Electric Charge on Sprayed Oil Droplets 6 (Relation between the largest charge and the size of droplets)

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(Relation between the largest charge and the size of droplets) +

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The largest electric chages on sprayed droplets of two kinds of lubricant, SAE 20W and Algol oil, were measured, using Millikan's oil drop method. The sizes of droplets were in the region from 3×10^{-13} cc to 6×10^{-11} cc in volume.

Experimental result showed good agreement with the relation between the size of the droplets and their largest charge calculated on the basis of the hypothesis that the electrification of sprayed droplets comes from the fluctuation of ion-denssity in lubricant.

1. Introduction

In the previous paper,[1] it was reported that the hypothesis that the electrification of sprayed droplets comes from the fluctuation of ion-density in lubricant could explain satisfactorily the relation between the observed largest charge on sprayed droplets and the specific electric conductivity of lubricant used.

In order to make sure whether the hypothesis is valid in the explanation of the relation between the observed largest charge and the size of sprayed oil droplets or not, the largest electric charge on sprayed droplets of the size from 3×10^{-13} cc to 6×10^{-11} cc in volume were measured, using Millikan's oil drop method. In the present work, SAE 20W and Algol oil which were employed in the previous work were used.

2. Experimentel

In the measurement of the electric charge on sprayed droplets of SAE 20W and Algol oil, the distance of the parallel electrodes was 0.636 cm and the voltages applied to the electrodes were 60V for SAE 20W and 400V for Algol oil,respectively. These voltages were large enough to make the droplets with large charge move upward against the gravitation, but too low for the droplets with small charge, therefore under the voltages all droplets with small charge were omitted favorably from the measurement.

The rise time t_u in which the droplet moving rapidly upward passes through the fixed virtical distance 0.082 cm under the voltage and fall time t_d in which the same droplet moves downward through the same distance under the gravity were measured in every spraying. By repeating spraying, many pairs of t_u and t_d of many droplets with large charge were obtained

