# Cutoff Frequencies of Circular Cylindrical Waveguides containing Ferrites

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By

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## 1. Introduction

Since the tensor premeability was introduced into a "Tellegan"(1) medium by Polder,(2) wave propagation problems in gyromagnetic medium have been treated by several authors.(3)-(7) The circular waveguide with the longitudinal D. C. magnetization was considered by Gamo(3) and Van Trier.(4) The mode behavior in the waveguides was discussed by Kales,(5) Chamber,(6) Suhl and Walker.(7) The parallel plane waveguide was studied by Brodwin and the more general theoretical treatment was done by Epstein(8) on the basis of the expression of the inverse tensor permeability.

This paper develops Epstein's analysis to the circular cylindrical waveguide completely filled with ferrite material and leads to the compatibility equations which determine the propagation constant as a function of the anisotripy in the ferrite. Also characteristic equations for cutoff frequencies are formulated by substituting the zero propagation constant to the compatibility equation.

Numerical calculations of the cutoff frequencies for the several lowest quasi-TE-and quasi-TM-modes, which are reduced to TE-and TM-modes in the limit of vanishing magnetic field, were done under the various values of the anisotropy of the ferrite.

Coaxial lines are usually operated with energy transmitted in the principal (TEM) mode and will transmit a wave of any frequency in this mode. But higher order modes can also exist. Later part of this paper discussed the cutoff frequencies in the coaxial line completely filled with gyromagnetic medium and characteristic equations for the determination of cutoff frequencies are formulated for both quasi-TE-and quasi-TM-modes. Finally the circular waveguide axially containing a concentric ferrite core was considered. However the compatibility condition leads to cumbersome equation.

The several assumptions are considered in the process of the derivation of the basic eqution in the gyromagnetic medium.

1. The electric displacement $D$ does not present anything unusual in the gyromagnetic medium and is related to the electric field strength $E$ by the familiar equation

$$\boldsymbol{D}=\varepsilon \boldsymbol{E} \tag{1}$$

where the dielectric constant $\varepsilon$ is an isotropic scalar either real or complex.

2. Let now a permanent magnetic field $H_0$ be produced in the medium by an external source. This field produces in the medium a magnetic anisotropy of such a nature that the magnetic induction $B$ and the magnetic field strength $H$ of an additional periodic magnetic field superposed upon $H_0$, stand in the following relation by denoting the Polder tensor permeability

$$\boldsymbol{B}=(\mu_T)\cdot\boldsymbol{H} \quad \text{where} \quad (\mu_T)=\begin{vmatrix} \mu_1 & j\mu_2 & 0 \\ -j\mu_2 & \mu_1 & 0 \\ 0 & 0 & \mu_3 \end{vmatrix} \tag{2}$$

3. For the inverse expression of the above equation (2), the reciprocal tensor permeability appears as follows;

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$$\boldsymbol{H}=(\mu_T)^{-1}\cdot\boldsymbol{B} \quad \text{where} \quad (\mu_T)^{-1}=\begin{vmatrix} M_1 & jK & 0 \\ -jK & M_1 & 0 \\ 0 & 0 & M_3 \end{vmatrix} \tag{3}$$

where the parameters in the above tensor are given by

$$M_1=\mu_1/(\mu_1^2-\mu_2^2),\ K=-\mu_2/(\mu_1^2-\mu_2^2),\ M_3=1/\mu_3 \tag{4}$$

4. The anisotiopy $\mu_a$, caused by the applied magnetic field in the gyromagnetic medium, is difined as the ratio of $\mu_1$ and $\mu_2$ or the ratio of $K$ and $M_1$

$$\mu_a=\mu_2/\mu_1 \quad \text{or} \quad \mu_a=-K/M_1 \tag{5}$$

## 2. Cylindrical Waveguide Completely filled with Ferrite

The field for the cylindrical wave in the gyromagnetic medium is obtained from a wave function as a solution of the following cylindrical wave equation

$$\frac{\partial^2\Psi}{\partial r^2}+\frac{1}{r^2}\frac{\partial\Psi}{\partial r}+\frac{1}{r^2}\frac{\partial\Psi}{\partial\varphi}+k_{\frac{1}{2}}{}^2\Psi=0 \tag{6}$$

where the wave function can be represented as the Bessel function of the first kind.

