The Effect of Walls on the Wake Behind a Circular Cylinder at Low Reynolds Numbers

THE EFFECT OF WALLS ON THE WAKE BEHIND A CIRCULAR CYLINDER AT LOW REYNOLDS NUMBERS

Giichi ARAKAKI

§ 1. Introduction

The effect of the walls of a channel on the characteristics and the stability of the wake behind a circular cylinder have been studied by several authors (Glauert[1] 1928; Rosenhead 1929; Tomotica[2] 1930; Thom[3] 1933, White 1945; Shair[4] 1963; Taneda[5] 1964) from both the theoretical and the experimental points of view.

In the present paper some observations developed by colour dye on the behavior of the wake behind a circular cylinder which is influenced by the proximity of the walls of a water tank are presented.

The purpose of this paper is to consider the wall-effect on the stability of the wake and the features of the flow behind a circular cylinder as the Reynolds number, ($R=Ud/\nu$, where U is the velocity of the cylinder, d the cylinder diameter and ν kinematic viscosity) changes over the range $10<R<150$.

§ 2. Experimental method

The experiments were made in a glass water tank of 75 cm in length, 50 cm in width and 35 cm in depth.

The experimental apparatus and method were the same as those used in a previous paper.[6]

In the present work, two parallel glass plates were used as the side walls. The distance between the two glass plates could vary to fit the desirable distance, so that it was possible to obtain a definite ratio of the diameter of the cylinder to the distance between the walls. This ratio will be referred to as d/H hereafter.

Various values of 0.02, 0.04, 0.1, 0.2, 0.3, 0.4 and 0.5 were used as the ratio d/H. The diameter of the cylinders used were 0.237 cm, 0.421 cm, 0.512 cm, 0.605cm, and 0.795cm, and the lengths of those were 25.0cm respectively.

Wake configurations behind a circular cylinder which is moved midway between two parallel plane walls were recorded with a still camera.

The experimental conditions were carefully controlled before beginning each experiment.

§ 3. The critical Reynolds numbers.

Up to the present, there are a few experimental investigations on the wall-effect on the stability of the wake behind a circular cylinder.

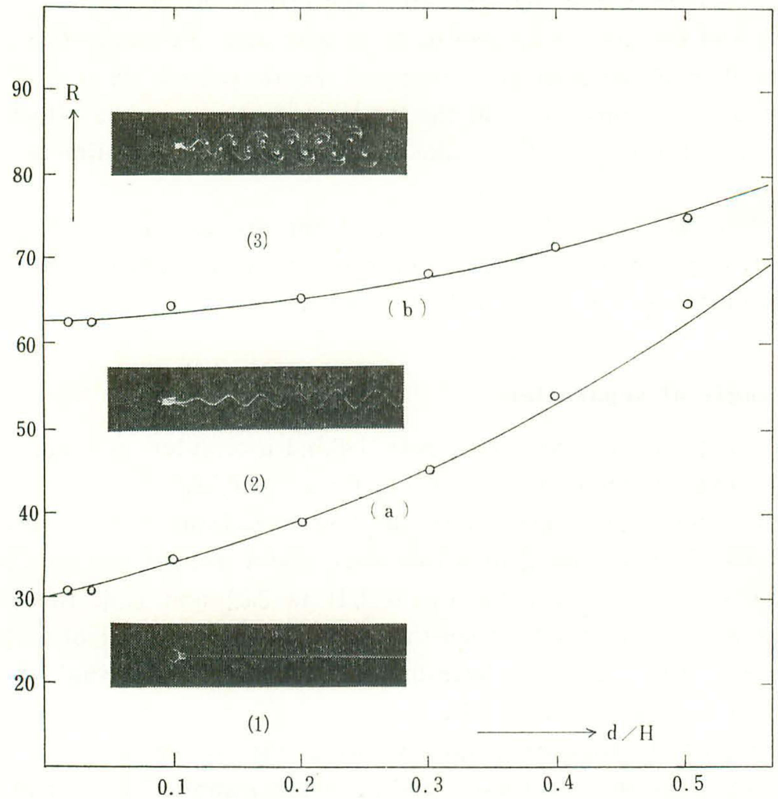


Fig. 1. Variation of the critical Reynolds numbers.

According to Thom[3] (1933), who made in the past one of the extensive investigations of the wake behind a circular cylinder at low Reynolds numbers, it was detected that when the ratio of cylinder diameter to channel width was 0.025 the critical value of Reynolds number was little more than 30, when it was 0.05 the critical value of Reynolds number was about 46, when it was 0.1 eddies did not come off below R=62.