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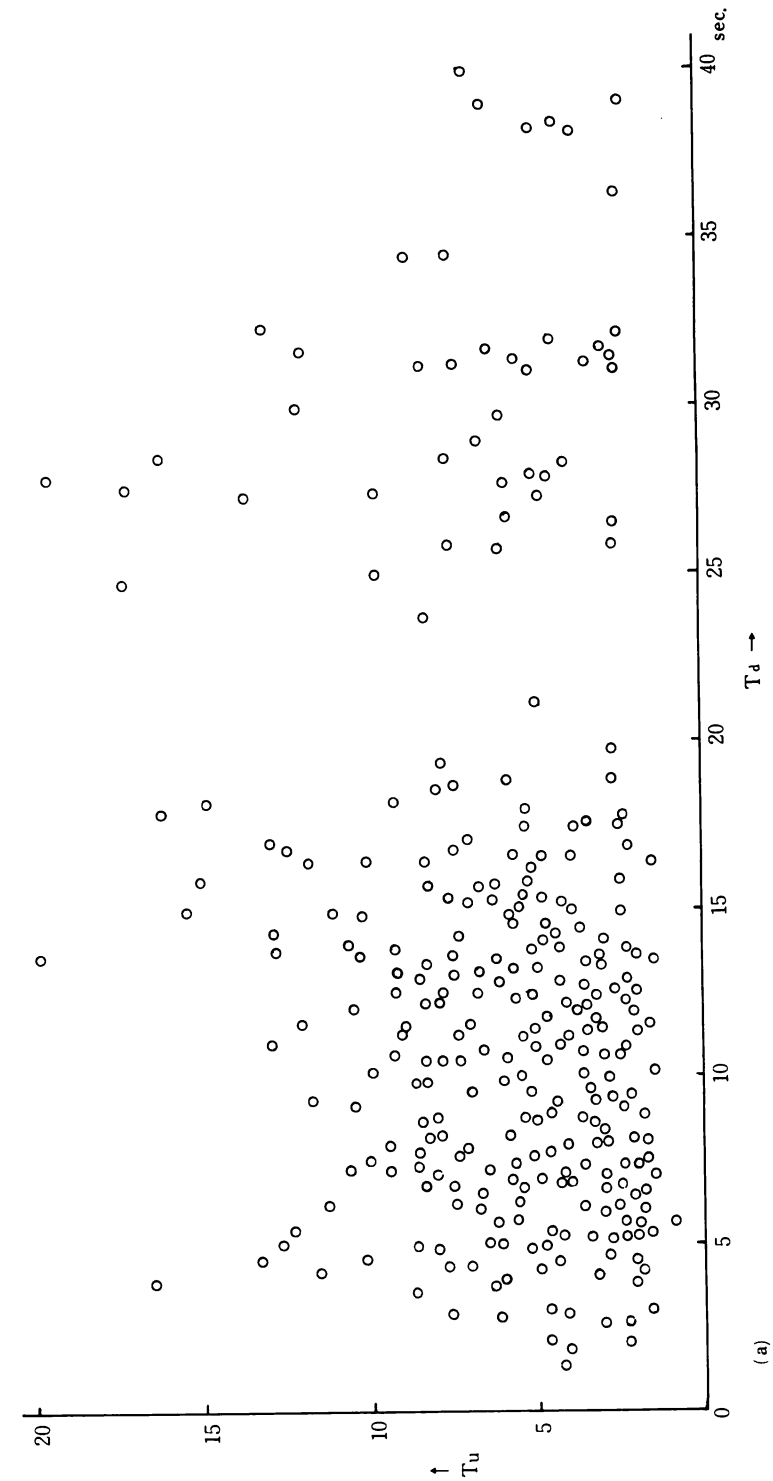