$$\Psi=J_n(k_{\frac{1}{2}}r)\cdot\exp\cdot j\cdot(n\varphi+\Gamma z) \tag{7}$$

Following Epstein, the general field expressions in the gyromagnetic medium are surmarized in the tensor equations. For the electric field,

$$E=(S_E)\cdot\nabla\Psi \quad \text{where} \quad (S_E)=\begin{vmatrix} i\sigma & \tau & 0 \\ -\tau & \sigma & 0 \\ 0 & 0 & -i\sigma k_{\frac{1}{2}}{}^2/\Gamma^2 \end{vmatrix} \tag{8}$$

where the parameters in the tensor are abbreviated as follaws

$$\begin{aligned} &\tau_{\frac{1}{2}}=M_1(k_{\frac{1}{2}}+\Gamma^2)-\varepsilon\cdot\omega^2, \quad \sigma=K\Gamma^2 \\ &k_{\frac{1}{2}}=[\varepsilon\omega^2-M_1\Gamma^2)\cdot(M_1+M_3)+K^2\Gamma^2\pm f]/2M_1M_3 \\ &f=[(M_1-M_3)^2-(\varepsilon\omega^2-M_1\Gamma^2)^2+2(M_1+M_3)\cdot(\varepsilon\omega^2-M_1\Gamma^2)\cdot K^2\Gamma^2+K^2(K^2+4M_1M_3)\Gamma^4]^{1/2} \end{aligned} \tag{9}$$

The above parameters $k$, $\sigma$ and $\tau$ can be written in terms of the propagation constant $\Gamma$ and the anisotropy $\mu_a$

$$\begin{aligned} &k_{\frac{1}{2}}=\omega\sqrt{\varepsilon\mu_1}[1-\Gamma_r{}^2-\mu_a{}^2/2\pm\mu_a\sqrt{(\mu_a/2)^2+\Gamma_r{}^2}]^{1/2} \\ &\tau_{\frac{1}{2}}=\omega^2\varepsilon\mu_a[\mu_a/2\pm\sqrt{(\mu_a/2)^2+\Gamma_r{}^2}]/(1-\mu_a{}^2) \\ &\sigma=\mu_a\omega^2/(1-\mu_a{}^2) \end{aligned} \tag{10}$$

where the normarized propagation constant $\Gamma_r$ is given by $\Gamma_r=\Gamma/\omega\sqrt{\varepsilon\mu_1}$ and the anisotropy $\mu_a$ is defined by $\mu_a=\mu_2/\mu_1=-K/M_1$.

The magetic field is given in the another tensor equation

$$H=\Gamma\cdot(S_H)\cdot\nabla\cdot\Psi \quad \text{where} \quad (S_H)=\begin{vmatrix} a & jb & 0 \\ -jb & a & 0 \\ 0 & 0 & g \end{vmatrix} \tag{11}$$

where the parameters in the tensor have the form

$$\begin{aligned} &a_{\frac{1}{2}}=[M_1\tau_{\frac{1}{2}}-K^2(\Gamma^2+k_{\frac{1}{2}}{}^2)]/k_0 \\ &b=-K\varepsilon k_0, \qquad g_{\frac{1}{2}}=-M_3k_{\frac{1}{2}}{}^2\tau_{\frac{1}{2}}/k_0\Gamma^2 \end{aligned} \tag{12}$$

By substituting Eq. (7) into Eq. (8), we can obtain all components of the electric field with the help of the idintity $J'_n(x)=nJ_n(x)/x-J_{n+1}(x)$.

$$E_{\substack{1\\2}r}(k_{\substack{1\\2}}r)=j[k_{\substack{1\\2}}\sigma J'_n(k_{\substack{1\\2}}r)+n\tau_{\substack{1\\2}}J_n(k_{\substack{1\\2}}r)/r]=j[-k_{\substack{1\\2}}\sigma J_{n+1}(k_{\substack{1\\2}}r)+n(\tau_{\substack{1\\2}}+\sigma)J_n(k_{\substack{1\\2}}r)/r]$$
$$E_{\substack{1\\2}\varphi}(k_{\substack{1\\2}}r)=-[k_{\substack{1\\2}}\tau_{\substack{1\\2}}J'_n(k_{\substack{1\\2}}r)+n\sigma J_n(k_{\substack{1\\2}}r)/r]=k_{\substack{1\\2}}\tau_{\substack{1\\2}}J_{n+1}(k_{\substack{1\\2}}r)-n(\tau_{\substack{1\\2}}+\sigma)J_n(k_{\substack{1\\2}}r)/r \quad (13)$$
$$E_{\substack{1\\2}z}(k_{\substack{1\\2}}r)=k_{\substack{1\\2}}^2\sigma J_n(k_{\substack{1\\2}}r)\cdot/\Gamma.$$

where the factor exp. $j(n\varphi+\Gamma z)$ is eliminated for each component. Also the substitution of Eq. (7) into Eq. (11) leads to the expressions for the magnetic field.

