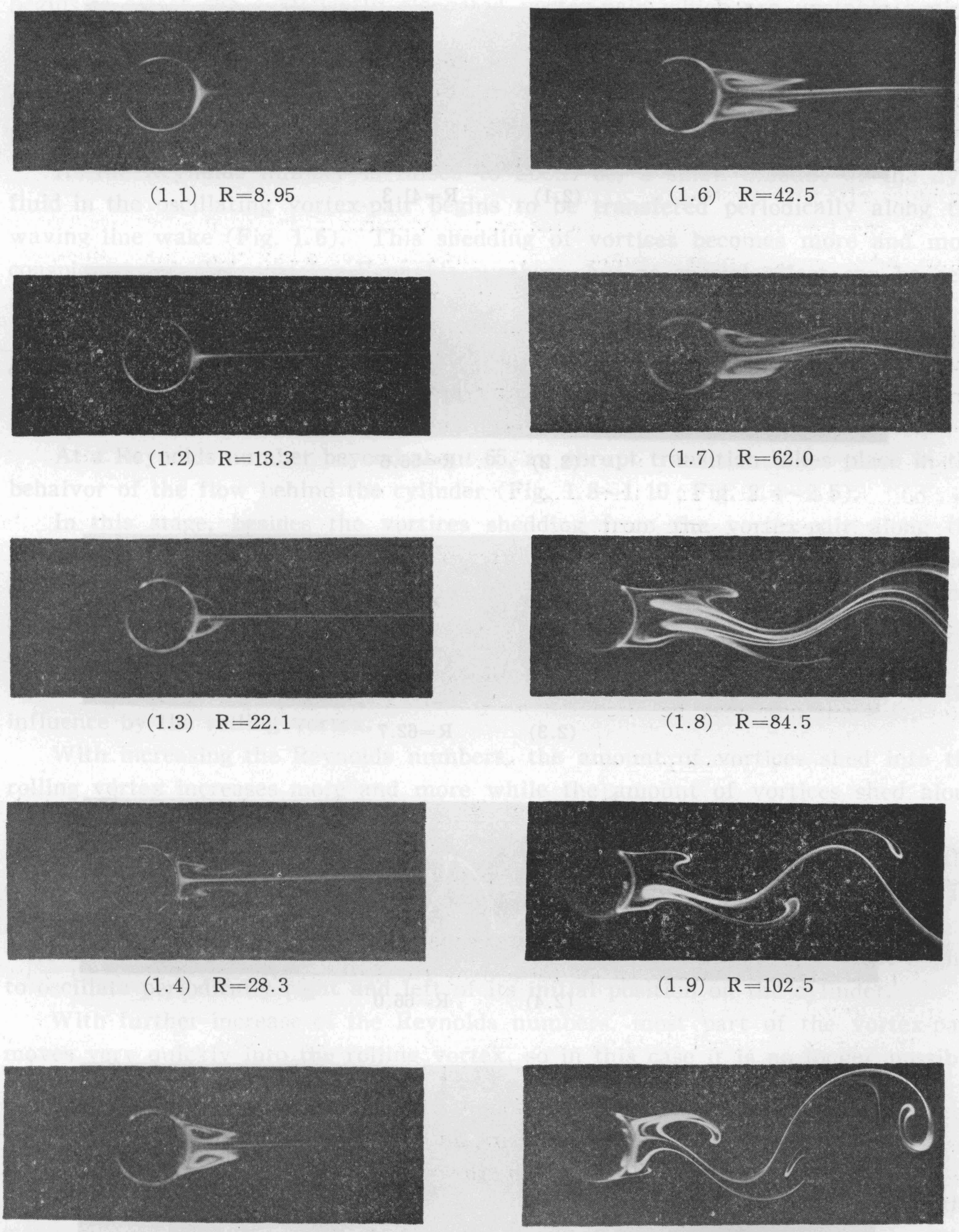
It is often observed that, when a small amount of dyed water is attached to the cylinder, only the standing vortex-pair which are fixed to the cylinder throughout are formed and no line wake can be found. In order to observe the line wake, it is always neccesary to put sufficient amount of dyed water ahead of the cylinder so that some separated portion of it flows back into the point some distance downstream of the cylinder.

In this stage, the flow field around the cylinder is steady.

(b) Oscillating vortex-pair and waving line wake.

