On the Cohomology of the mod 2 Steenrod Algebra

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by

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Introduction

Let A_p denote the mod p Steenrod algebra. J. P. May [1] constructed a spectral sequence, converging to $E^0H^*(A_p)$ and having $H^*(E^0A_p)$ as its E_2 term and made some computations of the algebra $E^0H^*(A_p)$ by using the imbedding method. Latter, for the case of the prime 2, M. C. Tangora [4] made extensive computations of this algebra for the range $t-s \leqq 70$. In his computations he used the imbedding method, manipulative methods, application of Adams' results on $H^*(A_2)$ and application of known results in homotopy theory.

The purpose of this paper is to compute the algebra $E^0H^*(A_2)$ for the range $71 \leqq t-s \leqq 77$, using the above methods together with the matric Massey product method, which was given in May's paper [2]. Our main results is Theorem 2.14.

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§ 1. Some known results on May spectral sequence

Since we are concerned only with the prime 2, from now on we will write A instead of A_2 for the mod 2 Steenrod algebra.

The following theorem is proved by J. P. May in his thesis.

THEOREM 1.1. (May) *There exists a spectral sequence (E_r, d_r), converging to $E^0H^*(A)$, and having as its E_2 term $H^*(E^0A)$. Each E_r is a tri-graded algebra, and each d_r is a homomorphism*

$$d_r:\ E_r^{u,v,t} \longrightarrow E_r^{u+r,v-r+1,t}$$

which is a derivation with respect to the algebra structure.

The algebra $H^*(E^0A)$,

$t-s < 165$ or $s < 4$, is calculated by J. P. May in his thesis. We quote May's results on $H^*(E^0A)$ for the range $t-s \leqq 80$.

THEOREM 1.2. (May) *$H^*(E^0A)$ is generated as an algebra over Z_2 by the following generators in Table I.A. for the range $t-s \leqq 80$. The relations in $H^*(E^0A)$ for the range $t-s \leqq 80$ are given in Table I.B.*

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Table I.A.

$t-s$	s	Name	Weight	$t-s$	s	Name	Weight
0	1	h_0	0	28	2	$b_{0,4}$	6
1	1	h_1	0	31	1	h_5	0
3	1	h_2	0	34	2	$h_2(1)$	2
4	2	$b_{0,2}$	2	38	3	$h_0(1,3)$	4
7	1	h_3	0	46	2	$b_{3,2}$	2
7	2	$h_0(1)$	2	46	3	$h_0(1,2)$	6
10	2	$b_{1,2}$	2	54	2	$b_{2,3}$	4
12	2	$b_{0,3}$	4	58	2	$b_{1,4}$	6
15	1	h_4	0	60	2	$b_{0,5}$	8
16	2	$h_1(1)$	2	63	1	h_6	0
22	2	$b_{2,2}$	2	70	2	$h_3(1)$	2
26	2	$b_{1,3}$	4	79	3	$h_1(1,3)$	4

Table I.B.

1	$h_ih_{i+1}=0 \ (i\geqq 0)$
2.1	$b_{i,2}b_{i+2,2}=h_i^2b_{i+1,3}+h_{i+3}^2b_{i,3} \ (i\geqq 0)$
2.2	$b_{0,3}b_{3,2}=b_{0,2}b_{2,3}+h_0^2b_{1,4}+h_4^2b_{0,4}$
3.1	$h_{i+2}b_{i,2}=h_ih_i(1) \ (i\geqq 0)$
3.2	$h_{i+2}h_i(1)=h_ib_{i+1,2} \ (i\geqq 0)$
3.3	$h_{i+3}h_i(1)=0 \ (i\geqq 0)$
3.4	$h_{i-1}h_i(1)=0 \ (i\geqq 1)$
4.1	$h_i(1)^2=b_{i,2}b_{i+1,2}+h_{i+1}^2b_{i,3} \ (i\geqq 0)$
4.2	$h_i(1)h_{i+1}(1)=0 \ (i\geqq 0)$
4.3	$h_0(1)h_2(1)=h_0h_4b_{1,3}$
4.4	$h_0(1)h_3(1)=0$
5.1	$b_{i,2}h_{i+1}(1)=h_{i+1}h_{i+3}b_{i,3} \ (i\geqq 0)$
5.2	$b_{i+2,2}h_i(1)=h_ih_{i+2}b_{i+1,3} \ (i\geqq 0)$
6.1	$b_{i,2}h_{i+2}(1)=h_ih_i(1,3) \ (i\geqq 0)$
6.2	$h_2h_0(1,3)=h_0h_4b_{1,3}$
6.3	$b_{3,2}h_0(1)=h_4h_0(1,3)$
6.4	$h_5h_0(1,3)=0$
6.5	$h_0h_1(1,3)=0$
7.1	$b_{0,3}h_2(1)=h_0h_0(1,2)+h_2h_4b_{0,4}$
7.2	$b_{2,3}h_0(1)=h_4h_0(1,2)+h_0h_2b_{1,4}$
7.3	$h_5h_0(1,2)=0$
8.1	$h_1(1)h_0(1,3)=h_1h_3h_0(1,2)$
8.2	$b_{1,2}h_0(1,3)=h_1^2h_0(1,2)+h_4b_{1,3}h_0(1)$
8.3	$b_{2,2}h_0(1,3)=h_3^2h_0(1,2)+h_0b_{1,3}h_2(1)$
8.4	$b_{0,3}h_0(1,3)=b_{0,2}h_0(1,2)+h_4b_{0,4}h_0(1)$