$$H_{\substack{1\\2}r}(k_{\substack{1\\2}}r)=\Gamma[k_{\substack{1\\2}}\cdot a_{\substack{1\\2}}\cdot J'_n(k_{\substack{1\\2}}r)-nbJ_n(k_{\substack{1\\2}}r)/r]=\Gamma[-a_{\substack{1\\2}}k_{\substack{1\\2}}J_{n+1}(k_{\substack{1\\2}}r)+n(a_{\substack{1\\2}}-b)J_n(k_{\substack{1\\2}}r)/r]$$
$$H_{\substack{1\\2}\varphi}(k_{\substack{1\\2}}r)=\Gamma[-jbk_{\substack{1\\2}}J'_n(k_{\substack{1\\2}}r)+jna_{\substack{1\\2}}J_n(k_{\substack{1\\2}}r)/r]=\Gamma[jbk_{\substack{1\\2}}J(k_{\substack{1\\2}}r)+jn(a_{\substack{1\\2}}-b)J_n(k_{\substack{1\\2}}r)/r]$$
$$H_{\substack{1\\2}z}(k_{\substack{1\\2}}r)=j\Gamma^2\cdot g_{\substack{1\\2}}\cdot J_n(k_{\substack{1\\2}}r) \quad (14)$$

Since the field is expressed as the sum of the two solutions $E_1$ and $E_2$, $H_1$ and $H_2$, which correspond to the parameters $k_1$, $\tau_1$ and $k_2$, $\tau_2$, the expressions for the electric field are given by

$$\begin{aligned} E_r&=A_1E_{1r}(k_1r)+A_2E_{2r}(k_2r)\\ E_\varphi&=A_1E_{1\varphi}(k_1r)+A_2E_{2\varphi}(k_2r)\\ E_z&=A_1E_{1z}(k_1r)+A_2E_{2z}(k_2r)\end{aligned} \quad (15)$$

Also for all components of the magnetie field

$$\begin{aligned} H_r&=A'_1H_{1r}(k_1r)+A'^2H_{2r}(k_2r)\\ H_\varphi&=A'_1H_{1\varphi}(k_1r)+A'_2H_{2\varphi}(k_2r)\\ H_z&=A'_1H_{1z}(k_1r)+A'_2\cdot H_{2z}(k_2r)\end{aligned} \quad (10)$$

where the constants $A_1$, $A'_1$, $A_2$ and $A'_2$ are determined from the boundary conditions at the wall of the waveguide.

For the convenience of the numerical calculations, the radius of the cylindrical waveguide is chosen as unity. Then the boundary conditions at $r=1$ are expressed by

$$E_\varphi(1)=E_z(1)=0, \quad \frac{\partial H_\varphi}{\partial r}\bigg|_{r=1}=\frac{\partial H_z}{\partial r}\bigg|_{r=1}=0 \quad (17)$$

Substitutions of Eq. (15) into Eq. (17) lead to the compatibility equations for the electric field.

$$\begin{aligned} &A_1[k_1\tau_1J_{n+1}(k_1)-n(\tau_1+\sigma)J_n(k_1)]+A_2[k_2\tau_2J_{n+1}(k_2)n(\tau_2+\sigma)\cdot J_n(k_2)]=0\\ &A_1\cdot k_1^2\cdot J_n(k_1)+A_2\cdot k_2^2\cdot J_n(k_2)=0\end{aligned} \quad (18)$$

Also by substituting Eq. (16) into Eq. (17) we obtain the following equations for the magnetic field.

$$\begin{aligned} &A'_1k_1^2\cdot J'_{n+1}(k_1)+A'_2k_2^2J'_{n+1}(k_2)=0\\ &A'_1k_1^3\tau_1J'_n(k_1)+A'_2k_2^3\tau_2\cdot J'_n(k_2)=0\end{aligned} \quad (19)$$

Since the above equations (18) and (19) must be compatible, the determinants, consisted by the parameters $A_1$, $A'_1$, $A_2$ and $A'_2$, should be zero., which leads to the transcendental equations for the field propagating in the ferrite filled cylindrical waveguide. For the electric field, we obtain from Eq. (18).

$$k_2^2J_n(k_2)\cdot[k_1\tau_1J_{n+1}(k_1)-n(\tau_1+\sigma)J_n(k_1)]=k_1^2J_n(k_1)[k_2\tau_2J_{n+1}(k_2)-n(\tau_2+\sigma)J_n(k_2)] \quad (20)$$

For the magnetic field in Eq. (19), we can reduce to the equation

$$k_2\tau_2J'_{n+1}(k_1)\cdot J'_n(k_2)=k_1\tau_1J'_{n+1}(k_2)\cdot J'_n(k_1) \quad (21)$$

The solutions of the above transcendental equation (20) and (21) give us the permissible propagation constant in the cylindrical waveguide which is completely filled with the gyromagnetic medium.