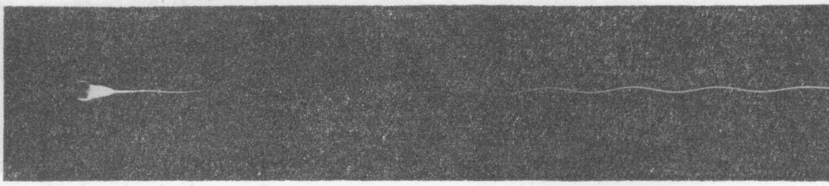
As the Reynolds number is raised beyond 30, the faint periodic oscillation begins alternately only at the rear of the vortex-pair but a large part of the vortex-pair remains undisturbed (Fig. 1.5).

It would seem that a cross flow from the laminar region outside the wake

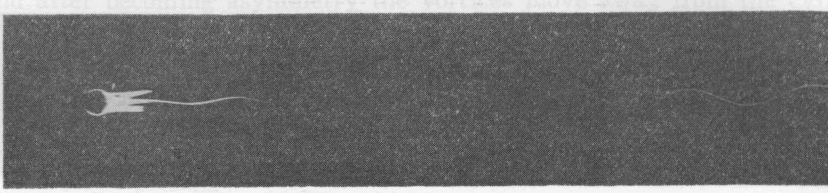


(1.1) R=8.95
(1.2) R=13.3
(1.3) R=22.1
(1.4) R=28.3
(1.5) R=33.2
(1.6) R=42.5
(1.7) R=62.0
(1.8) R=84.5
(1.9) R=102.5
(1.10) R=125.0

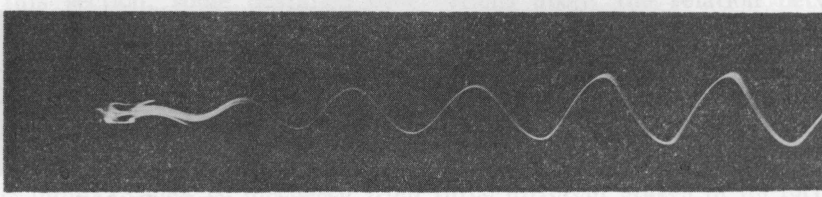
Fig. 1 Development of the vortex-pair immediately behind a circular cylinder.



(2.1) R=41.3



(2.2) R=56.6



(2.3) R=62.7

(2.4) R=66.0

(2.5) R=72.0

Fig. 2 The downstream development of the wake behind a circular cylinder.

begins to affect the sufficiently elongated vortex-pair which are unstable against such forces.

Even in this transition stage, no vortex shedding occurs but the line wake behind the vortex-pair begins to oscillate very faintly corresponding to the oscillation of the vortex-pair.

As the Reynolds number is raised to about 35, a small amount of the dyed fluid in the oscillating vortex-pair begins to be transfered periodically along the waving line wake (Fig. 1.6). This shedding of vortices becomes more and more conspicuous whith increasing Reynolds numbers, but no special effect can be seen upon the oscillation of the line wake (Fig. 1.7).

In the Reynolds numbers range from 30 to 65, the amplitude of the oscillation of line wake increases with increasing Reynolds numbers (Fig. 2.1, 2.2, 2.3).

(c) Degenerated vortex-pair and vortex street.

At a Reynolds number beyond about 65, an abrupt transition takes place in the behaivor of the flow behind the cylinder (Fig. 1.8~1.10 ; Fig. 2.4~2.5).

In this stage, besides the vortices shedding from the vortex-pair along the waving line wake, there is another fluid springing alternately from the outer side of the vortex-pair, rolling up into vortices, and moving downstream. The vortices arrange themselves in a double row and the typical Karman vortex street is observed.

The amplitude of waving line wake becomes larger abruptly owing to the influence by the rolling vortex.

With increasing the Reynolds numbers, the amount of vortices shed into the rolling vortex increases more and more while the amount of vortices shed along the line wake decreases.

The region of flow inside the vortex-pair between the separation points on the cylinder and the first appearance of the periodic rolling vortex decreases in its portion with increasing the Reynolds number.

As the Reynolds number of about 90 is reached, the rear stagnation point begins to oscillate periodically right and left of its initial position on the cylinder.