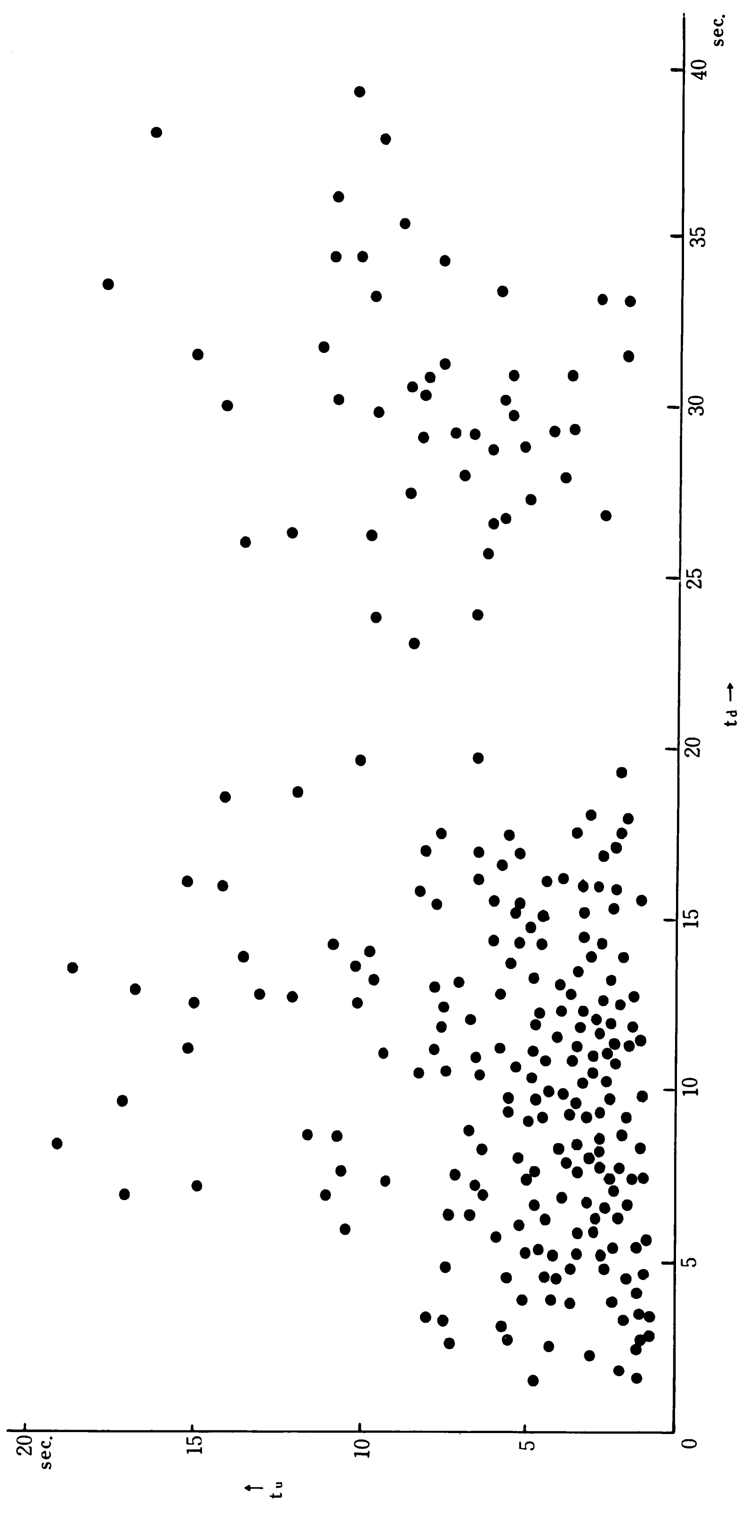
(see Fig. 1) .