In order to obtain the characteristic equations for the determination of cutoff frequencies, we substitute $\Gamma_r=0$ to the transcendental equations (20) and (21). For the zero propagation constant, the parameters $k_{\substack{1\\2}}$, $\tau_{\substack{1\\2}}$ and $\sigma$ for the electric field in Eq. (10) can be written as follows

$$k_1=\omega\sqrt{\varepsilon\mu_1}\equiv k_c,\quad k_2=\omega\sqrt{\varepsilon\mu_1}\sqrt{1-(\mu_a/2)^2}\equiv\beta k_c$$
$$\tau_1=\omega^2\varepsilon\mu_a/(1-\mu_a^2),\quad \tau_2=0,\quad \sigma=0$$
$$\text{where}\quad \beta=\sqrt{1-(\mu_a/2)^2} \tag{22}$$

Also the parameters in the tensor for the magnetic field in Eq. (12) become the following forms by substituting Eq. (21) into Eq. (12)

$$a_1=[M_1\tau_1-K^2k_1^2]/k_0,\quad a_2=-K^2\cdot k_2^2\cdot/k_0.$$
$$\Gamma^2g_1=-M_3k_1^2\tau_1/k_0,\qquad \Gamma^2g_2=0. \tag{23}$$

By substituting Eq. (21) into Eq. (18), we obtain the equation

$$J_n(\beta k_c)\cdot[k_1J_{n+1}(k_c)-nJ_n(k_c)]=0$$

which is either $J_n(\beta k_c)=0$ or $k_c\cdot J_{n+1}(k_c)=nJ_n(k_c)$

Since the second relation does not contain the magnetization dependency, it is not applicable for the equation for the determination of the cutoff frequencies. The first equation becomes

$$J_n(\sqrt{1-(\mu_a/\mu)^2}\cdot k_c)=0 \tag{24}$$

which is identical with the equation for the determination of the cutoff frequencies of the transverse-magnetic modes in the isotropic medium.

Also by substituting Eq. (21) into Eq. (14), we obtain the equation for the determination of the cutoff frequencies for the magnetic wave.

$$J'_n(\sqrt{1-(\mu_a/\mu)^2}\cdot k_c)=0 \tag{25}$$

which leads to the identical equation $J'_n(k_c)=0$ for the determination of the cutoff frequencies of the transverse-electric modes in the isotropic medium, so that the two types of modes, determined by the equation (24) and (25), belong to the modes corresponding respectively to the *TM*-and *TE*-modes. As Gamo states, they are called respectively the quasi-*TM*-modes and quasi-*TE*-modes, because they have the $z$-components of both electric and magnetic fields.

As shown in Fig. (1) and Fig. (2), the results of the numerical calculations of Eq. (23) and Eq. (24) for the several lowest modes state that the cutoff frequencies for both quasi-*TE*-and quasi-*TM*-modes increase as the magnetization in the ferrite increases.

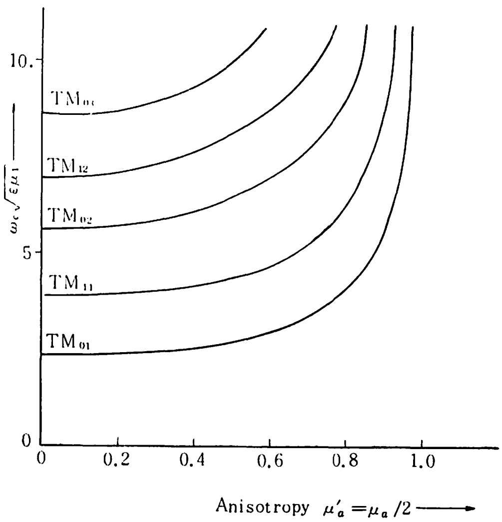


Fig. (1). The cutoff frequencies for quasi-TM-modes against the anisotrophy $\mu_a'=\mu_a/2$.

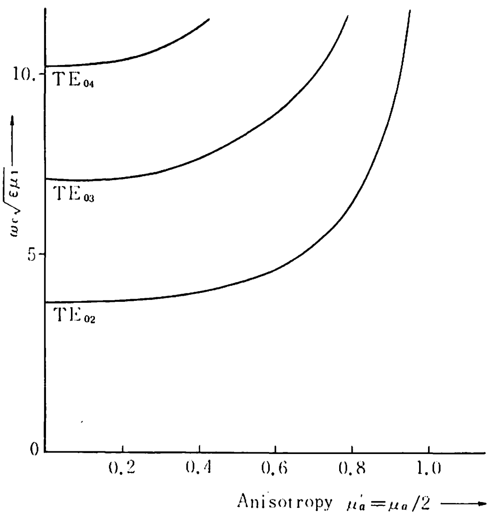


Fig. (2). The cutoff frequencies for quasi-TE-modes against the anisotropy $\mu_a'=\mu_a/2$.

