Study on the Rheological Behavior for Molasses

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ABSTRACT

Heat transfer coefficients in laminar flow of molasses and rheological constant were conducted on this investigation. The rheological constant "n" and viscosity of molasses were obtained from the results of this experiments, also the heat conductivity and specific heats are came up from the results on this investigation.

For the flow of molasses, the following equations were obtained as the results of this experiments:

$$(h_m D/L)=1.92(D^{1/n}U^{2-1/n}\rho/\eta^*)^{0.3}[(C_p\eta^*/L)(D/U)^{1-1/n}(n+3)/4]^{0.3}(D/L)^{0.3}(\eta/\eta^*)^{0.17}$$
$$=2.04[(GC_p/L\cdot 1)(n+3)/4]^{0.3}(\eta/\eta^*)^{0.17}$$

where,

$(Re)<1800$, $[(GC_p/L\cdot1)(n+3)/4]>70$, and $D=1.18\times10^{-2}$ (m)

I. INTRODUCTION

This is the study on the rheological behavior of molasses which is by-product of sugar plants in Okinawa. The greatest loss of sugar in the factory or refinary is that, which goes to the residual molasses, and the extent to which this sugar may be recovered economically has been the subject of many studies.

This investigation was treated to make the rheological behavior of molasses to be clear from theview point of heat transfer concerened.

II. DIMENSIONLESS NUMBER AND BASIC EQUATION:

On molassis fluid, which is quite high viscus flow, is shown as the following equation;[1),2),4)]

$$Q=(D\cdot g_c/4\pi h\eta)(1/D^2_i+1nD_i-1nr^2_y-1/r^2y-1nr^2_y\ln/r^2_y) \quad (1)$$

Heat transfer concerned for Molasses Fluid is expressed as the following equatinn,[3),5),6)]

$$u_y(\partial\theta/\partial y)=k_d(\partial^2\theta/\partial r^2+1/r\cdot\partial\theta/\partial r) \quad (2)$$

as the following condition,

$$\left.\begin{array}{l} y=0,\ \rightarrow\ \theta=1,\ (t=t_1) \\ r=R\ \rightarrow\ \theta=0,\ (t=t_w) \end{array}\right\} \quad (3)$$

where

$$u_y=(n+3)/(n+1)\cdot U\{1-(r/R)^{n+1}\} \quad (4)$$

then

$$(n+3)U\{[(R-r)/R]-n/2!\cdot[(R-r)/R]^2+n(n/1)/3!\cdot[(R-r)/R]^3\cdots\cdots\}\partial\theta/\partial y$$
$$=k_d(\partial^2\theta/\partial r^2+1/r\cdot\partial\theta/\partial r) \quad (5)$$

For the special solution for the equation (5) is,

$$\theta=A\exp-(2k_y)(n+3)^{-1}(U)^{-1}(b)^{-2}\cdot p \quad (6)$$

from the equation (6) and (5),

$$(d^2o/d\mu^2)+(1/\mu)(dp/d\mu)+2\{(\beta-\mu)\beta^{-1}-n/2!\cdot(\beta\cdot-\mu)^2\cdot\beta^{-2}$$
$$+n(n-1)3!(\beta-\mu)3\beta^{-3}-\cdots\cdots\}p=0 \qquad (7)$$

where, $r/b=\mu$ and $R/b=\beta$ (8)

