Study on the Rheological Constants for Molasses

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ABSTRUCT

On molasses, which is the very important by-product from the project of cane-sugar industry on Okinawa, the following rheological constants are obtained by using the BL-type viscometer:

	Revolution per minute (r.p.m.)	Viscosity (c.p.)	Moment (Kg·m)	Depth of cylinder (m) × 10^{-2}	t_y (Kg/m²)
A	6	35,960	5.73	5	4.76
B	6	36,800	5.80	5	4.91
C	6	35,900	5.85	5	4.73
D	6	28,430	3.52	5	2.80
E	6	19,750	2.26	5	2.02
Average		31,368	4.63	5	3.84

Table 1

where,

A: Ryukyu Seitoh K.K.
B: Nishihara Seitoh K.K.
C: Dai-ichi Seitoh K.K.
D: Nohren-Daini Seitoh K.K.
E: Hokubu Seitoh K.K.

1. INTRODUCTION

Molasses is the very important by-product from the sugar product process which is the top project in Okinawa. From the view point of viscosity, this investigation was done to obtaine rheological constant of molasses by means of BL-type viscometer which made by Tokyo Keiki K.K.

The fluid of molasses is not like a ordinary Newtonian-liquid and it is quite high viscus fluid, then the theoretical treatment is complicated.[i)ii)]

Since rheological constants were not obtained yet for molasses on the cane sugar industry in Okinawa, this investigation was tried to make them clear.

2. BASIC THEORY

The BL-type viscometer, as known[iii)] widly, sample was puted in a beaker or a cylinder and clumped the handle to the stand and hanged the viscometer then operated,[iii)] viscosity was obtained through readings on the scale of the viscometer.

For the very high viscus flow,[iv),v)]

$$\frac{dw}{ar}=\frac{(t-t_y)g_c}{e\times r} \tag{1}$$

where,

r: radius of particular layer under consideration. (m)

w: angular velocity of particular layer under consideration. (1/sec)

t: shearing stress at r. (Kg/m)

t_y: yield value of the high viscus materials. (Kg/m)

e: Plastic viscosity. (Kg/m·sec)

g_c: Conversion factor. (Kg·m/Kg·sec^2)

Let,

D_i: radius of inner cylinder. (m)

D_O: radius of outer cylinder. (m)

h: depth of immersion of inner cylinder. (m)

M: moment due to external force. (Kg·m)

then, moment was calculated as follow;

$$M=t\times 2\pi rh\times r$$

hence,

$$t=M/2\pi r^2h \tag{2}$$

at the edge of inner cylinder, shearing stress will be maximum values,

$$r=D_O \tag{3}$$

$$t_{min}=M/2\pi D_O^2h \tag{4}$$

where, r_y (m) is the radius at the point of the following condition,

$$t=t_y$$

so, $t_y=M/2\pi r_y^2h$ (5)

Substitute (1) by (2) and (5),

$$dw=\frac{Mg_c}{2\pi he}(1/r^3-1/r\cdot r_y^2)dr \tag{6}$$

Actualy the moment is not a constant number in case of moment is changed, then t_{max} and t_{min} are also changed.[vi),vii)] On the other hand, t_y is the constant which depend on the materials.[viii)]

Therefore the relationship among t_{max}, t_{min}, and t_y are;

I) $t_{max}\leqq t_y$

II) $t_{min}\leqq t_y\leqq t_{max}$

III) $0\leqq t_y\leqq t_{min}$

I) On the high viscus fluid, there is no slip in construction, if $t<t_y$, hence in case of $t_{max}\leqq t_y$, from the equation (3) and (5) $D_i\geqq r_y$, then slipping is never happened in every parts of the liquid. Fluid showes the property of solid.[ix)]

II) Under the condition of $t_{min}\leqq t_y<t_{max}$, from the equation (3), (4) & (5) $D_O\leqq r_y>D_i$, the slipping will be occured under the conditions of $D_i\leqq r\leqq r_y$ and in the range of $r_y\leqq r\leqq r_y$, in this case slipping never take place and the outer cylinder will be moved with the roter.[ix)]