The values of t_d ranged from 1.3 sec to 40 sec, which correspond to

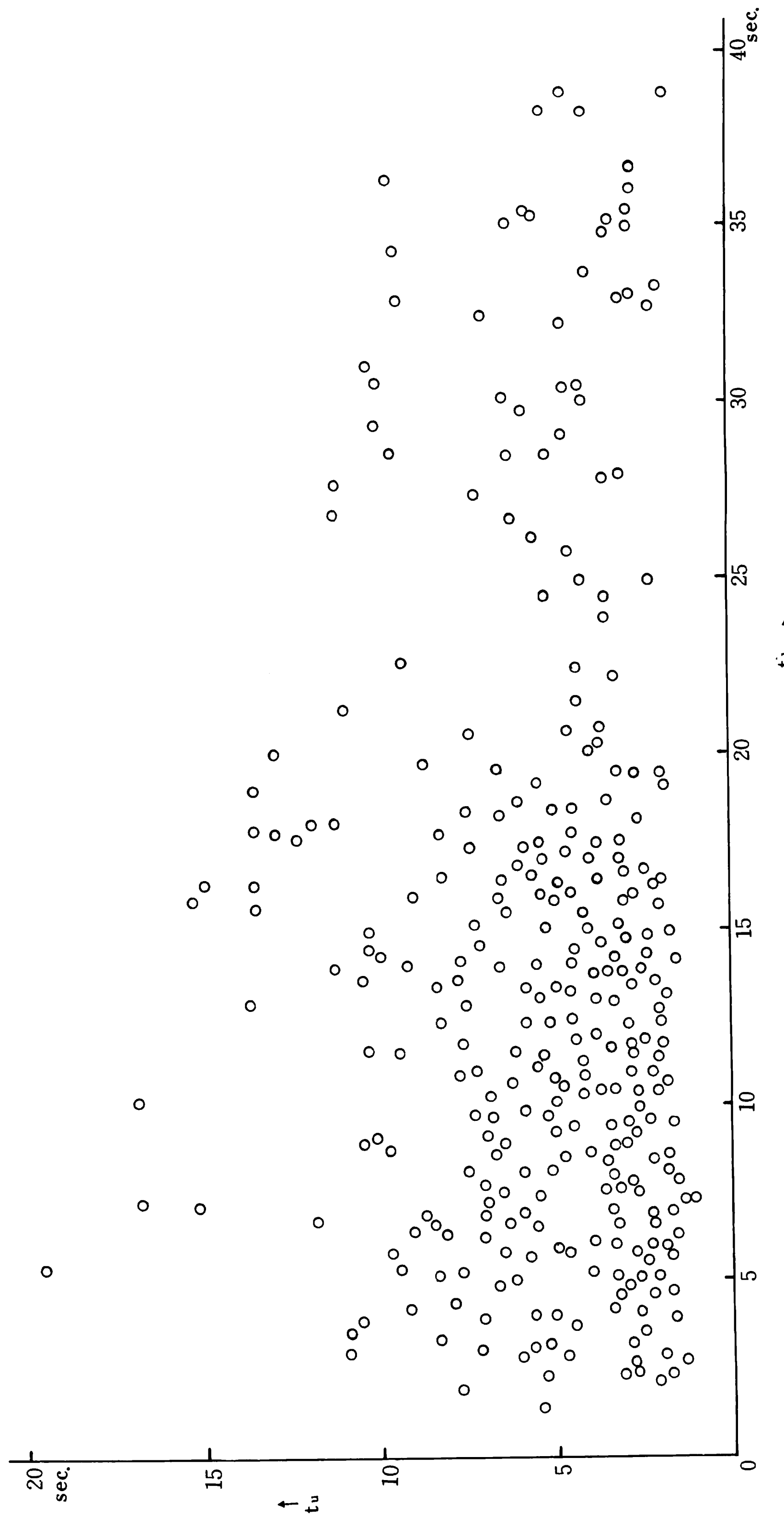


Fing. 1 (a) Positively charged droplets of SAE 20w.