Table I.B. (continued)

10.1	$h_0(1)h_0(1,3)=h_1^2h_4b_{0,4}+h_4b_{0,2}b_{1,3}$
10.2	$h_2(1)h_0(1,3)=h_0h_3^2b_{1,4}+h_0b_{1,3}b_{3,2}$
11.1	$h_0(1)h_0(1,2)=h_1b_{0,4}b_{1,2}+h_4b_{0,3}b_{1,3}$
11.2	$h_2(1)h_0(1,2)=h_0b_{1,4}b_{2,2}+h_0b_{1,3}b_{2,3}$
12	$h_0(1,3)^2=h_1^2h_3^2b_{0,5}+h_1^2b_{0,4}b_{3,2}+h_3^2b_{0,2}b_{0,4}+b_{0,2}b_{1,3}b_{3,2}$

At least for the range $t\text{-}s \leqq 71$, the differentials d_r in May spectral sequence were determined by J. P. May [1] and M. C. Tangora [4]. Their methods of proof are the following:

1. *The Imbedding Method.*
2. *Manipulative Methods.*
3. *Application of Adams' Results on $H^*(A)$.*
4. *Application of Known Results in Homotopy Theory.*

THEOREM 1.3. (May and Tangora) *For the range $t-s \leqq 71$. d_r is given as follows:*

1. $d_r(h_i)=0 \ (r \geqq 2)$
2. $d_2(b_{i,2})=h_{i+1}^3+h_i^2h_{i+2}$,

   $d_{2^{k+1}}(b_{0,2}^{2^k})=h_0^{2^{k+1}}h_{k+2} \ (k \geqq 1)$;
3. $d_{2^{k+1}}(b_{i,j}^{2^k})=h_{i+j+k}b_{i,j-1}^{2^k}+h_{i+k+1}b_{i+1,j-1}^{2^k} (k \geqq 0, j \geqq 3)$;
4. $d_2(h_i(1))=h_ih_{i+2}^2$;
5. $d_2(h_i(1,3))=h_i(1)h_{i+4}^2+h_ih_{i+2}(1)$;
6. $d_2(h_i(1,2))=h_{i+3}h_i(1,3)$;
7. $b_{2^{k+2}}(h_{i+k+2}b_{i,3}^{2^k})=h_i^{2^{k+1}}h_{i+k+3}^2 \ (k \geqq 0)$;
8. $d_6(b_{0,2}^2 i)=h_0^5 s$,

   here $i=h_0b_{0,2}b_{0,3}h_0(1)$ *and* $s=h_4b_{0,2}^2h_0(1)+h_0^3b_{0,2}b_{1,3}$;
9. $d_4(h_0h_3b_{0,4})=h_0^3h_2(1)+h_0h_4^2b_{0,2}$;
10. $d_6(b_{0,2}^2 r)=h_0^6 x$,