In the previous paper,[6] the author classified the formation of the wake behind a circular cylinder into the following three stages: (1) the standing vortex-pair and straight line wake, (2) the oscillating vortex-pair and waving line wake, and (3) the degenerated vortex-pair and vortex street.

Coresponding to the stages described above, in the present work two kinds of critical Reynolds number were determined by repeating careful inspections of the wake: the one is of the onset of wake oscillations and the other is of the first appearance of rolling vortex street.

The results of the experiments are presented in Fugure 1, which illustrates the wall-effect on the stability of the wake by showing the variation of the critical Reynolds number with the ratio d/H.

In Fugure 1, the curve (a) represents the critical Reynolds number at which the oscillation of the line wake begins to appear and the curve (b) the critical Reynolds number of the first appearance of the rolling vortex street.

These results demonstrate that the walls evidently have the effect to prevent the formation of the waving line wake and to delay the formation of the vortex street.

It would appear that the cross flow from the laminar region outside the wake should be prevented by the presence of the walls since it is known that the wake is unstable against such disturbances.

§ 4. The angle of separation

Many photographs of the vortex-pair behind a circular cylinder were taken at different Reynolds numbers by varying the ratio d/H.

The separation angles were determinded by examining the photographs. The results are shown in Figure 2 in which each point represents the averages of the data thus obtained. When the raitio d/H is 0.02 and 0.04, there is hardly any difference which exists between the line of 0.02 and that of 0.04. It can be seen from the figure that for a definite Reynolds number the angle of separation decreases with increasing ratio d/H.

Two examples of the photographs showing the wall-effect on the standing vortex-pair are illustrated in Plate I and 2. Plate I shows the variation of the standing vortex-pair at the same Reynolds number of 14.0 with the different ratios d/H, and plate 2, the variation of those at the same ratio of 0.3 with the different Reynolds numbers.

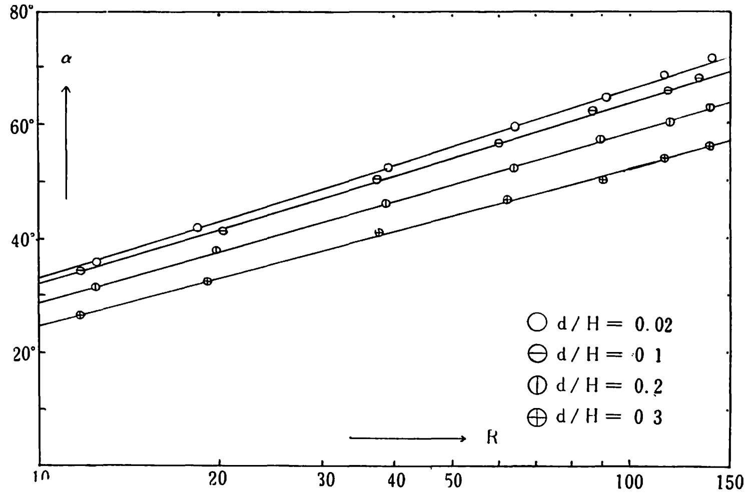
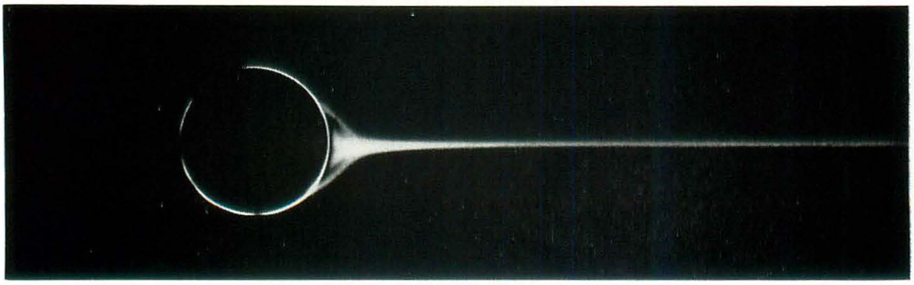
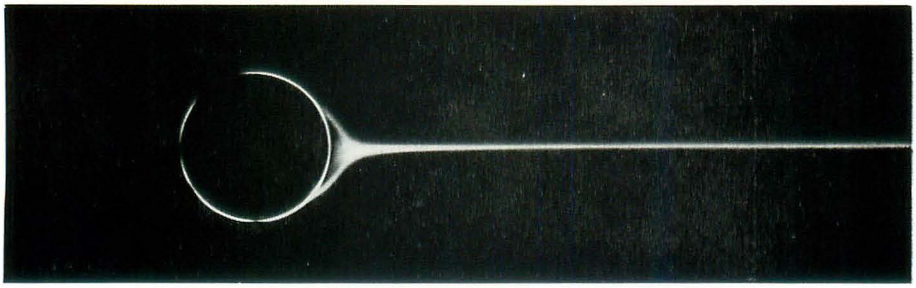