Then the special solution of (5) is shown as the following equation:[6),7),14)]

$$P(\mu)=\sum_{m=0}^{\infty}\beta_m\mu^m \qquad (9)$$

In case of $r=R$, $\mu=\beta$ and $\theta=0$

$$P(\beta)=0 \qquad (10)$$

hence,

$$\theta=\sum_{m=0}^{\infty}A_m\exp\{-(2\beta_m{}^2K_y)/(n+3)UR^2\}P_m \qquad (11)$$

where,

$$A_m=-\{-1/\beta_m{}^2(dp_m/d\xi)_{\xi=1}\}/\int_0^1 2P_m\xi\{(1-\xi)-n/2!\cdot(1-\xi)^2+n)n-1)/3!\cdot(1-\xi)^3\cdots\}d\xi \qquad (12)$$

and,

$$2[\beta_m{}^2k_y/(n+3)UR^2]=\pi\beta_m{}^2/2\cdot(L/GC_p)(4/n+3)=E_m \qquad (13)$$

therefore

$$\theta=(t-t_w)/(t_1-t_w)=\sum_{m=0}^{\infty}A_me^{-E_my}p_m \qquad (14)$$

at $y=y_1$, and the average temperature t_m the equation expressed as;[8),11),12)]

$$\theta_m=(t_m-t_w)/(t_1-t_w)=1/\pi R^2U\int_0^R 2\,rU_y\theta\ dr=\sum_{m=0}^{\infty}F_mr^{-E_m\cdot y} \qquad (15)$$

where,

$$F_m=-(n+3)A_m(dp_m/d\xi)/\beta^2{}_m \qquad (16)$$

then,

$$Q_y=-L(t_1-t_w)(\partial\theta/\partial r)_{r=R}=(t_1-t_w)\cdot h_y\theta_m \qquad (17)$$

Therefore heat coefficient is expressed as,

$$h_y=-(L/R)\cdot\sum_{m=0}^{\infty}A_me^{-E_m\cdot y}(dp\ _m/d\xi)_{\xi=1}/\sum_{m=0}^{\infty}F_me^{-E_m\cdot y}$$

hence,

$$(h_yD/L)=1/\pi\cdot(GC_p/L\cdot 1)\ln[1/\sum_{m=0}^{\infty}F_me^{-E_m1}]=1/\pi[(GC_p/L\cdot 1)\cdot\ln(1/\theta\cdot M_1)] \qquad (18)$$

Here,

$$\theta=(t_{M1}-t_w)/(t_1-t_w)=\sum_{m=0}^{\infty}F_me^{-E_m\cdot 1} \qquad (19)$$

In case of $n=\infty$,

$$U(\partial\theta/\partial y)=k(\partial^2\theta/\partial r^2+1/r\cdot\ \partial\theta/\partial r) \qquad (20)$$

and,

$$\theta=2\sum_{S=1}^{\infty}\exp[-\phi^2{}_S(L\cdot y/G\cdot C_p)]\cdot J_0(\phi_s\cdot r/R)/\phi_s\cdot J_1(\phi_s) \qquad (21)$$

where,

ϕ_s = roots of $I_0(\phi_0)$

and the dimensionless number is expressed under the average temperature, as the followings;[4),9),13)]

$$(h_m D/L) = 1/\pi \cdot (GC_p/L \cdot l) \ln(1/\theta_{average}) \quad (22)$$

Under the condition fo $U_y = U$,

$$(h_m D/L) = 4/\pi (GC_p/L \cdot l)^{1/2} \quad (23)$$

Between the range of $2 < n < 3$,

$$(h_m D/L) = 1.89\{(GC_p/L \cdot l)(n+3)/4\}^{1/3} \quad (24)$$

The relation-shipb etween $(h_m D/L)$ and $(GC_p/L \cdot l)$ are illustrated as Figure 1;

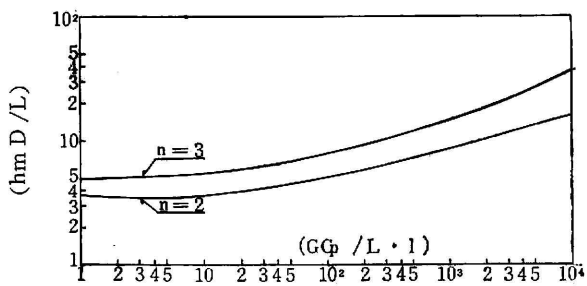

Fig.1 Tne relationship between $(h_m D/L)$ and $(GC_p/L \cdot l)$ which related in the equation (24).