    here $r=h_2^2b_{0,3}^2$ *and* $x=h_3b_{0,2}b_{1,3}+h_1^2h_3b_{0,4}$;
11. $d_6((b_{0,3}b_{1,3}+b_{0,4}b_{1,2})h_0(1))=h_0^3b_{2,2}^2$;
12. $d_4(h_2b_{0,4}h_1(1))=h_1^2b_{2,2}^2$;
13. $d_4(h_2h_0(1,2))=h_0h_3^2h_2(1)$;
14. $d_8(h_3b_{0,2}^6b_{0,3}^2)=h_0^{10}b_{0,2}b_{0,3}(b_{0,3}b_{1,3}+b_{0,4}b_{1,2})$;
15. $d_{12}(h_4b_{0,2}^{12})=h_0^{14}h_5b_{0,2}^2 i$;
16. $d_{12}(h_3b_{0,2}^8b_{0,3}^2)=h_0^{10}h_5b_{0,2}^2 i$;
17. $d_4(h_2b_{0,3}b_{0,4}h_1(1))=b_{1,2}^2b_{2,2}h_1(1)$;
18. $d_6(h_0b_{0,2}b_{0,3}(b_{0,3}b_{1,3}+b_{0,4}b_{1,2}))=h_0h_5b_{0,2}^3b_{1,2}$;

19. $d_6(h_0b_{0,3}(b_{0,3}b_{1,3}+b_{0,4}b_{1,2}))h_0(1))=h_0h_5b_{0,2}^2b_{1,2}h_0(1)$;
$d_6(b_{0,2}^2b_{1,2}(b_{0,3}b_{1,3}+b_{0,4}b_{1,2}))=h_1h_3h_5b_{0,2}^4$;
$d_6(b_{0,2}^4b_{1,2}(b_{0,3}b_{1,3}+b_{0,4}b_{1,2}))=h_1h_3h_5b_{0,2}^6$;
$d_6(h_0b_{0,3}b_{0,2}^2(b_{0,3}b_{1,3}+b_{0,4}b_{1,2}))=h_0h_5b_{0,2}^4b_{1,2}h_0(1)$;

20. $d_6(h_1b_{1,2}(b_{0,3}b_{1,3}+b_{0,4}b_{1,2}))=h_1^2h_3h_5b_{0,2}^2$;

21. $d_4(b_{1,3}^2h_1(1)+h_3^2b_{0,4}b_{1,3})=h_5(b_{1,2}^2h_1(1)+h_1^2b_{0,3}b_{2,2})$;

22. $d_4(h_3b_{0,4}h_2(1))=h_0^2h_2(1)^2$,
$d_6(b_{0,2}^2b_{0,3}^2x)=h_0^6b_{0,3}^2,b_{2,2}^2$,
$d_6(h_0^2b_{0,3}^2(b_{0,3}b_{1,3}+b_{0,4}b_{1,2})h_0(1))=h_0^5b_{0,3}^2b_{2,2}^2$;

23. $d_8(h_0h_3^7b_{0,4}^2)=h_0^8h_4b_{2,3}$,
$d_8(h_0^2h_2h_6b_{0,3}^3)=d_8(h_3h_5b_{0,2}^2b_{0,3}^2)=h_0^8h_4b_{2,3}$;

24. $d_8(h_0^2h_4b_{0,4}^2)=h_0^2h_5^2b_{0,2}^2$;

25. $d_4(h_0h_3b_{0,2}b_{0,4}b_{1,3})=h_0^4b_{1,3}h_0(1,3)$.

Furthermore the following two methods of proof are known for the various differentials in May spectral sequence.

5. *The Matric Massey Products Method.* The matric Massey products in spectral sequences were studied by J. P. May [2]. We quote some of his results which we use in this paper.

THEOREM 1.4. (May) *Let $\langle v_1, v_2, v_3\rangle$ be defined in E_{r+1}. Assume that $v_i \in E_{r-1}^{p_i, q_i, t_i}$ and that v_i converges to w_i, where $\langle w_1, w_2, w_3\rangle$ is defined in $H^*(A)$. Assume further that the following condition (*) is satisfied.*

(*) *If $(p, q, t)=(p_1+p_2, q_1+q_2, t_1+t_2)$ or $(p_2+p_3, q_2+q_3, t_2+t_3)$, then*
$E_{r+u+1}^{p-r-u, q+r+u-1, t} \subset E_{r+u+1,\infty}$ *for* $u \geq 0$.