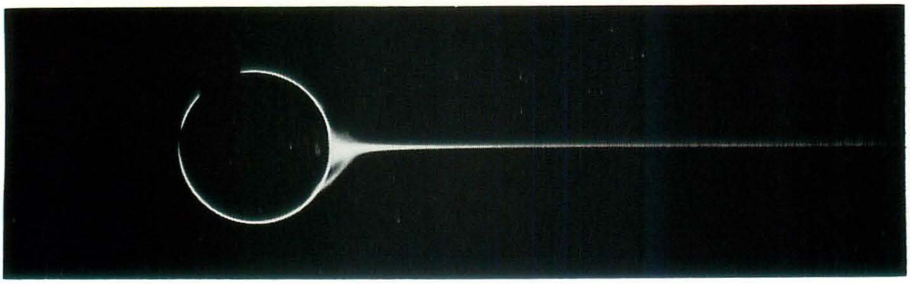
Fig. 2. Separation angles plotted against the Reynolds numbers.



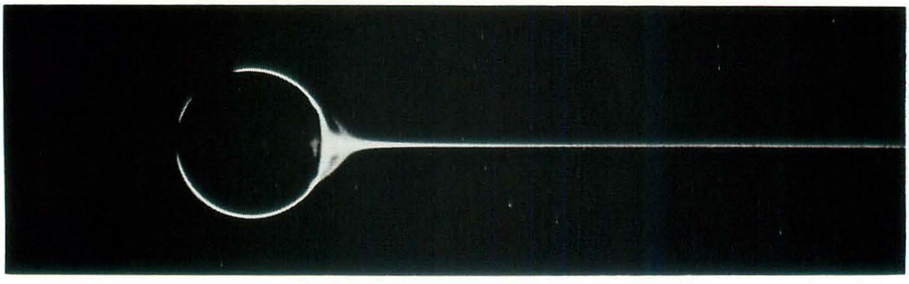
d/H = 0.02



d/H = 0.04

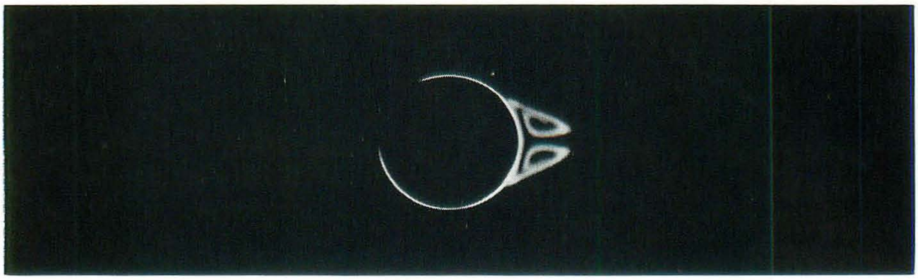


d/H = 0.1

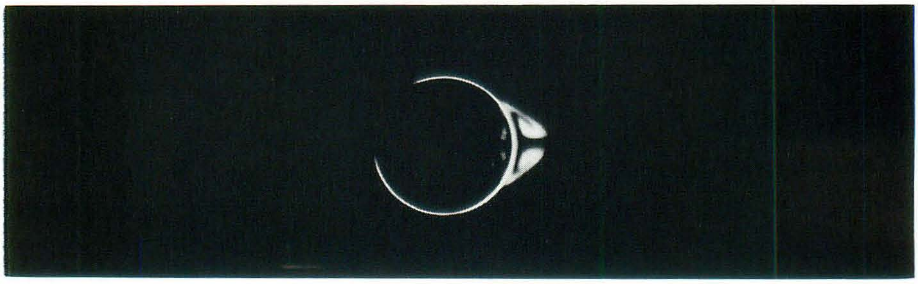


d/H = 0.2

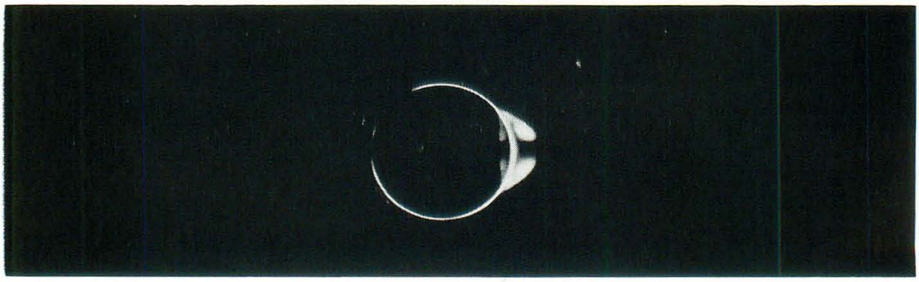
Plate 1. Variation of the standing vortex-pair with d/H. (R=14.0)