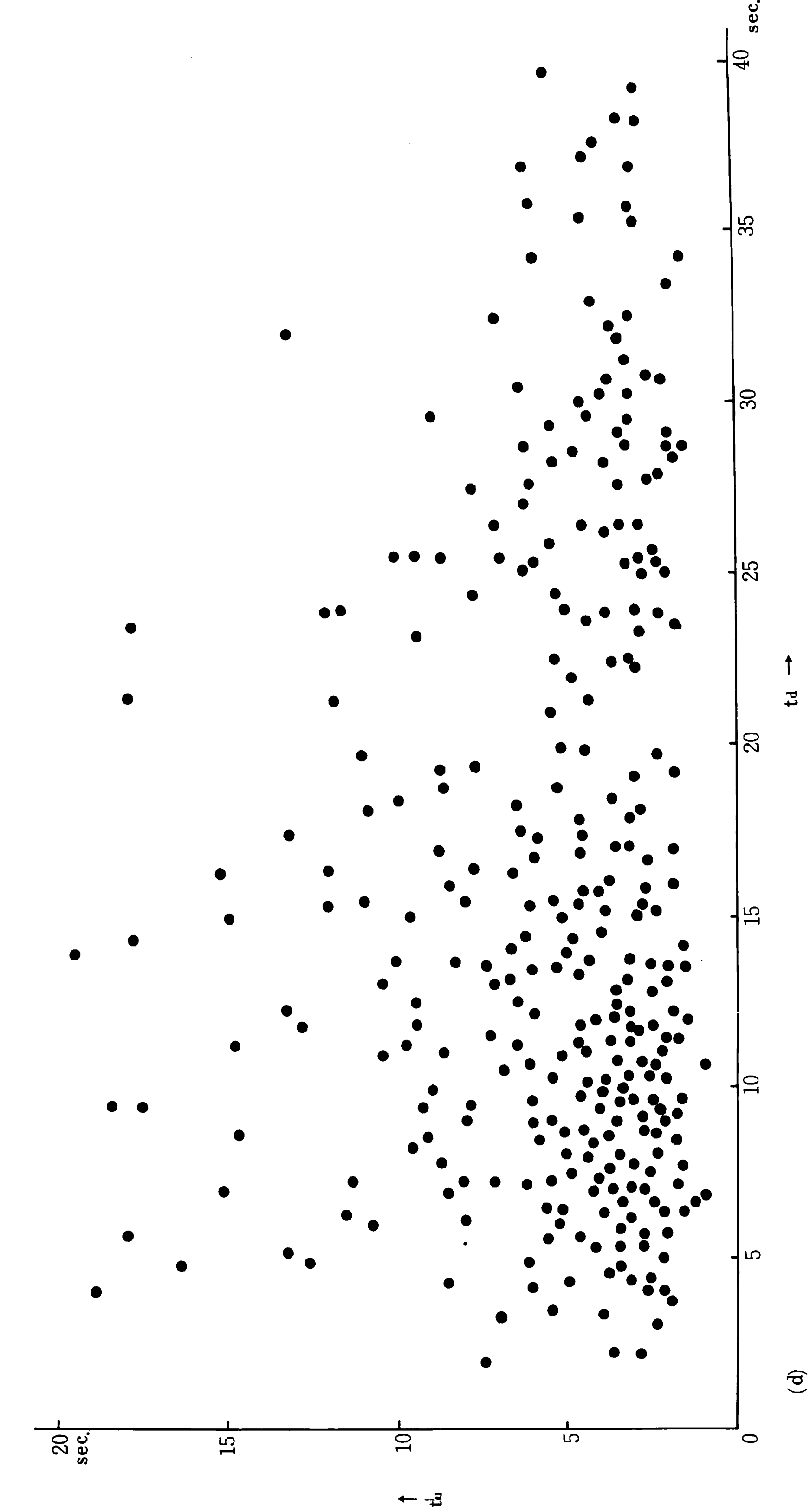


(b)

(b) Negatively charged droplets of SAE 20w.



(c) Positively charged droplets of Algol oil.



(d) Negatively charged droplets of Algol oil.

6×10^{-11} cc and 3×10^{-13} cc in volume. The observed droplets were classified into several size groups in accordance with the value of t_d and then the largest charge of every size group was calculated by putting the smallest value of t_u of every size group and it's pair t_d into the following equation:

$$t_u=\frac{(K/q)\cdot t_d}{(t_d)^{3/2}-(K/q)},$$

where

$$K=9\sqrt{2}\frac{\pi\cdot\eta^{3/2}\cdot\ell^{3/2}}{E(\rho-\rho')^{1/2}\cdot g^{1/2}}$$

η : viscosity of the air ($=1.79\times10^{-4}$ g/cm sec)
ℓ : moved distance of droplet ($=0.082$ cm)
ρ' : density of the air ($=0.001$ g/cc)
ρ : density of lubricant
q : electric charge on droplet in esu
E : electric field strength in esu.

The measurement of t_u and t_d carried out in the room-temperature between 18.3°C and 23.8°C for SAE 20W and between 15.4°C and 22.8°C

Table I. The largest charge of droplet in every size group and their volumes

Algol oil							
td sec	tu sec	volume cc	positive charge	td sec	tu sec	volume cc	negative charge
1.4	5.4	4.8×10^{-11}	58 e	1.9	7.4	3.1×10^{-11}	37 e
2.1	2.1	2.8 〃	50〃	2.2	2.9	2.2 〃	30〃
2.7	1.3	1.9 〃	53〃	3.7	2.0	1.2 〃	30〃
7.3	1.0	4.4×10^{-12}	32〃	6.6	1.3	5.2×10^{-12}	27〃
7.3	1.3	4.4 〃	26〃	6.8	1.0	4.8 〃	34〃
14.1	1.5	1.6 〃	15〃	10.6	1.0	2.5 〃	26〃
14.9	1.7	1.4 〃	13〃	13.5	1.6	1.7 〃	14〃
19.1	1.8	1.05 〃	11〃	15.9	1.9	1.3 〃	11〃
19.4	1.9	1.0 〃	10〃	19.2	1.9	1.03 〃	10〃
24.9	2.2	6.8×10^{-13}	8〃	23.5	1.8	7.6×10^{-13}	9〃
32.7	2.2	4.6 〃	6〃	28.7	1.6	5.6 〃	9〃
33.3	2.0	4.4 〃	7〃	33.4	2.0	4.4 〃	7〃
38.9	1.8	3.6 〃	4〃	34.6	1.6	4.2 〃	9〃