III. EXPERIMENT AND DATA:

1. Deformation of the equation(24):

Because of high viscus flow of molasses fluid, deformation of the epuation was done for the purpose of experiment as the followings,

$$(Re) = D^{-n} U^{2-1/n} \eta^{*-1} \quad (25)$$

hence

$$(h_m D/L) = 1.62[(D^{-1} U^{2-1/n} \rho \cdot \eta^{*-1})\{(C_p \eta^* L^{-1})(D/U)^{1-1/n}(n+3)/4\}(D/1)]^{1/3} \quad (26)$$

On this equation, the second tarm is also dimension-less tarm,[4),5)] therefore

$$(h_m D/L) = C(D^{1/n} U^{2-1/n} \rho \cdot \eta^*)^p \{(C_p \eta^*/L)(D/U)^{1-1/n}(n+3)/4\}^p (D/1)^r \quad (27)$$

The relation between η and η^* are shown as,

$$\eta^* = \{2(n+3)\eta\}^{1/n}/8 \quad (28)$$

hence,

$$(h_m D/L) = C(D^{-1} U^{2-1/n} \rho/\eta^*)^p \{(C_p \eta^*/L)(D/U)^{1-1/n}(n+3)/4\}^b (D/1)^r (\eta/\eta^*)^s \quad (29)$$

where, C, P, q, r, and s are able to determined by experiments.

2. Experimental data;

The following apparatus was set up for this experiments; special attention was paid to keep the temperature constant.

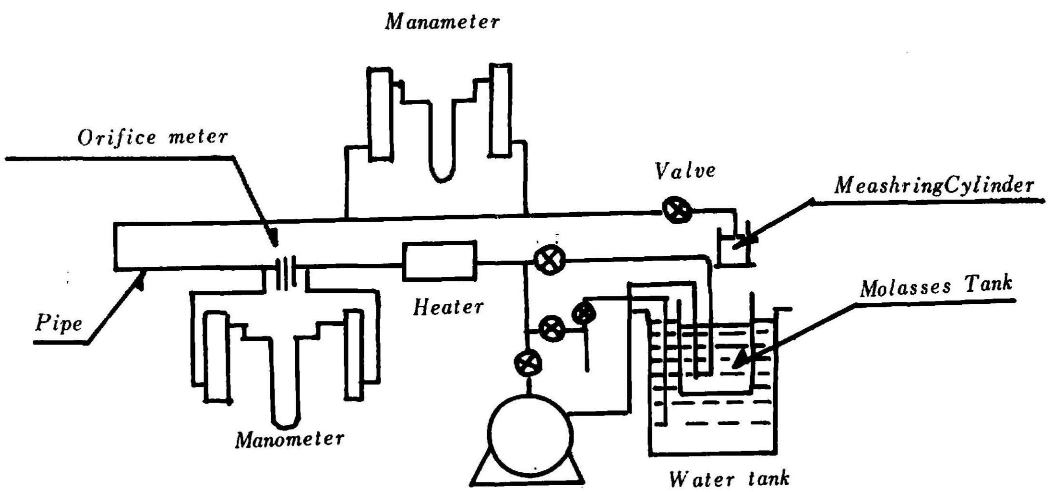

Fig. 2 Apparatus for Experiment.

1) Rheological constants for molasses:

On this experiments, and formar experiments, following data were obtained;

Table 1. The rheological constant for molasses and the Glucose Coefficlents

No.	Molasses	Constant "n"	Glucose Ceefficients	Symbols
1	HKB	1.61	3.62	●
2	DTO	1.62	3.41	×
3	CBU	1.83	3.08	▲
4	IMN	2.51	2.55	■
5	RTO	1.59	2.93	□
6	FPN	1.93	2.67	△
7	TWN	2.91	2.29	○
8	KDT	2.31	2.58	[illegible]
9	IBU	2.34	2.59	[illegible]
10	IGK	2.47	2.50	[illegible]
11	MKO	2.08	2.60	⊕
12	MTA	2.62	2.35	[illegible]
13	KMA	2.67	2.26	⊠