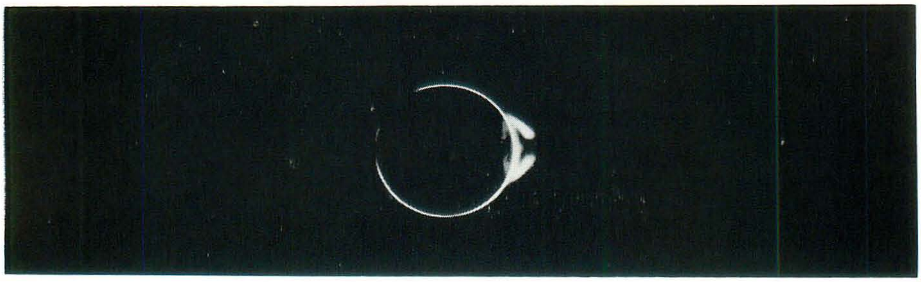
R = 25.4



R = 17.0



R = 15.5



R = 14.6

Plate 2. Variation of the standing vortex-pair with the Reynolds numbers. (d/H=0.3)

§ 5. The wave length

The wave length of the wake was also determined by examining the photographs.

As is shown in the previous paper, the length of the wave increases gradually with distance downstream from the cylinder as far as the region where a constant value of the wave length is maintained before it increases futher in its value beyond this stable region.

The variation of wave length with distance downstream from the cylinder for a Reynolds number of 69.5 is illustrated in Figure 3 with the ratio d/H.

It is evident from this figure that the distance from the cylinder up to the stable region decreases with increasing ratio d/H. Further, the stable region increases in its length with increasing ratio d/H.

For instance, when the ratio d/H is raised up to 0.3, the wake behind the cylinder becomes stable at the beginning of its appearance and persists the status for a considerably long distance downstream from the cylinder.

Thus, the proximity of the walls enhances greatly the stability of the wake. The variation of the wave length which is measured at the stable region with the ratio d/H is shown in Figure 4. In the case that the ratio d/H is 0.02 and 0.04, it is still difficult to find the difference which exists between the curve of 0.02 and that of 0.04. It is found from this figure that the presence of the walls causes a decrease in the length of the waves.

The length of the wave is considerably affected by the value of the ratio d/H, and as the value of the ratio d/H increases the effect becomes larger and more noticeable. It is also found that, as the ratio d/H increases, the length of the waves becomes constant for the Reynolds numbers beyond a certain value.

In the case that the ratio d/H reaches as high as 0.5, the value of the wave length becomes shorter greatly and almost constant of $\lambda/d=2.7$ for the Reynolds number range from about 70 to 150. It seems likely that there exists an upper and a lower liniting values of the wave length for a given ratio d/H and for a sufficiently high ratio, the length of the waves becomes constant with the Reynolds numbers.

The photographs presented in Plate 3 anb 4 illustrate the effect of the walls on the downstream development of the wake behind a circular cylinder. Both of them show the variation of the dimension of the wake at the same Reynolds number with the different ratios d/H.

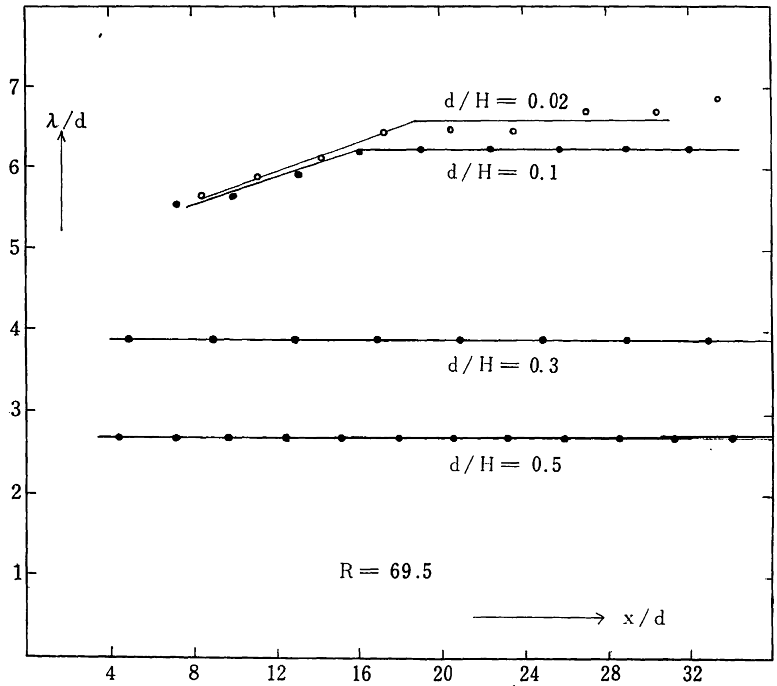


Fig. 3. Wave-length plotted against the distance downstream.
(x is the distance from the center of the circular cylinder.)