## 3. Coaxial Lines Completely filled with Ferrite

We consider the coaxial lines completely filled with gyromagnetic medium between the inner and the outer cylinders. For the convenience of calculations, we assume that the lines have the outer radius 1 and the inner radius $R$. The boundary conditions at the surface of the inner and outer cylinder are

$$E_\varphi=E_z=0,\quad \frac{\partial H_\varphi}{\partial r}=\frac{\partial H_z}{\partial r}=0 \quad \text{at } r=1 \text{ and } r=R \tag{26}$$

Since the singular line $r=0$ of the scalar wave equation (6) is now excluded from the range of the variable, the following solution can be used in addition to Eq. (7).

$$\Psi=N_n(k_{\substack{1\\2}}r)\cdot\exp\cdot j(n\varphi+\Gamma z) \tag{27}$$

By adding the following equation (28) and (29) into Eq. (13) and (14). We can obtain all components of $E$ and $H$

$$\begin{aligned}
&E_{\substack{1\\2}r}(k_{\substack{1\\2}}r)=j[k_{\substack{1\\2}}\sigma N'_n(k_{\substack{1\\2}}r)+n\tau_{\substack{1\\2}}N_n(k_{\substack{1\\2}}r)/r]=j[-k_{\substack{1\\2}}\sigma N_{n+1}(k_{\substack{1\\2}}r)+n(\tau_{\substack{1\\2}}+\sigma)N(k_{\substack{1\\2}}r)/r]\\
&E_{\substack{1\\2}\varphi}(k_{\substack{1\\2}}r)=-[k_{\substack{1\\2}}\tau_{\substack{1\\2}}N'_n(k_{\substack{1\\2}}r)+n\sigma N_n(k_{\substack{1\\2}}r)/r]=k_{\substack{1\\2}}\cdot\tau_{\substack{1\\2}}N_{n+1}(k_{\substack{1\\2}}r)-n(\tau_{\substack{1\\2}}+\sigma)N_n(k_{\substack{1\\2}}r)/r]\\
&E_{\substack{1\\2}z}(k_{\substack{1\\2}}r)=k_{\substack{1\\2}}\sigma N_n(k_{\substack{1\\2}}r)/\Gamma.
\end{aligned} \tag{28}$$

and for the magnetic field, we obtain the following equations.

$$\begin{aligned}
&H_{\substack{1\\2}q}(k_{\substack{1\\2}}r)=\Gamma[k_{\substack{1\\2}}a_{\substack{1\\2}}N'_n(k_{\substack{1\\2}}r)-nbN_n(k_{\substack{1\\2}}r)/r]=\Gamma[n(a_{\substack{1\\2}}-b)N_n(k_{\substack{1\\2}}r)/r-a_{\substack{1\\2}}k_{\substack{1\\2}}N_{n+1}(k_{\substack{1\\2}}r)]\\
&H_{\substack{1\\2}\varphi}(k_{\substack{1\\2}}r)=-j\Gamma[k_{\substack{1\\2}}b\cdot N'_n(k_{\substack{1\\2}}r)-nbN_n(k_{\substack{1\\2}}r)/r]=j\Gamma b\cdot k_{\substack{1\\2}}N_{n+1}(k_{\substack{1\\2}}r)\\
&H_{\substack{1\\2}z}(k_{\substack{1\\2}}r)=j\Gamma^2\cdot g\cdot N_n(k_{\substack{1\\2}}r)
\end{aligned} \tag{29}$$

The two values of $k$ being always available, the complete expressions of $E_\varphi$ and $E_z$ become

$$\begin{aligned}
E_\varphi&=A_1\cdot E_{1\varphi}+A_2\cdot E_{2\varphi}+A_1\cdot E_{1\varphi}+A_2\cdot E_{2\varphi}\\
E_z&=A_1\cdot E_{1z}+A_2\cdot E_{2z}+A_1\cdot E_{1z}+A_2\cdot E_{2z}
\end{aligned} \tag{30}$$

also for $H_\varphi$ and $H_z$

$$H_\varphi = A'_1 \cdot H_{1\varphi} + A'^2_2 H_{2\varphi} + A'_1 \cdot H_{1\varphi} + A'_2 \cdot H_{2\varphi}$$
$$H_z = A'_1 \cdot H_{1z} + A'_2 \cdot H_{2z} + A'_1 \cdot H_{1z} + A'_2 \cdot H_{2z} \quad (31)$$