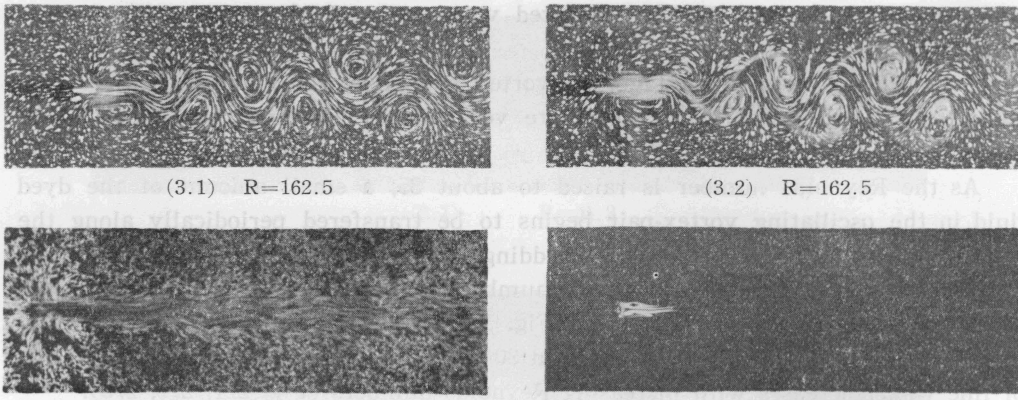
With further increase of the Reynolds numbers, most part of the vortex-pair moves very quickly into the rolling vortex, so in this case it is no longer possible to speak of vortex-pair.

§3.2. Streamlines and streak lines

In order to observe the streamlines corresponding to the streak lines in the wake, the aluminium powder method is used besides the dyed method. The aluminium powder which is scattered on the free surface shows the streamlines in the wake behind a moving cylinder.

Some of the photographs which show the relation between the streamlines and the streak lines in the wake are illustrated in the figures 3.

In obtainning the picture shown in Fig. 3.2, two methods are used at the same time, that is, the aluminium powder method in which the aluminium powder is



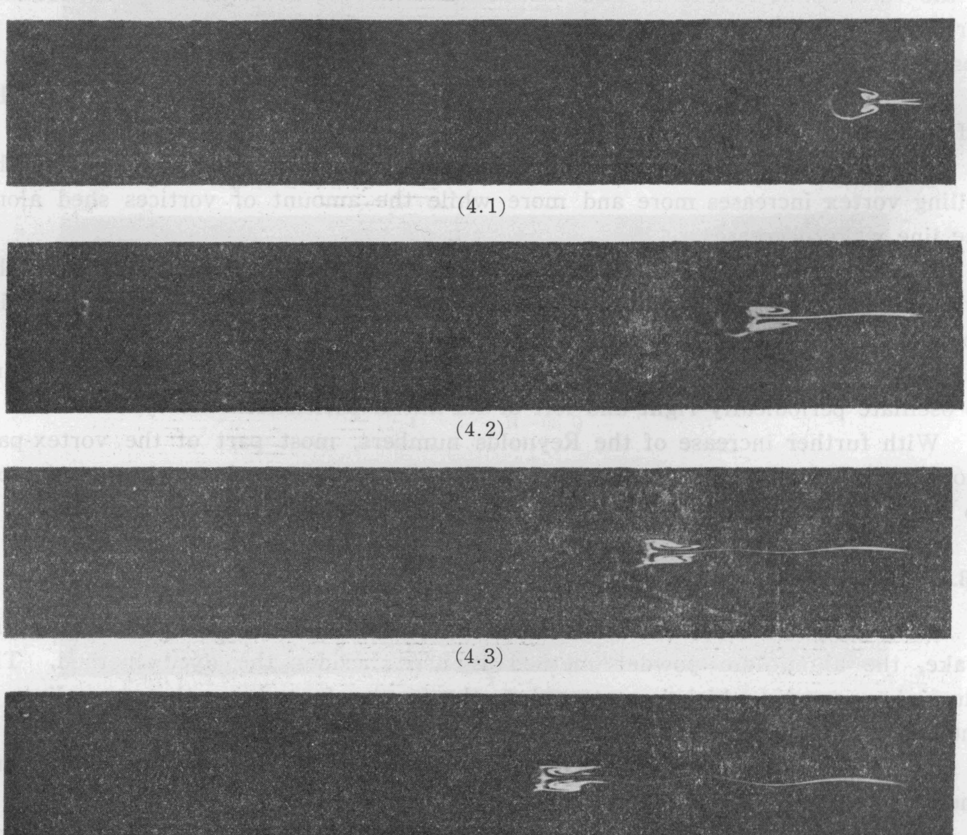
(3.1) R=162.5 (3.2) R=162.5

(3.3) R=51.1 (3.4) R=52.2

Fig. 3. Streamlines and streak lines in the wake behind a circular cylinder.

scattered on the free surface and the dyed method in which the dyed water is put into the water below the free surface.