Let angular velocity of outer cylinder Q (1/sec) and then integrate the equation (6) with condition of;

$$r=D_i \qquad w=0$$

$$r=r_y \qquad w=Q$$

$$\int_{w=0}^{w=Q} dw=\frac{Mg_c}{2he}\int_{r=D_i}^{r=r_y}(1/r^3-1/r\cdot r_y^2)dr$$

$$Q=\frac{M\cdot g_c}{4\pi he}\left(1/D_i^2+ln\frac{D_i}{r_y^2}-1/r_y^2-\frac{ln\, r_y^2}{r_y^2}\right) \qquad (7)$$

III) In case of $0\leqq t_y\leqq t_{min}$, the following condition may be obtained; $r_y\geqq D_O$, so slipping will be occured and integrate the equation (6) with following conditions;

$$r=D_i \qquad w=0$$

$$r=D_O \qquad w=Q$$

$$\int_{w=O}^{w=Q} dw=\int_{r=D_i}^{r=D_O}\frac{M\cdot g_c}{2he}(1/r^3-1/r\cdot r_y^2)dr$$

$$Q=\frac{M\cdot g_c}{4\pi he}\left(1/D_i^2+\frac{ln\, D_i^1}{r_y^2}-1/D_0^2-\frac{ln\, D_0^2}{r_y^2}\right) \qquad (8)$$

several dimensionless terms were applied, like Bingham fluid in the pipe;[x),xi)].

$$t_y/t_{max}=a \qquad \text{or} \qquad t_y=t_{max}\cdot a \qquad (9)$$

$$t_{min}/t_{max}=d \qquad \text{or} \qquad t_{min}=d\cdot t_{max} \qquad (10)$$

from the equation (3), (4) and (5)

$$D_i^2/r_y^2=a \qquad (11)$$

$$D_i^2/D_0^2=d \qquad (12)$$

Therefore, the equation (7) are shown as the following;

$$Q=\frac{M\cdot g_c}{4\pi D_i^2he}(1+a\cdot ln\, a-a) \qquad (13)$$

Beside, on this condition will be denoted as

$$t_{min}\leqq t_y\leqq t_{max}$$

$d \leq a \leq 1$

then, the equation (8) may be express as the following,

$$Q = \frac{M \cdot g_c}{4\pi D_i^2 he}(1 + a \cdot ln\, d - d) \quad (14)$$

On this condition, $0 \leq t_y \leq t_{min}$ is re-noted as $0 \leq a \leq d$ if conditions are,

$$d \leq a \leq 1 \qquad L = 1 + a \cdot lna - a \quad (15)$$

$$0 \leq a \leq d \qquad L = 1 + a \cdot lnd - d \quad (16)$$

therefore the equation (13) and (14) will be shown,[xii]

$$Q = \frac{M \cdot g_c}{4\pi D_i^2 he}(L) \quad (17)$$

Let N is revolution of outer cylinder per minute (r.p.m.),

$$Q = \pi N/30$$

the equation (17) is writen as

$$N = \frac{15M \cdot g_c}{2\pi^2 h D_i^2 e}(L) \quad (18)$$

From the equation (3) and (9),

$$t_y = a \cdot M/(2\pi D_i^2 h) \quad (19)$$

d, which involve the equation (16) is the constant which determined by the radius of inner cylinder, then L of the equation (16) is the function of a:

$$L = f(a)$$

a, d, and L will be very important term to decide rheological constants of high viscus materials, especially for the determination of e and t_y.

On the experimentation to measure viscosity, for the two pairs of M and N the following equations are obtained;

$$N_1/N_2 = M_1 L_1/M_2 L_2$$

then,

$$M_1 N_2/M_2 N_1 = k \quad (20)$$

hence,

$$L_2 = L_1 k \quad (21)$$

where k is the dimensionless constant, and from the equation (19),

$$a_1 M_1 = a_2 M_2 \quad (22)$$

and,

$$a_1 = a_2 j \quad (23)$$

also j is the dimensionless constant.