This Glucose Coefficient was calculated as follows:

(Percent Reducing sugar)x100/{Per cent Sucrose(pol.)}=Glucose Coefficient

2) Specific heat and Thermal conductivity[3), 10)] for molasses:

The apparatus, which was illustrated on Figure 2, was used for mesurement of thermal conductivity for molasses, as shown in the following table;

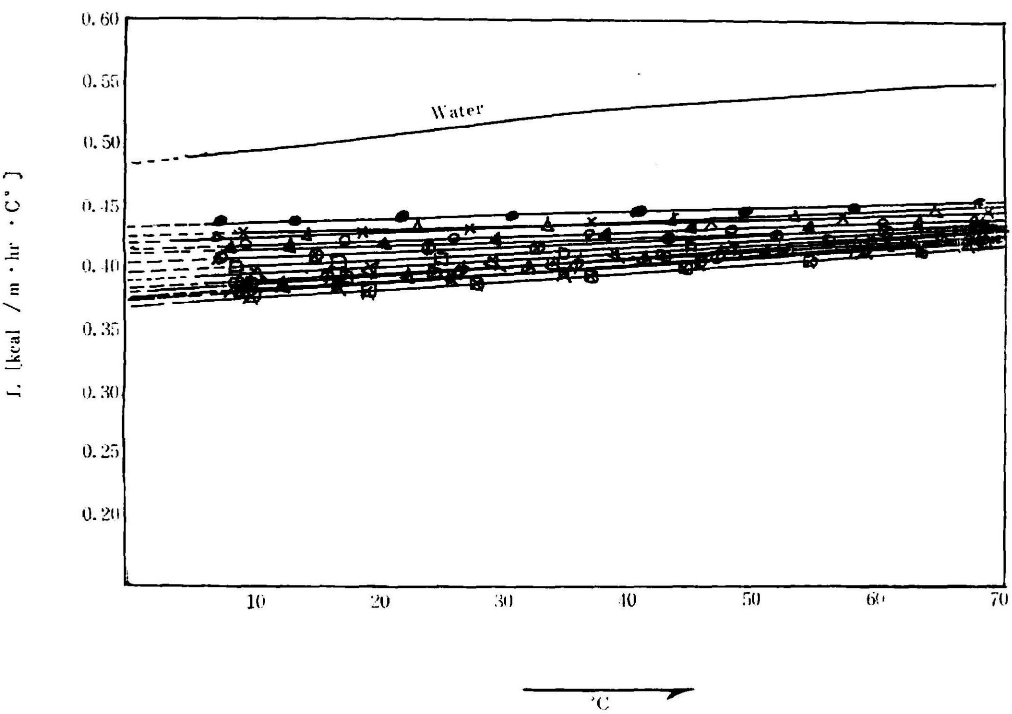

Fig. 3 The Relationship between Thermal Conductivity and Temperature for molasses.

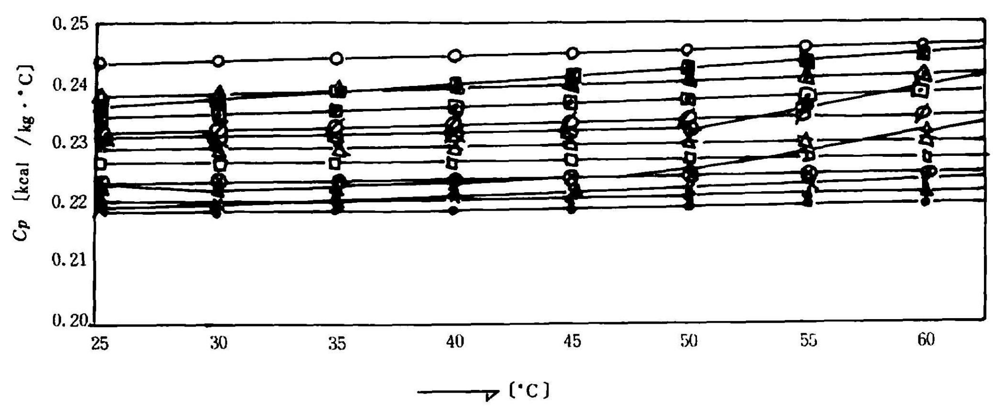

Fig. 4 Specific Heat of Molasses.