Substituting the above equation (29) and (30) into the boundary conditions (25), these expressions lead to a system of four simultaneous equations which are linear and homogenious in the four coefficients $A$ and $A'$. Then the compatibility condition for the quasi-$TM$-modes has the form

$$\begin{vmatrix} E_{1\varphi}(1) & E_{2\varphi}(1) & E_{1\varphi}(1) & E_{2\varphi}(1) \\ E_{1z}(1) & E_{2z}(1) & E_{1z}(1) & E_{2z}(1) \\ E_{1\varphi}(R) & E_{2\varphi}(R) & E_{1\varphi}(R) & E_{2\varphi}(R) \\ E_{1z}(R) & E_{2z}(R) & E_{1z}(R) & E_{2z}(R) \end{vmatrix} = 0 \quad (32)$$

and for the quasi-$TE$-modes, we have the following equation.

$$\begin{vmatrix} H'_{1\varphi}(1) & H'_{2\varphi}(1) & H'_{1\varphi}(1) & H'_{2\varphi}(1) \\ H'_{1z}(1) & H'_{2z}(1) & H'_{1z}(1) & H'_{2z}(1) \\ H'_{1\varphi}(R) & H'_{2\varphi}(R) & H'_{1\varphi}(R) & H'_{2\varphi}(R) \\ H'_{1z}(R) & H'_{2z}(R) & H'_{1z}(R) & H'_{2z}(R) \end{vmatrix} = 0 \quad (33)$$

These are the transendental equations for the determination of the permissible propagation constant $\Gamma$ for the quasi-$TM$-and the quasi-$TE$-modes.

In order to obtain the characteristic equation for the determination of the cutoff frequencies, we substitute Eq. (21) into Eq. (31), then

$$\begin{vmatrix} k_c J_{n+1}(k_c) - nJ_n(k_c) & 0 & k_c N_{n+1}(k_c) - nN_n(k_c) & 0 \\ k_c J_{n+1}(k_c R) - nJ_n(k_c R) & 0 & k_c N_{n+1}(k_c R) - nN_n(k_c R) & 0 \\ J_n(k_c) & J_n(\beta k_c) & N_n(k_c) & N_n(\beta k_c) \\ J_n(k_c R) & J_n(\beta k_c R) & N_n(k_c R) & N_n(\beta k_c R) \end{vmatrix} = 0 \quad (34)$$

This equation leads to practical difficulty for numerical calculation to obtain the cutoff frequencies of the quasi-$TM$-modes. However we consider the simplist case for the dominant mode by substituting $n=0$ into Eq. (34), then

$$\begin{vmatrix} J_1(k_c) & 0 & N_1(k_c) & 0 \\ J_1(k_c R) & 0 & N_1(k_c R) & 0 \\ J_0(k_c) & J_0(\beta k_c) & N_0(k_c) & N_0(\beta k_c) \\ J_0(k_c R) & J_0(\beta k_c R) & N_0(k_c R) & N_0(\beta k_c R) \end{vmatrix} = 0 \quad (35)$$

This equrtion leads to the following form,

$$\begin{vmatrix} J_1(k_c) & N_1(k_c) \\ J_1(k_c R) & N_1(k_c R) \end{vmatrix} \cdot \begin{vmatrix} J_0(\beta k_c) & N_0(\beta k_c) \\ J_0(\beta k_c R) & N_0(\beta k_c R) \end{vmatrix} = 0 \quad (36)$$

Finaly we obtain the characteristic equation for the cutoff frequencies of the dominant quasi-$TM$-mode as follows.

$$J_0(\beta k_c) \cdot N_0(\beta k_c R) = J_0(\beta k_c R) \cdot N_0(\beta k_c) \quad \text{and} \quad J_1(k_c) \cdot N_1(k_c R) = J_1(\beta k_c R) \cdot N_1(k_c) \quad (37)$$

Since the second relation does not include the magnetization dependency, it is not applicable for the equation of the determination of cutoff frequencies.

Now we consider the equation for the quasi-$TE$-modes. By substituting Eq. (14) and Eq. (28) into Eq. (32) we obtain the following form with the help of Eq. (22)

$$\begin{vmatrix} J'_{n+1}(k_c) & J'_{n+1}(\beta k_c) & N'_{n+1}(k_c) & N'_{n+1}(\beta k_c) \\ J'_{n+1}(k_c R) & J'_{n+1}(\beta k_c R) & N'_{n+1}(k_c R) & N'_{n+1}(\beta k_c R) \\ J'_n(k_c) & 0 & N'_n(k_c) & 0 \\ J'_n(k_c R) & 0 & N'_n(k_c R) & 0 \end{vmatrix} = 0 \quad (38)$$

which leads to the equations

$$\begin{aligned} J'_n(k_c) \cdot N'_n(k_c R) &= J'_n(k_c R) \cdot N'_n(k_c) \\ J'_{n+1}(\beta k_c) \cdot N'_{n+1}(\beta k_c R) &= J'_{n+1}(\beta k_c R) \cdot N'_{n+1}(\beta k_c) \end{aligned} \quad (39)$$