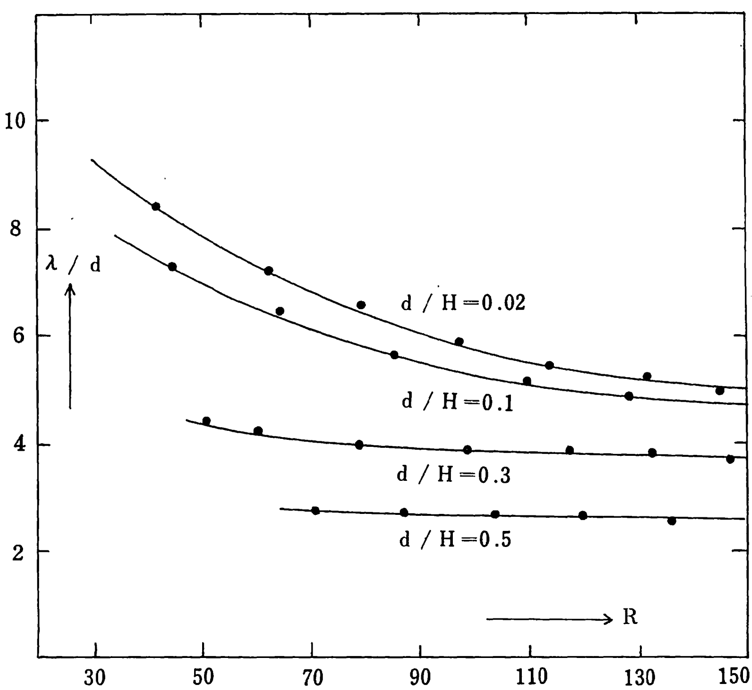
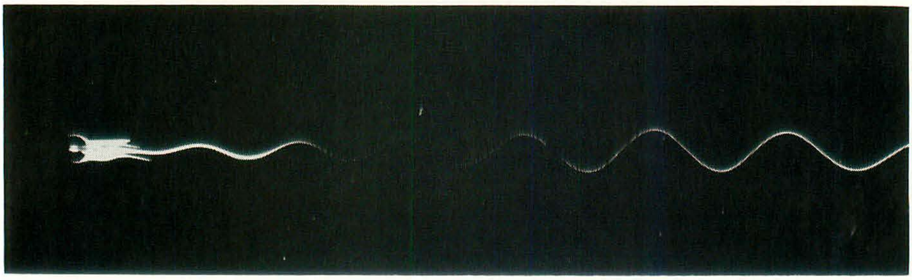
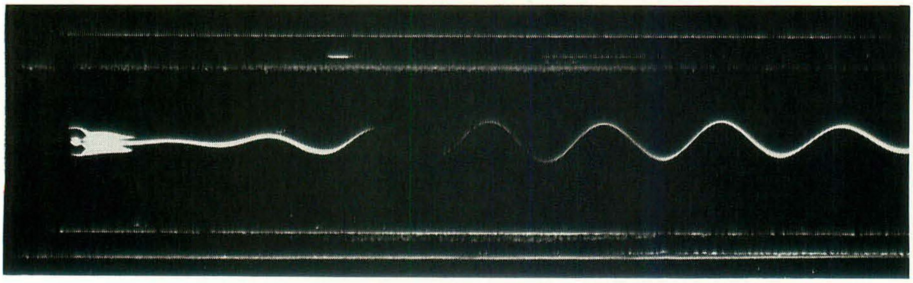


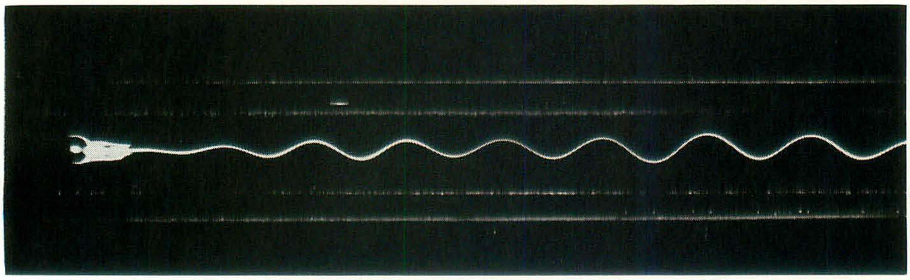
Fig. 4. Wave-length plotted against the Reynolds numbers.