SAE 20W							
td sec	tu sec	volume cc	positive charge	td sec	tu sec	volume cc	negative charge
1.3	4.3	6.0×10^{-11}	432 e	1.6	1.6	4.4×10^{-11}	509 e
2.0	2.3	3.1 〃	340〃	2.8	1.3	1.9 〃	351〃
3.0	1.6	1.7 〃	285〃	3.4	1.3	1.4 〃	297〃
5.6	0.9	6.8×10^{-12}	281〃	4.4	1.5	9.8×10^{-12}	219〃
7.0	1.5	4.8 〃	158〃	5.7	1.4	6.6 〃	192〃
10.1	1.5	2.7 〃	124〃	7.6	1.6	4.3 〃	141〃
11.5	1.6	2.3 〃	108〃	10.0	1.5	2.8 〃	125〃
13.4	1.5	1.8 〃	104〃	12.9	1.8	1.9 〃	91〃
16.3	1.5	1.35 〃	93〃	15.8	1.6	1.4 〃	89〃
25.8	2.6	6.8×10^{-13}	43〃	19.6	2.1	1.03 〃	61〃
32.2	2.4	4.8 〃	41〃	27.2	2.6	6.2×10^{-13}	42〃
39.0	2.3	3.8 〃	48〃	32.0	2.0	4.9 〃	48〃
				33.6	1.9	4.6 〃	52〃

for Algol oil, therefore the mean values of density, 0.884 g/cc for SAE 20W and 0.909 g/cc for Algol oil, were employed in the calculation. The result is shown in Table I.

3. Theoretical

Suppose that there are N positive ions and same number of negative ions in unit volume of lubricant and they are all univalent, then the probability that m positive or negative ions are found in a very small volume υ which is equal to the volume of sprayed droplet will be as follows:

$$P = {}_N C_m \left[(V-\upsilon)/V \right]^{N-m} \cdot (\upsilon/V)^m, \tag{3.1}$$

where V=1 cc.

If $N \gg m$, equation (3.1) can be written as, using Stirling's formula,

$$P \fallingdotseq \frac{1}{\sqrt{2\pi m_o}} e^{-\frac{x^2}{2m_o}}, \tag{3.2}$$

where m_o is the value of m satisfying $m/\upsilon = N/V$, and $x = m - m_o$.
Therefore the probability that there are m positive and n negative ions in volume υ simultaneously will be as follows:

$$P = \frac{1}{\sqrt{2\pi m_o}} e^{-\frac{x^2}{2m_o}} \cdot \frac{1}{\sqrt{2\pi m_o}} e^{-\frac{y^2}{2m_o}},$$

where $y = n - m_o$.

When $x-y=q$, the net electric charge in volume υ is equal to qe. There are many pairs of values of x and y satisfying $x-y=q$.
Therefore the probability that the charge quantity in volume υ is qe will be as follows:

$$P \doteqdot \sum_{x,y} \frac{1}{\sqrt{2\pi m_o}} e^{-\frac{x^2}{2m_o}} \frac{1}{\sqrt{2\pi m_o}} e^{-\frac{y^2}{2m_o}}, \qquad (3.3)$$

where $(x,y) = (q,0), (q+1,1), (q+2,2), (q+3,3), \ldots\ldots,$
$(q-1,-1), (q-2,-2), (q-3,-3), \ldots\ldots.$

Equation (3.3) can be written as,

$$P \doteqdot 2\frac{1}{\sqrt{4\pi m_o}} e^{-\frac{q^2}{4m_o}} \sum_{x+y} \frac{1}{\sqrt{4\pi m_o}} e^{-\frac{(y+x)^2}{4m_o}}.$$

As $x+y=$ takes every other integer, $\sum_{x+y} \frac{1}{\sqrt{4\pi m_o}} e^{-\frac{(x+y)^2}{4m_o}} \doteqdot 1/2.$

$$\therefore\ P = \frac{1}{\sqrt{4\pi m_o}} e^{-\frac{q^2}{4m_o}}. \qquad (3.4)$$

Table II. The probability that the net electric charge in υ is qe.