Table 2. Thermal conductivity of Molasses.

No.	Molesses	Thermal Conductivity (L)	Temperature (°C)
1	HKB	7.5	0.440
		13.5	0.443
		22.0	0.445
		31.0	0.447
		41.5	0.450
		50.0	0.452
		58.3	0.454
		69.0	0.456
2	DTO	9.4	0.425
		18.5	0.426
		27.5	0.431
		36.9	0.440
		47.0	0.442
		57.5	0.445
		69.5	0.447
3	CBU	8.1	0.415
		13.0	0.420
		20.5	0.425
		29.8	0.425
		38.5	0.425
		45.5	0.433
		55.5	0.435
		63.5	0.437
4	IMN	7.5	0.405
		13.2	0.413
		24.1	0.415
		33.0	0.418
		43.5	0.425
		52.8	0.425
		61.3	0.431
		69.0	0.435
5	RTO	8.0	0.400
		17.5	0.403
		21.0	0.407
		35.0	0.415
		45.2	0.423
		57.5	0.445
		67.0	0.430
6	FPN	7.0	0.425
		14.5	0.428
		23.3	0.429
		34.0	0.435
		44.0	0.435

		53.5	0.440
		65.0	0.455
7	TWN	9.5	0.424
		17.5	0.425
		26.3	0.425
		37.0	0.428
		48.1	0.430
		61.0	0.435
		68.5	0.440
8	KDT	10.5	0.397
		19.5	0.400
		29.3	0.405
		39.2	0.413
		48.9	0.420
		60.0	0.424
		67.5	0.427
9	IBU	8.0	0.385
		16.3	0.390
		36.0	0.405
		47.0	0.412
		58.3	0.420
		69.2	0.425
10	IGK	10.0	0.390
		17.5	0.395
		24.5	0.400
		34.0	0.405
		43.5	0.410
		53.0	0.418
		63.2	0.425
11	MKO	9.0	0.378
		16.5	0.380
		26.0	0.385
		35.1	0.394
		41.3	0.406
		59.4	0.418
		67.8	0.423
12	MTA	12.5	0.385
		25.5	0.394
		32.5	0.400
		41.7	0.408
		51.5	0.425
		62.5	0.430
		69.0	0.435
13	KMA	10.0	0.375
		09.5	0.380
		27.6	0.380

		37.3	0.382
		45.0	0.400
		55.0	0.405
		64.0	0.415

Table 3. Specific heat of Molasses

Temp. (°C)	HKB	DTO	CBU	IMN	RTO	FPN	TWN
25	0.285	0.220	0.220	0.224	0.227	0.223	0.244
30	0.285	0.221	0.220	0.224	0.228	0.221	0.245
35	0.290	0.225	0.221	0.225	0.228	0.224	0.245
40	0.295	0.225	0.222	0.225	0.229	0.225	0.246
45	0.290	0.230	0.226	0.225	0.229	0.225	0.245
50	0.295	0.232	0.230	0.226	0.229	0.226	0.246
55	0.295	0.233	0.235	0.227	0.229	0.228	0.247
60	0.295	0.234	0.240	0.228	0.229	0.237	0.248