The magnetic field dependency is included in the second relation in Eq. (39), which will be applicable for the characteristic equation of cutoff frequencies of the quasi-$TE$-modes. This relation is written as follows for the dominant mode.

$$J_1(\beta k_c)\cdot N_1(\beta k_c R)=J_1(\beta k_c R)\cdot N_1(\beta k_c) \tag{40}$$

This equation was solved for $R=1.5$ with the help of Jahnke and Emde[8]. The result of numerical calculation is shown in Fig. (3). As shown in Fig. (1) and (2), the cutoff frequencies both for the quasi-$TE$-and quasi-$TM$-modes show to increase as the anisotropy increases.

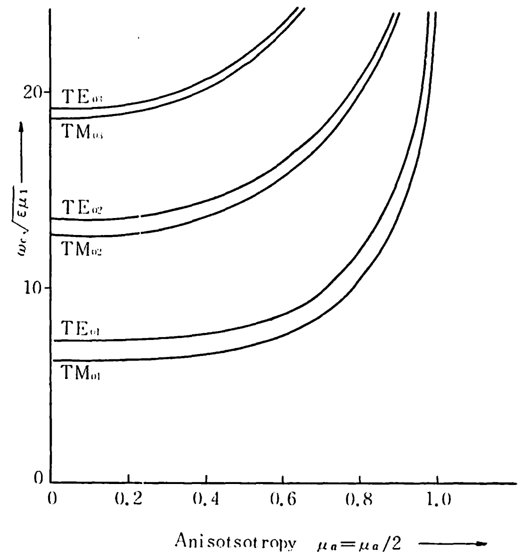


Fig. (3). The cutoff frequencies for quasi-$TE_{0n}$-and quasi-$TM_{0n}$-modes against the anisotropy $\mu_a'=\mu_a/2$.

## 4. Circular Waveguide containing Coaxial Core

We consider an infinite circular waveguide inside which are loaded with an axial ferrite cylinder of the radius $R$ and filled with air around the axial ferrite cylinder outside up to the radius $r=1$. The inner ferrite cylinder is longitudinally magnetized. Let the inner ferrite cylinder be characterized by $k_1$ and $k_2$, the outer medium by $k$.

Following Epstein, the field in the medium outside the inner cylinder is expressed by omitting the factor expj. $(n\varphi+\Gamma z)$.

$$\begin{aligned}
&E_{1r}=-jnJ_n(kr)/r && E_{1\varphi}=kJ'_n(kr) && E_{1z}=0 \\
&H_{1r}=-kr\cdot J'_n(kr)/\mu k_0 && H_{1\varphi}=-n\Gamma J_n(kr)/\mu k_0 r && H_{1z}=jk^2J_n(kr)/\Gamma \\
&E_{2r}=kJ'_n(kr) && E_{2\varphi}=jnJ_n(kr)/r && E_{2z}=-k^2J_n(kr)/\Gamma \\
&H_{2r}=-jnk_0\varepsilon J_n(kr)\cdot/\Gamma r && H_{2\varphi}=kk_0\varepsilon\cdot J'_n(kr)/\Gamma && H_{2z}=0
\end{aligned} \tag{44}$$

The field in the inner ferrite cylinder $(r<R)$, which we denote by the dotted letters, is represented by Eq. (13) and (14) being characterized the wave number by $k_1$ and $k_2$.

$$E=A_1\cdot E_1+A_2E_2, \qquad H=A_1H_1+A_2H_2 \tag{43}$$

The boundary conditions follow two at the outer boundary $r=1$.

$$E_\varphi(1)=E_z(1)=0 \tag{44}$$

and four at the interface $r=R$.

$$E_\varphi(R)=E_\varphi(R),\quad E_z(R)=E_z(R),\quad H_\varphi(R)=H_\varphi(R),\quad H_z(R)=H_z(R) \tag{45}$$

Thus the complete set of condition becomes.