(1) $N = 2.5\times10^{14}$ (SAE 20W)

q (e)	Probability		
	$\upsilon=10^{-12}$cc	$\upsilon=10^{-11}$cc	$\upsilon=10^{-10}$cc
0	1.784×10^{-2}	5.642×10^{-3}	1.784×10^{-3}
10	1.614 〃		
20	1.196 〃		
30	7.254×10^{-3}		
40	3.602 〃		
50	1.464 〃	4.394 〃	
60	4.875×10^{-4}		
70	1.329 〃		
80	2.964×10^{-5}		
90	5.416×10^{-6}		
100	8.100×10^{-7}	2.076×10^{-3}	
150		5.947×10^{-4}	
200		1.033 〃	
250		1.089×10^{-5}	
300		6.963×10^{-6}	7.254×10^{-4}
400			3.602 〃
500			1.464 〃
600			4.875×10^{-5}
700			1.329 〃
800			2.964×10^{-6}

(2) $N=3\times10^{12}$ (Algol oil)

q (e)	Probability		
	$\upsilon=10^{-12}$cc	$\upsilon=10^{-11}$cc	$\upsilon=10^{-10}$cc
0	1.629×10^{-1}	5.150×10^{-2}	1.629×10^{-2}
2	1.167 〃		
4	4.293×10^{-2}		
6	8.109×10^{-3}		
8	7.863×10^{-4}	3.021 〃	
10	3.915×10^{-5}		
12	1.001×10^{-6}		
16		6.100×10^{-3}	
20			1.167×10^{-2}
24		4.239×10^{-4}	
32		1.014×10^{-5}	
40		8.341×10^{-8}	4.293×10^{-3}
60			8.109×10^{-4}
80			7.863×10^{-5}
100			3.915×10^{-6}
120			1.001×10^{-7}

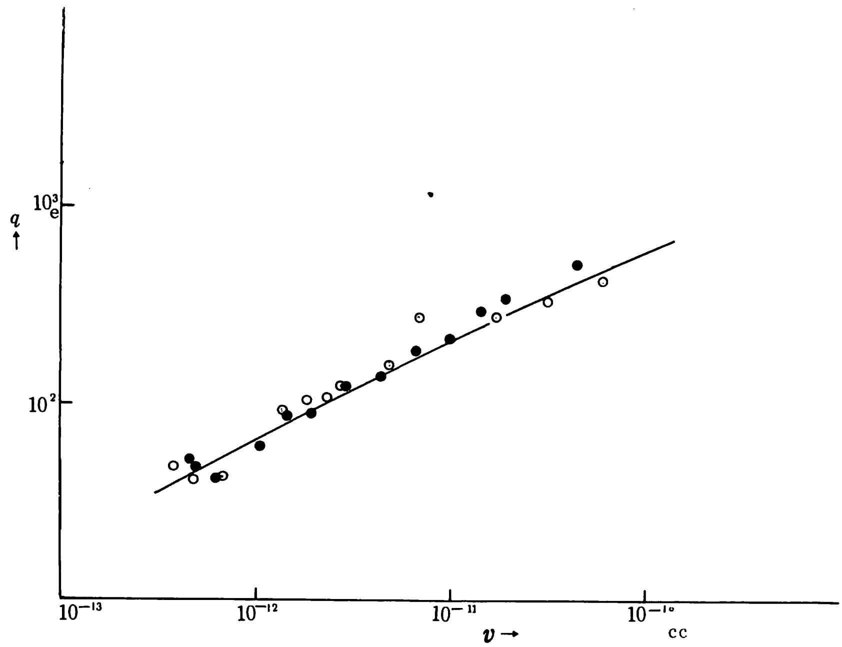
As $m_o=N.\upsilon$, when any two of N,P,q and υ in the equation (3.4) are fixed, the relation between the two remains can be obtained by using the equation. The relation between q and P is shown in Table II.