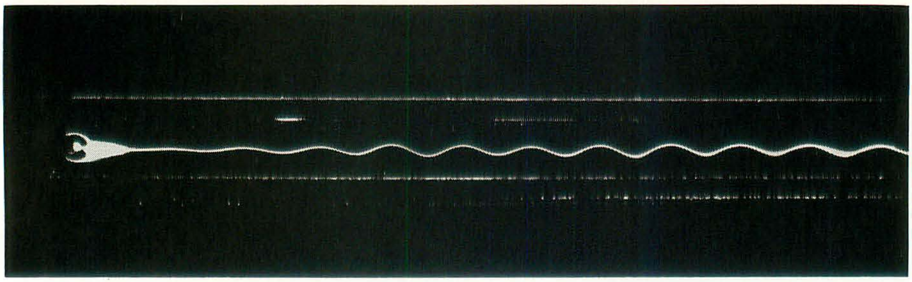
d/H = 0.02



d/H = 0.1



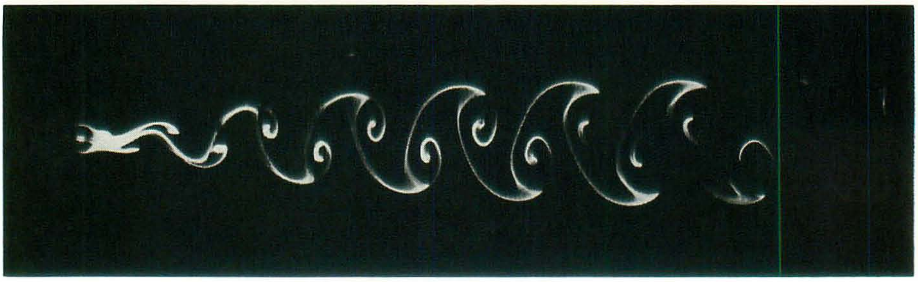
d/H = 0.2



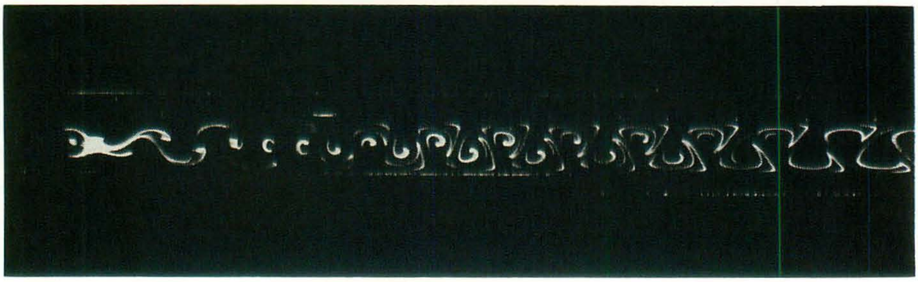
d/H = 0.3

Plate 3. Variation of the waving line wake with d/H. (R=57.4)

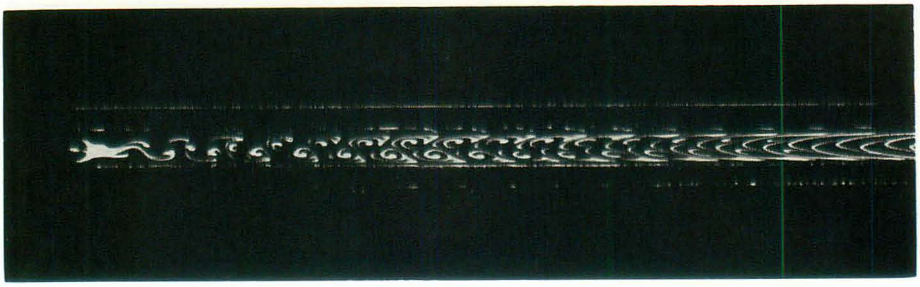
d/H = 0.02



d/H = 0.1



d/H = 0.3



d/H = 0.5

Plate 4. Variation of the vortex street with d/H. (R=77.8)

§ 6. Frequency of the waves

The most dominant effect of the walls is to increase the shedding frequency of the vortex which in turn decreases the length of the wave.

The frequency of the wave can be defined in the nondimentional form as

$$F = Nd/U$$

where N is the frequency of a wave, d the diameter of the cylinder, and U the velocity of the circular cylinder.