Temp. (°C)	KDT	IBU	IGK	MKO	MTA	KMA
25	0.239	0.233	0.235	0.231	0.238	0.237
30	0.239	0.234	0.236	0.232	0.239	0.238
35	0.240	0.234	0.236	0.233	0.239	0.239
40	0.240	0.234	0.237	0.233	0.239	0.241
45	0.241	0.235	0.238	0.234	0.240	0.242
50	0.242	0.235	0.239	0.235	0.241	0.242
55	0.243	0.235	0.240	0.238	0.241	0.244
60	0.240	0.236	0.241	0.239	0.242	0.244

3) Determination of Constants:

By the graphycal method, the constants of C, p, r, and s were determined under the condition of;

$(Re) < 1800, [(GC_p/KL)(n+3)/4] > 70$ aod $D = 1.18 \times 10^{-2}$ (m).

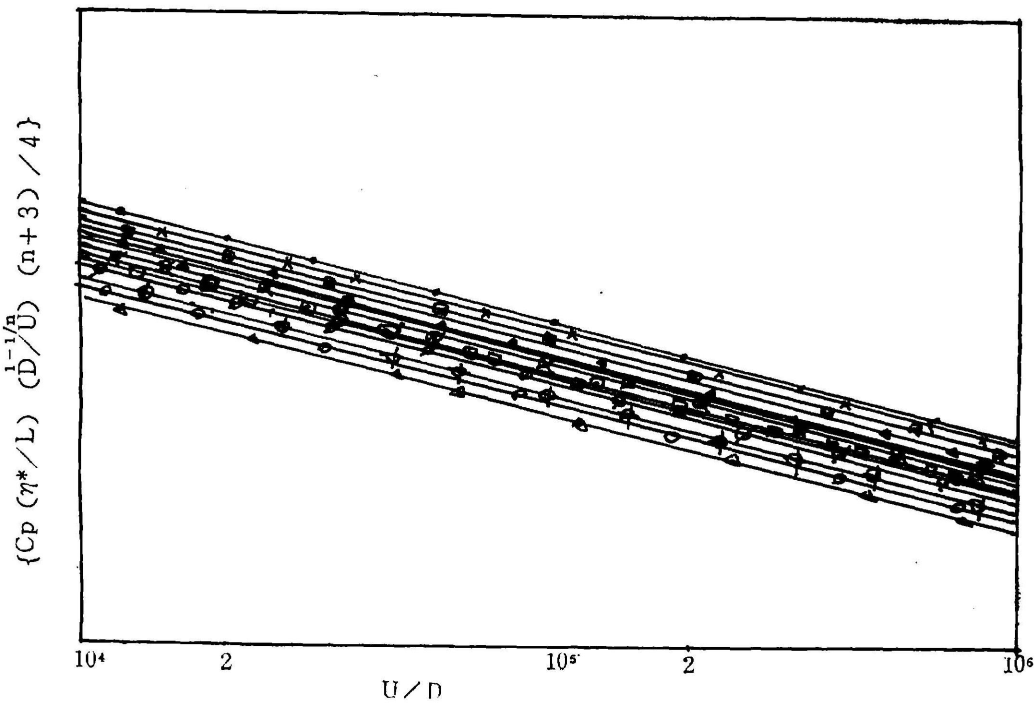

Fig. 5 The Relationship between
$\{Cp(\eta^*/L)(D/U)^{1-1/n}(n+3)/4\}$ and(U/D)