It can be seen from the picture that the centres of the vortex shown by the streamlines coincide with those of the vortex shown by the streak lines.



(4.1)

(4.2)

(4.3)

(4.4)

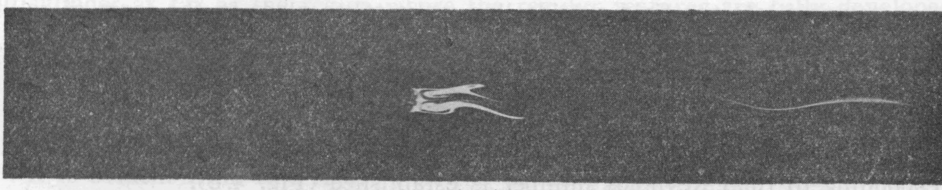
As is shown in Fig. 3.3, the row of vortices is not formed clearly, but it may be thought that there exist the weak vortex generated by the flow downstream in the neighborhood of the waving wake, though the positions of the centre of the vortex are not clear.

However, it is evident from the picture shown in Fig. 3.4 that in this stage of the Reynolds numbers no vortices are shed in the form of rolling vortex from the vortex-pair.

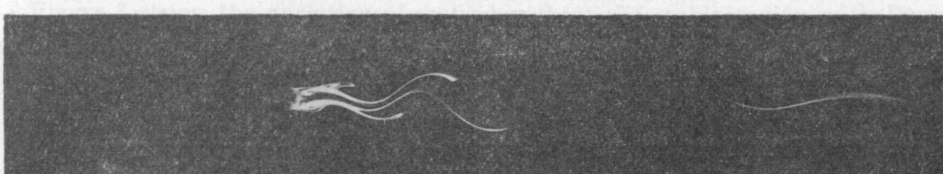
§4. Evolution of the wake

Observations are made toexamine the flow phenomena behind a circular cylinder starting from rest. Photographs of the progressive development of the wake formed behind a circular cylinder which is accelerated from rest to a uniform velocity were taken.

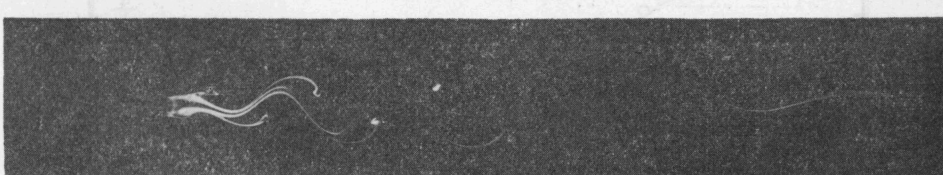
Figures 4.1 to 4.8 show a typical sequence of instantaneous pictures illustrating



(4.5)



(4.6)



(4.7)

(4.8)

Fig. 4. Consecutive pictures of the evolution of the wake behind a circular cylinder. (The final Reynolds number is 79.5)

the development of the wake behind a circular cylinder corresponding to final Reynolds number of 79.5.

Figure 4.1 shows a pair of axisymmetric standing vortex and the straight line wake shortly after the boundary layer has separated from the surface of the cylinder.

The standing vortex, after elongating more in the direction of flow for a short time, becomes unsymmetry and the line wake begins to oscillate faintly (Fig. 4.2).

As the speed of cylinder is increased the rear portion of the vortex-pair begins to be disturbed by the crossflow from outside the wake and is devided into inner and outer parts (Fig. 4.3).

Up to this stage, no vortices shedding from the vortex-pair can be observed.

The oscillation of vortex-pair becomes more conspicuous and the amplitude of waving line wake becomes larger and larger.

With further increase of the speed of cylinder, some amount of vortex element begins to be shed along the line wake (Fig. 4.4, 4.5) and then the vortex shedding from vortex-pair into the field outside line wake takes place in the form of rolling vortex (Fig. 4.6, 4.7).

The amplitude of waving line wake is increased abruptly by the influence of rolling vortex.