To obtain the frequency of a wave, it is necessary to know the ratio V/U, where V is the velocity of the wave form relative to the undisturbed water. The velocity of the wave form was obtained by taking successively the pictures of a definite part of the wave in the wake behind a circular cylinder which moved a distance for a konwn time interval.

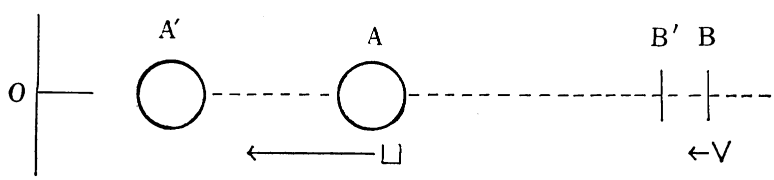


Fig. 5

d=0.512 cm			R=65.0			
d/H	x cm	x/d	λ cm	λ/d	V/U	F
0.5	2.2	4.30	1.39	2.71	0.208	0.292
	3.6	7.03	1.39	2.71	0.214	0.290
	5.0	9.75	1.39	2.71	0.214	0.290
	6.4	12.5	1.39	2.71	0.214	0.290
	7.8	15.2	1.39	2.71	0.214	0.290
	9.1	17.8	1.39	2.71	0.214	0.290
	10.5	20.5	1.39	2.71	0.214	0.290
	12.0	23.4	1.39	2.71	0.214	0.290
0.3	5.0	9.75	2.00	3.91	0.100	0.230
	7.0	13.7	2.05	4.00	0.088	0.228
	9.0	17.6	2.05	4.00	0.088	0.228
	11.1	21.7	2.05	4.00	0.088	0.228
	13.1	25.6	2.05	4.00	0.088	0.228
	15.2	29.7	2.05	4.00	0.088	0.228
	17.2	33.6	2.05	4.00	0.080	0.230
0.1	4.2	8.2	3.00	5.86	0.101	0.153
	5.6	10.9	3.20	6.25	0.081	0.147
	7.2	14.0	3.30	6.44	0.081	0.143
	8.9	17.4	3.40	6.63	0.071	0.140
	10.5	20.5	3.50	6.82	0.066	0.137
	12.2	23.8	3.60	7.03	0.061	0.133
0.02	5.10	9.95	3.25	6.34	0.166	0.132
	6.85	13.4	3.45	6.54	0.152	0.130
	8.50	16.6	3.65	7.12	0.146	0.120
	10.5	20.5	3.70	7.22	0.141	0.119
	12.2	23.8	3.90	7.61	0.121	0.115
	14.4	28.1	4.20	8.20	0.111	0.108

Table 1.

d=0.512 cm			R=75.5			
d/H	x cm	x/d	λ cm	λ/d	V/U	F
0.5	2.3	4.49	1.37	2.67	0.216	0.293
	3.7	7.22	1.37	2.67	0.220	0.292
	5.1	9.95	1.37	2.67	0.220	0.292
	6.4	12.5	1.37	2.67	0.220	0.292
	7.8	15.2	1.37	2.67	0.220	0.292
	9.2	17.9	1.37	2.67	0.220	0.292
	10.5	20.5	1.37	2.67	0.220	0.292
	11.9	23.2	1.37	2.67	0.220	0.292
0.3	4.1	8.01	1.95	3.81	0.102	0.236
	6.0	11.7	2.00	3.91	0.092	0.232
	7.9	15.4	2.00	3.91	0.092	0.232
	9.9	19.3	2.00	3.91	0.092	0.232
	12.0	23.4	2.00	3.91	0.092	0.232
	13.9	27.2	2.00	3.91	0.092	0.232
	15.8	30.8	2.00	3.91	0.087	0.233
0.1	4.65	9.1	2.85	5.56	0.111	0.158
	6.10	11.9	2.95	5.76	0.089	0.158
	7.60	14.8	3.05	5.95	0.083	0.154
	9.15	17.9	3.20	6.25	0.072	0.149
	10.9	21.3	3.30	6.50	0.067	0.144
	12.4	24.2	3.35	6.54	0.061	0.144
0.02	4.75	9.27	3.05	5.95	0.157	0.142
	6.15	12.0	3.10	6.05	0.152	0.140
	7.80	15.2	3.25	6.34	0.147	0.135
	9.40	18.4	3.40	6.63	0.142	0.130
	11.2	21.9	3.50	6.83	0.132	0.127
	12.9	25.2	3.80	7.42	0.123	0.118

Table 2.

In Figure 5, the poind A and A′ represent the center of a circular cylinder with time interval Δt for which a definite point of the wave form, B moves to the point B′. The coordinates of these points A, A′, B and B′ can be measured by the photographs.