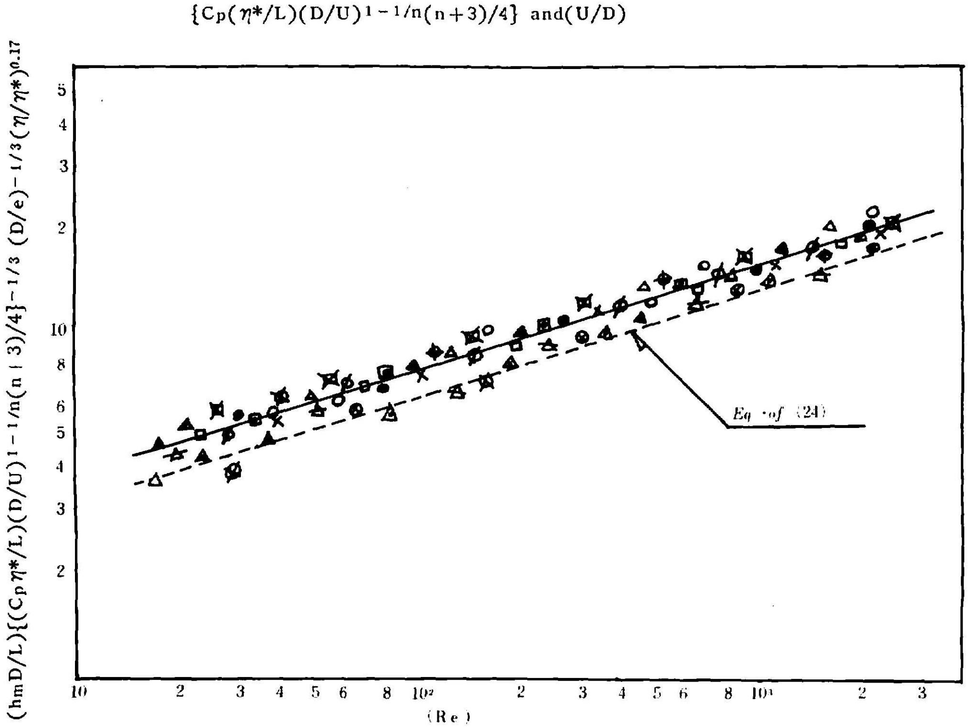

Fig. 6 Correlation of Heat Transfor

The following constant were able to be determined;

$c \equiv 1.92$, $p=0.3$, $p=0.3$, $r=0.3$ and $s=0.17$

IV. CONCLUSION:

For the low of molasses, the following epuation was obtained from the view point of rheological constant "n" and specific heat of molasses;

$$(h_m D/L)=1.92(D^{1/n}U^{2-1/n}\rho/*\eta)^{0.3}\{(C_p\eta*/L)(D/U)^{1-1/n}(n+3)/4\}^{0.3}(D/L)^{0.3}(\eta/\eta*)^{0.17}$$

Under the condition of,

$(Re) < 1800$, $\{(GC_p/L\cdot 1)(n+3)/4\} > 70$ and $D=1.18\times10^{-2}$(m)

NOMENCLATURE

C_p	: Specific heat	(Kcal/kg. °C)
D	: Diameter of pipe	(m)
G	: Mass Rate of Flow	(kg/nr)
h_m	: Average coefficient of heat transfer	(kcal/m^2. hr. °C)
k_d	: Thermal diffusivity	(m^2/hr)
l	: length of pipe heated	(m)
n	: Rheological constant	(−)
L	: Thermal conductivity	(Kcal/m.hr.°C)
Re	: Reynolds number for molasses	(−)
η	: Viscosity of molasses	($kg_n/m^n\cdot hr^{2n-1}$)
$\eta*$	: geneneralized viscosity	($kg/m\cdot hr^{2-1/n}$)
ρ	: density	(kg/m^3)
C, p, q, r, cs	: Constants of dimensional analysis.	
A	: constant	
A_m	: defined by epuation (12)	
E_m	: defined by epuation (13)	
b	: consrant	
F_m	: defined by equation (16)	
J_o, J_1	: Bessel functions of Oth and 1 st order.	
p_m	: defined by equation (11)	
Q_y	: local heat transfer rate	(kcal/m$^2\cdot$hr)
t, t_1, t_w	: local temperature	(°C)
U	: average velocity	(m/hr)
u_y	: velocity at y	(m/hr)
β	: R/b	(m)
ξ	: r/R	(−)
β_m	: roots of $P(\beta)=0$	(m)
θ	: defined by equation (11)	
θ_m	: defined by equation (15)	
ϕ	: roots of J_o	
μ	: r/b	(m)

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