Then, the same periodic behavior persists and the fully developed wake corresponding to final Reynolds number is established (Fig. 4.8).

Throughout the observations, it was found that the most of the dye leaving the cylinder on one side of the wake flowed into the vortices on that side, but a small amount of the dye remained within the rotational region of the fixed vortices was transfered little by little to the other side through the rear stagnation point.

§5. The angle of separation

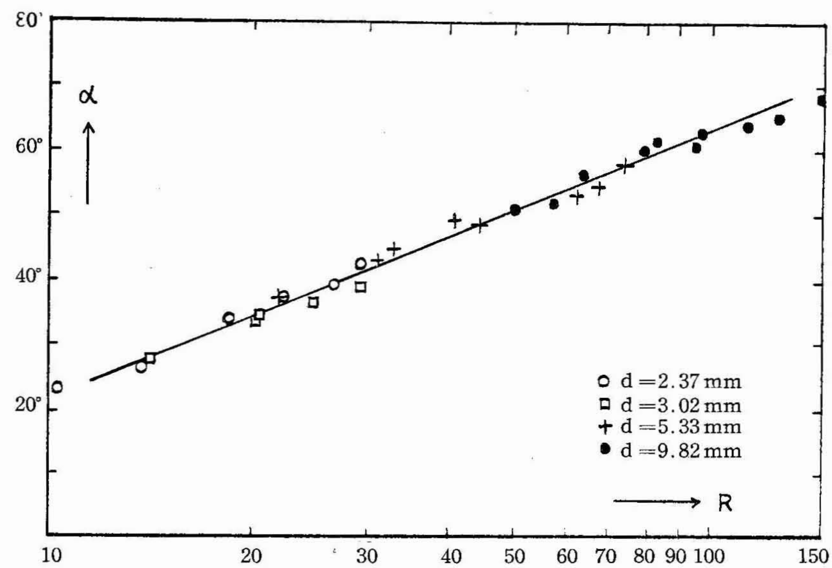


Fig. 5. Separation angles plotted against Reynolds numbers.

Many photographs of the vortex-pair behind a circular cylinder were taken at different Reynolds numbers.

The separation angles are determined by examining the photographs in the Reynolds number range from about 10 to 150.

Below the Reynolds number of about 10, it is considerably difficult to estimate the separation angles since the dye accumulated in the attached eddies becomes vague probably by diffusion.

The angles of separation increase linearly with the logarithm of Reynolds number as shown in figure 5.

§6. The wave length and amplitude

The wave length and amplitude of wake are also determined by examining the photographs.

Both the wave length and amplitude increase with distance downstream from the cylinder as far as the region where the regular features are fully developed.

In the fully developed stable region, both of them are found to be independent of distance for a definite Reynolds number.

The variation of wave length with distance downstream from the cylinder for different Reynolds numbers is illustrated in figure 6.

It is evident from this figure that the distance downstream from the cylinder up to the stable region decreases with increasing Reynolds number.

Figure 7 shows the variation of wave length at the stable region with Reynolds numbers.

The variation of amplitude which is determined at the stable region is shown

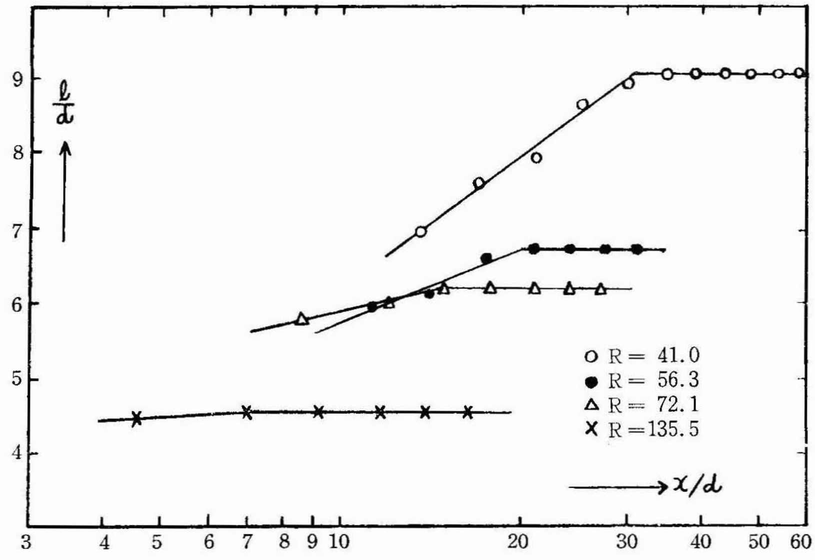


Fig. 6. Wave-length plotted against distance downstream.
(x is the distance from the centre of the cylinder)