$$\begin{aligned}
&A_1E_{1\varphi}(1)+A_2E_{2\varphi}(1)+A_1E_{1\varphi}(1)+A_2E_{2\varphi}(1)=0\\
&A_1E_{1z}(1)+A_zE_{2z}(1)+A_1E_{1z}(1)+A_2E_{2z}(1)=0\\
&A_1E_{1\varphi}(R)+A_2E_{2\varphi}(R)+A_1E_{1\varphi}(R)+A_2E_{2\varphi}(R)-A_1E_{1\varphi}(R)-A_2E_{2\varphi}(R)=0\\
&A_1E_{1z}(R)+A_2E_{2z}(R)+A_1E_{1z}(R)+A_2E_{2z}(R)-A_1E_{1z}(R)-A_2E_{2z}(R)=0\\
&A_1H_{1\varphi}(R)+A_2H_{2\varphi}(R)+A_1H_{1\varphi}(R)+A_2H_{2\varphi}(R)-A_1H_{1\varphi}-A_2H_{2\varphi}(R)=0\\
&A_1H_{1z}(R)+A_2H_{2z}(R)+A_1H_{1z}(R)+A_2H_{2z}(R)-A_1H_{1z}(R)-A_2H_{2z}(R)=0
\end{aligned} \tag{46}$$

The determination of the roots of the compatibility condition for the above equations requires very cumbersome numerical calculations. Even for the dominant mode $n=0$, it reduces the following complicate condition.

$$\begin{vmatrix}
J_1(k) & N_1(k) & 0 & 0 & 0 & 0\\
0 & 0 & J_0(kR) & N_0(kR) & 0 & 0\\
kJ_1(k) & kN_1(k) & 0 & 0 & -k_1\tau_1J_1(k_1R) & -k_2\tau_2J_1(k_2R)\\
0 & 0 & k^2J_0(kR) & k^2N_0(kR) & k_1{}^2\sigma J_0(k_1R) & k_2{}^2\sigma J_0(k_2R)\\
0 & 0 & -kk_0\varepsilon J_1(kR)/\Gamma & -kk_0\varepsilon N_1(kR)/\Gamma & j\Gamma bk_1J_0(k_1R) & j\Gamma bk_2J_0(k_2R)\\
k^2J_0(kR)/\mu k_0 & k^2N_0(kR)/\mu k_0 & 0 & 0 & j\Gamma^2f_1J_0(k_1R) & j\Gamma^2g_2J_0(k_2R)
\end{vmatrix}=0 \tag{47}$$

The numerical calculation of the above determinant would be relyed upon the digital computer.

## 5. Conclusion

The cutoff frequencies of the circular waveguide and the coaxial lines filled with ferrite material were calculated for both quasi-$TE$ and quasi-$TM$ modes. As shown in Fig. (1)-(3), the cutoff frequencies increase as the magnetization of the ferrite increases. For the circular waveguide containing the coaxial ferrite core, the compatibility condition becomes cumbersome, which is shown in Eq. (47).

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# フェライトを含む円筒状導波管の遮断周波数（摘要）

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フェライトを含む円筒状導波管を，その軸方向に磁化させると，それと同方向に伝播する電磁波の伝播定数及び遮断周波数はフェライトの直流磁化の強さで変ってくる。

本論文では，Epstein の解析手段を発展させ，フェライトで充された円筒導波管にその境界条件を適用し，直流磁化をパラメータとして含む伝播定数の満足すべき超越方程式を導き，更にその式中の伝播定数を零とおいて得られる遮断周波数を求める特性式を導いた。伝播定数を決定する超越方程式には，E 波に対して式(20)，又 H 波に対しては式(21) を得，尚遮断周波数を定める特性式に式 (24) と式 (25) を得た。又式 (24) と式 (25) で，フェライトの非等方性係数 $\mu_a$ を零とおいた時，即ち外部磁場が無くなってフェライトが等方性媒質となったときには，式 (24) と式 (25) はそれぞれ TM 波及び TE 波の遮断周波数を決定する式と一致する。従って式 (24) 及び式 (25)，又それに対応する超越方程式 (20) 及び (21) はそれぞれ準 TM 波及び準 TE 波の遮断周波数及び伝播定数を決定する式といえる。フェライトの磁化の程度を示す非等方性係数 $\mu_a$ の値を，0 から 2.0 まで変えてそれに対する準 TM 波及び準 TE 波の遮断周波数が計算され，第1図と第2図に示された。これは Gamo の得た結果と一致し，非等方性係数 $\mu_a$ が増加するにつれて遮断周波数は高くなる事を示す。

同軸線路は TEM 波以外に高次の伝送姿態をもつが，第3章ではそれらの遮断周波数を，フェライトの充った同軸線路について導き出し，準 TM 波の基本姿態に対して式 (37)，又準 TE 波の基本姿態に対して式 (40) を得た。それらの式について二，三の低次姿態に対する遮断周波数を計算したがその結果は第3図に示される如く，円筒導波管の場合と同様，非等方性係数 $\mu_a$ の増加と共に遮断周波数は高くなる事がわかった。

第4章では，同軸的に円筒状フェライトを含む円筒導波管についてその伝播定数を定める超越方程式を式 (47) に導いたが，その行列式の根を求めるには，デジタル電子計算機に依る事になる。