From the relationship of (15), (16), (21) and (23), a_1 and L_1 are calculated; if a_1 and L_1 are decided,

$$e=\frac{15M_1g_cL_1}{2\pi^2D_i^2hN_1} \tag{24}$$

From the equation (19),

$$t_y=a_1M_1/2\pi D_i^2h \tag{25}$$

t_y is obtained, under the condition of

$$0\leqq a_1\leqq d \qquad \text{and} \qquad 0\leqq a_2\leqq d \tag{26}$$

and

$$L_1=1+a_1\,ln\,d-d \tag{27}$$

$$L_2=1+a_2\,ln\,d-d \tag{28}$$

From the equation of (21), (23), (27) and (28)

$$a_1=\frac{(1-d)(1-k)}{[k-(1/j)]ln\,d} \tag{29}$$

$$a_2=\frac{(1-d)(1-k)}{(kj-1)ln\,d} \tag{30}$$

$$L_1=(1-d)[1-(1/j)]\times[k-(1/j)] \tag{31}$$

Substitute the equation (20) and (22) into the equation (29), (30) and (31),

$$a_1=\frac{(1-d)(M_2N_1-M_1N_2)}{M_1(N_2-N_1)ln\,d} \tag{32}$$

$$a_2=\frac{(1-d)(M_2N_1-M_1N_2)}{M_1(N_2-N_1)ln\,d} \tag{33}$$

$$L_1=\frac{N_1(M_2-M_1)(1-d)}{M_1(N_2-N_1)} \tag{34}$$

From the equation (34) and (24),

$$e=\frac{15(M_2-M_1)(1-d)}{2\pi^2D_i^2h(N_2-N_1)} \tag{35}$$

And, substite the equation (32) to (25)

$$t_y=\frac{(1-d)(M_2N_1-M_1M_2)}{2\pi D_i^2h(N_2-N_1)ln\,d} \tag{36}$$

Also, from the equation (32), (33) and (26),

$$0\leqq\frac{(1-d)(M_2N_1-M_1N_2)}{M_1(N_2-N_1)ln\,d}\leqq d \tag{37}$$

$$0 \leqq \frac{(1-d)(M_2N_1-M_1N_2)}{M_2(N_2-N_1)ln\, d} \leqq d \qquad (37)'$$

These are the method to solve e and t_y without using the trial and error method. In case of Newtonian liquid,

$$a_1 = a_2 = 0$$

Then, from the equation (16)

$$L_1 = L_2 = 1-d \qquad (38)$$

and,

$$M_1N_2 = M_2N_2 = 1 \qquad (39)$$

This is special characteristic for the Newtonian liquid, and from the equation (24) and (25)

$$e = \frac{15M_1g_c}{2\pi^2D_i^2hN_1}(1-d) \qquad (40)$$

then,

$$t_y = 0 \qquad (41)$$

These equations, (40) and (41) may be obtained from the equation (35) and (36).[ii]

3. EXPERIMENTAL DATA

Samples of molasses were taken from Ryukyu Seitoh, Nishihara Seitoh, Daiichi Seitoh, Nohren Daini Seitoh, Hokubu Seitoh, the following datas were obtained.

A) Ryukyu Seitoh K.K.

No.	Revolutions per minute (r.p.m.)	Viscosity (c.p.)	Moment (Kg·m) $\times 10^{-5}$	Depth of cylinder (m) $\times 10^{-2}$	t_y (Kg/m^2)
1	6	36,200	5.98	5	4.95
2	6	35,300	4.88	5	4.25
3	6	36,200	5.92	5	4.93
4	6	36,000	5.82	5	4.73
5	6	35,700	5.54	5	4.38
6	6	35,900	5.74	5	4.80
7	6	35,900	5.72	5	4.81
8	6	36,100	5.90	5	4.90
9	6	36,200	5.93	5	4.91
10	6	36,100	5.92	5	4.90
Average	6	35,960	5.73	5	4.76

Table 2

B) Nishihara Seitoh K.K.