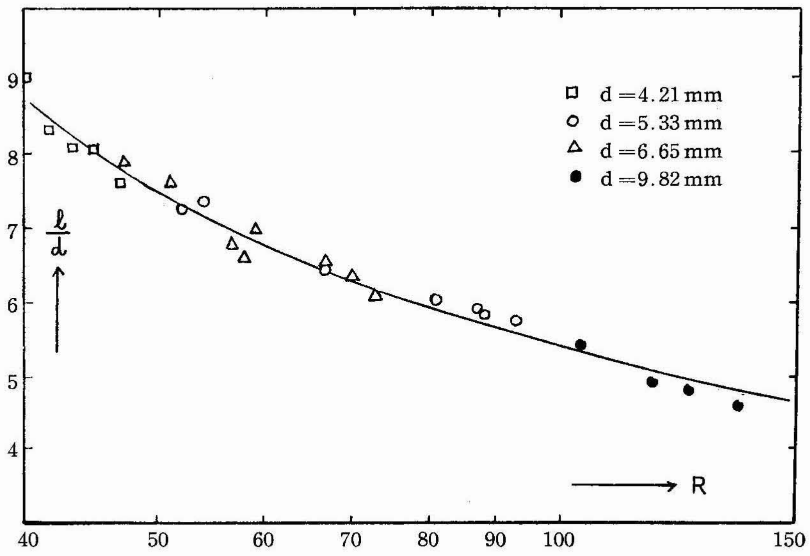


Fig. 7. Wave-length plotted against Reynolds numbers.

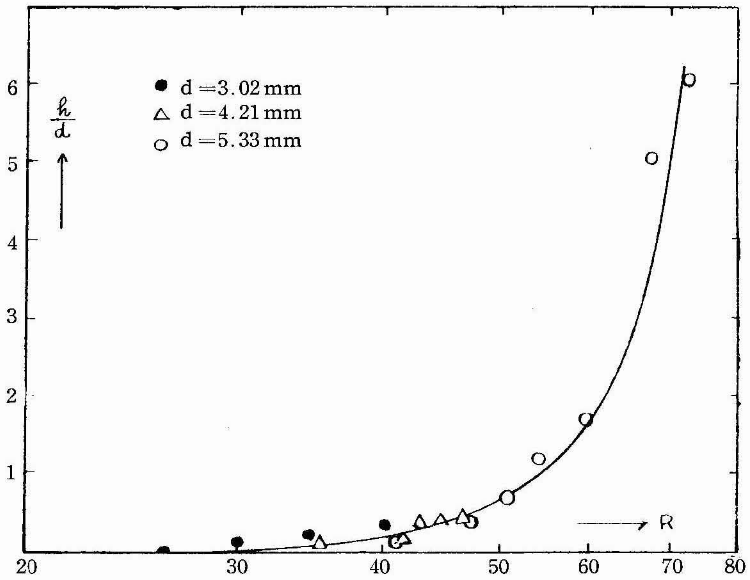


Fig. 8. Amplitude plotted against Reynolds numbers.

in figure 8.

As is found from the figure, the oscillation of line wake begins at the Reynolds number of slightly lower than 30.

The amplitude of wake increases gradually with increasing Reynolds number: suddenly there is a jump in amplitude between the Reynolds number 60 and 70, which means the rolling vortex begins to be shed from the vortex-pair at a certain Reynolds number in this range.

§7. Summary

An experimental investigation was carried out for the purpose of studing certain characteristics of the wake behind a circular cylinder at an intermediate Reynolds number ($10 < R < 150$).

Special attention was paid to the variation of these characteristics with increasing Reynolds number associating with three different stages in its development.

An illustration of wake evolution behind a circular cylinder accelerated from rest to a uniform velocity was also presented.

Some details of the wake geometry were obtained.

Throughout the series of observations, it was found that no turbulent motion was generated in this Reynolds number range.

In conclusion the author would like to express his sincere thanks to Professor T. Maekawa of Hiroshima University for his suggestions and kind encouragement.

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