Then, the dimentionless frequency of the wave can be obtained from the following relations:

$$F=\frac{1-q}{P}$$

where

$$P=\frac{\lambda}{d}$$

$$q=\frac{V}{U}=\frac{a-a'}{b-b'}$$

and λ is the length of the wave, d the diameter of the cylinder, V the velocity of the wave relative to the undisturved water, U the velocity of the cylinder. and a, a′, b and b′ the coordinates of the points A, A′, B and B′, respectively.

The results thus obtained are listed in Table 1 and 2 in the cases of Reynolds number of 65.0 and 75.5 and shown graphically in Figure 6.

The length of the waves increases as they recede from the cylinder, and this effect produces a retardation on the relative velocity V of the waves with respect to the undisterved water. For this reason the frequency of the waves decreases as the distance downstream is increased. Thus, in a real fluid the frequency of the waves is the function of not only the Reynolds number but also the distance downstream. However, for a sufficiently high ratio d/H, the frequency of the waves becomes constant with the distance downstream from the cylinder for a definite Reynolds number.

In Figure 6 it can be seen that when the ratio d/H is as low as 0.02 and 0.1 the frequency of the waves still decreases gradually with increasing distance downstream, but when the ratio is raised up to 0.3 the frequency becomes independent of the distance downstream from the cylinder. As for the shedding frequency of the vortex (Strauhal number), it can be estimated by extending the lines in Figure 6. It appears therefore from Figure 6 that when the ratio d/H are 0.02 and 0.1 the Strauhal numbers within the Reynolds numbers ranging from 65.0 to 75.5 are still increasing with the Reynolds number, in the case that the value of the ratio d/H reaches as high as 0.5, those numbers become almost constant with the Reynolds number beyond about R=70.

Thus, the proximity of the walls not only increases greatly the Strauhal number, but also by raising the ratio d/H up to a sufficiently high value, makes the frequency of the waves independent of the distance downstream from

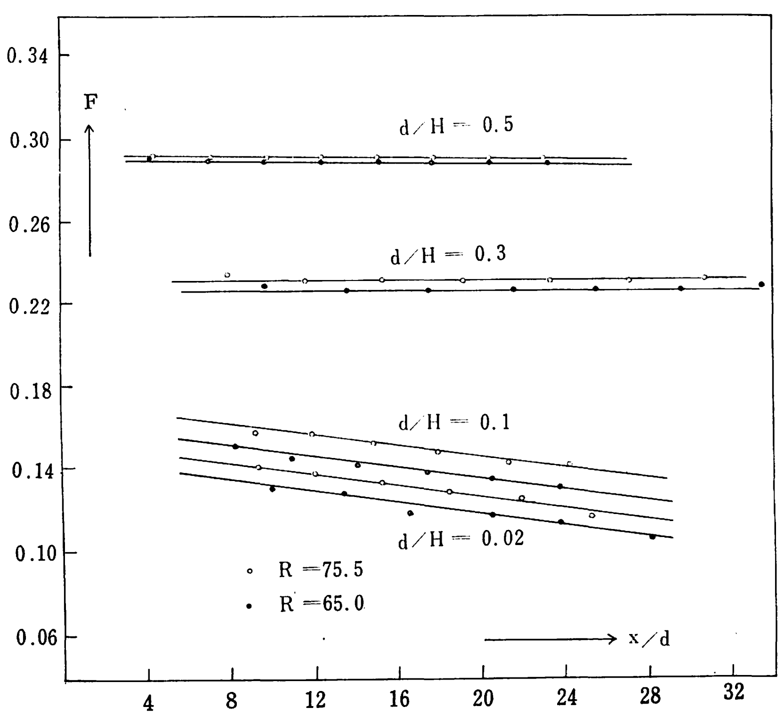


Fig. 6. Frequency of the waves plotted against the distance douwnstream.

the cylinder.

§ 7. Summury

An experimental investigation was made for the purpose of studing the effect of walls on the wake behind a circular cylinder of the Reynolds numbers range from about 10 to 150.

The main results obtained are the followings: The stability of the wake is increased by the proximity of the walls. The walls have a lateral and longitudinal compressing effect on the wake. The angle of separation and the length of the wave for a definite Reynolds number decrease with increasing the ratio of the cylinder diameter to the distance between two walls. The frequncy of the waves is much influenced by the proximity of the walls and increases with inereasing the ratio of the cylinder diameter to the width of the walls.

In conclusion, the author would like to express his sincere thanks to Professor T. Maekawa of Hiroshima University for his suggestions and kind encouragement.

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