No.	Revolutions per minute (r.p.m.)	Viscosity (c.p.)	Moment (Kg·m) $\times 10^{-5}$	Depth of cylinder (m) $\times 10^{-2}$	t_y (Kg/m^2)
1	6	36,000	5.82	5	4.73
2	6	36,100	5.90	5	4.78
3	6	35,700	5.80	5	4.65
4	6	35,800	5.84	5	4.73
5	6	35,900	5.91	5	4.80
6	6	35,700	5.82	5	4.66
7	6	36,200	5.91	5	4.83
8	6	35,900	5.83	5	4.70
9	6	35,900	5.85	5	4.78
10	6	35,800	5.80	5	4.65
Average	6	35,900	5.85	5	4.73

Table 3

In this case, the molasses was taken from the original Nishihara Seitoh factory, not from the old Nohren Seitoh K.K..

C) Daiichi Seitoh K.K.

No.	Revolution per minute (r.p.m.)	Viscosity (c.p.)	Moment (Kg·m) $\times 10^{-5}$	Depth of cylinder (m) $\times 10^{-2}$	t_y (Kg/m^2)
1	6	36,000	5.82	5	4.73
2	6	36,100	5.90	5	4.78
3	6	35,700	5.80	5	4.65
4	6	35,800	5.84	5	4.73
5	6	35,900	5.91	5	4.80
6	6	35,700	5.82	5	4.66
7	6	36,200	5.91	5	4.83
8	6	35,900	5.83	5	4.70
9	6	35,900	5.85	5	4.78
10	6	35,800	5.80	5	4.65
Average	6	35,900	5.85	5	4.73

Table 4

D) Nohren Dai-ni Seitoh K.K.

No.	Revolution per minute (r.p.m.)	Viscosity (c.p.)	Moment (Kg·m) $\times 10^{-5}$	Depth of cylinder (m) $\times 10^{-2}$	t_y (Kg/m^2)
1	6	29,000	3.89	5	2.88
2	6	27,400	2.93	5	2.50
3	6	28,200	3.50	5	2.75
4	6	28,600	3.76	5	2.83
5	6	28,300	3.69	5	2.82
6	6	28,800	3.80	5	2.87
7	6	27,900	3.48	5	2.79
8	6	28,500	3.75	5	2.88
9	6	28,900	2.55	5	2.82
10	6	28,700	3.80	5	2.86
Average	6	28,430	3.52	5	2.80

Table 5

E) Hokubu Seitoh K.K.

No.	Revolution per minute (r.p.m.)	Viscosity (c.p.)	Moment (Kg·m) $\times 10^{-5}$	Depth of cylinder (m) $\times 10^{-2}$	t_y (Kg/m²)
1	6	19,800	2.21	5	1.98
2	6	19,500	2.15	5	1.86
3	6	20,000	2.53	5	2.23
4	6	19,700	2.19	5	1.90
5	6	19,500	2.18	5	1.89
6	6	19,900	2.29	5	2.05
7	6	19,800	2.20	5	1.93
8	6	19,900	2.30	5	2.10
9	6	19,600	2.27	5	2.14
10	6	19,800	2.31	5	2.12
Average	6	19,750	2.26	5	2.02

Table 6

4. CONCLUSION

On this investigation, study on the rheological constants for the molasses in Okinawa, the following results were obtained.

	Revolution per minute (r.p.m.)	Viscosity (c.p.)	Moment (Kg·m) $\times 10^{-5}$	Depth of cylinder (m) $\times 10^{-2}$	t_y (Kg/m²)
A	6	35,960	5.73	5	4.76
B	6	36,800	5.80	5	4.91
C	6	35,900	5.85	5	4.73
D	6	28,430	3.52	5	2.80
E	6	19,750	2.26	5	2.02

Table 7

For this experiment, there is a pattern, the viscosity and other rheological constants are higher in southern part of Okinawa than in northern part in Okinawa. Where,

A: Ryukyu Seitoh K.K.
B: Nishihara Seitoh K.K.
C: Dai-ichi Seitoh K.K.
D: Nohren-Daini Seitoh K.K.
E: Hokubu Seitoh K.K.

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