4. Discussion

In order to make sure whether the hypothesis that the charging of sprayed droplets comes from the fluctuation of ion-density in lubricant can explain the experimental result or not, the theoretical relation between the charge quantity q and the volume υ must be examined, fixing the number of ion N in unit volume and the appearance probability P. When $N=2.5\times10^{14}$ for SAE 20W and 3×10^{12} for Algol oil and $P=5\times10^{-5}$, the largest charges that appear in 10^{-12} cc, 10^{-11} cc and 10^{-10} cc in volume are about ±80e, ±220e, ±600e for SAE 20W and about ±10e, ±28e, ±85e for Algol oil (see Table II).

In the measurement of the largest charge on droplet of every size group, a large number of droplets with various charge-quantities were seen through the microscope. $P=5\times10^{-5}$ is due to the rough number of the droplets. No measurement was made to determine the number of the ion-pairs in unit volume of the lubricant, but those values mentioned above would not be unresonable. The values of N differ very much from those shown in Fig. 5

of the previous paper in spite of using of identical lubricant. The difference comes from the value of v. In the previous paper, 10^{-12} cc was employed as the mean volume of droplets, but in fact the size of the droplet



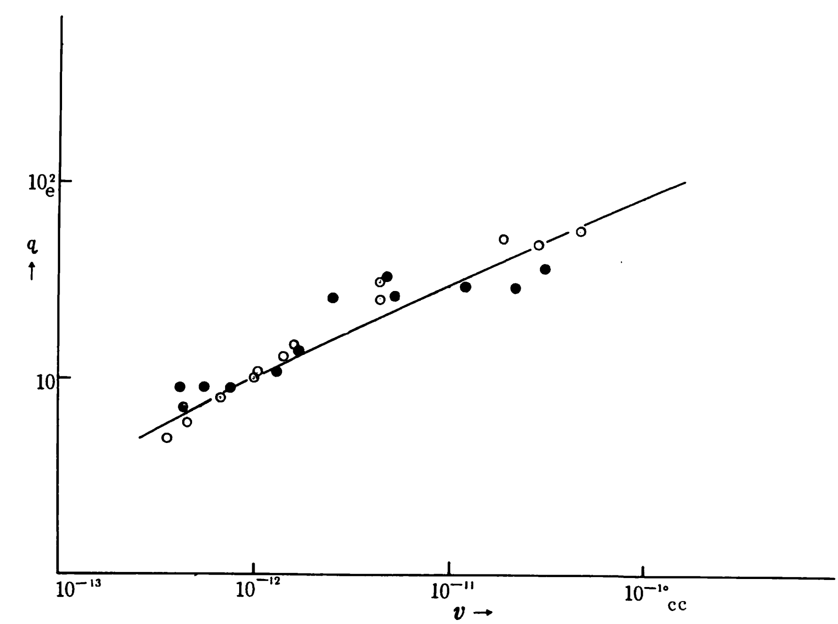


Fig. 2 (a) Observed largest charge and size of droplets of SAE 20w.
(b) Observed largest charge and size of droplets of Algol oil.
Solid curves represent theoretical relation.

with the largest charge was much greater than 10^{-12} cc, Therefore if a suitable value was employed as the mean volume of droplets, the difference would be disappeared.

The theoretical and experimental relations between q and υ are shown in Fig. 2. The solid curve representing the theoretical relation agrees satisfactorily with the experimental result. The fact supports the hypothesis that the electrification of sprayed droplets of 10^{-5} cm $\sim 10^{-4}$ cm in diameter comes from the fluctuation of ion-density in lubricant.

It can be shown easily that from the equation (3.4) , the mean value of electric charge on droplets of same size is proportional to $\upsilon^{1/2}$, which was mentioned by Dodd(2, because the precision is $(4m_{\circ})^{-1/2}$.

In conclusion, the writer wishes to take this opportunity to express his best thanks to Mr. G. Maeshiro and Mr. M. Kakihana, for their valuable assistance throuout the whole course of this work.

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