Then any element of $\langle v_1, v_2, v_3\rangle$ is a permanent cocycle which converges to an element of $\langle w_1, w_2, w_3\rangle$.

THEOREM 1.5. (May) *Let $\langle v_1, v_2, v_3\rangle$ be defined in E_{r+1}, where $v_i \in E_{r+1}^{p_i, q_i, t_i}$. Let $n > r$ be given such that $d_m(v_i)=0$ for $m < n$ and $1 \leq i \leq 3$ and such that the following condition (*) is satisfied.*

(*) *If $(p, q, t)=(p_1+p_2, q_1+q_2, t_1+t_2)$ or $(p_2+p_3, q_2+q_3, t_2+t_3)$, then, for each m such that $r < m < n$, $E_m^{p-r+m, q+r-m, t}=0$ and $E_{r+n-m}^{p-r+m, q+r-m, t}=0$.*

Assume in addition to above hypotheses that for $1 \leq i \leq 3$ there is just one element $y_i \in E_{r+1}$ which survives to $d_n(v_i)$ and that $\langle d_n(v_1), v_2, v_3\rangle$, $\langle v_1, d_n(v_2), v_3\rangle$ and $\langle v_1, v_2, d_n(v_3)\rangle$ are defined in E_{r+1}. Assume further that all Massey products in sight have zero indeterminancy. Then

$d_n(\langle v_1, v_2, v_3\rangle)=\langle d_n(v_1), v_2, v_3\rangle + \langle v_1, d_n(v_2), v_3\rangle + \langle v_1, v_2, d_n(v_3)\rangle$

REMARK. In adove two theorems we consider only the Massey products in May spectral sequence. They turn out to be powerful tool in studying differentials in May spectral sequence.

6. *Application of Effect of Squaring Operations in May Spectral Sequence.* I have studied differentials in May spectral sequence by applying the effect of the squaring operations [3]. However, we will not use this method in this paper.

§ 2. Calculations

In this section we indicate the calculation of $E^0 H^*(A)$ for the range $71 \leqq t-s \leqq 77$. It is impossible to give all details, of course. We consider only the new independent differentials for the range $72 \leqq t-s \leqq 78$. After each new independent differential is obtained, we must do a certain amount of routine calculation. We omit this part of calculation. We now begin the calculation.

PROPOSITION 2.1. $d_6(h_0^2 b_{1,2} b_{0,3}^2(B)) = h_0 h_5 d_0 j$.

Here $B = b_{0,3} b_{1,3} + b_{0,4} b_{1,2}$.

PROOF. We make use of Theorem 1.5. Since $d_4(h_2 b_{0,3}) = h_0^2 h_3^2$ and $d_6(h_0 h_0(1) b_{0,3}(B)) = h_0 h_5 h_0(1)^3 b_{0,2}$, Massey product $\langle h_0 h_0(1) b_{0,3}(B), h_0 h_3, h_0 h_3 \rangle$ is defined in E_5 and is equal to $h_0^2 b_{1,2} b_{0,3}^2(B)$. Furthermore it is easy to see that this Massey product satisfies all conditions of Theorem 1.5 for $n=6$. Then we have $d_6(\langle h_0 h_0(1) b_{0,3}(B),\ h_0 h_3,\ h_0 h_3 \rangle) = \langle h_0 h_5 h_0(1)^3 b_{0,2},\ h_0 h_3,\ h_0 h_3 \rangle$. Since $\langle h_0 h_5 h_0(1)^3 b_{0,2},\ h_0 h_3,\ h_0 h_3 \rangle$ is equal to $h_0 h_5 d_0 j$, we have the result.

PROPOSITION 2.2. $d_6((b_{0,2} b_{1,3} + h_1^2 b_{0,4}) b_{2,2}^2 + h_2(1) b_{1,3} h_0(1)) = 0$.

PROOF. We make use of Theorem 1.4. Let α be a cochain $(b_{0,2} b_{1,3} + h_1^2 b_{0,4}) b_{2,2}^2 + h_2(1) b_{1,3} h_0(1)^2$. Since $d_2(b_{0,2} b_{1,3} + h_1^2 b_{0,4}) = h_4 h_0(1)^2$ and $d_2(h_2(1) b_{1,3}) = h_4 b_{2,2}^2$, Massey product $\langle h_0(1)^2, h_4, b_{2,2}^2 \rangle$ is defined in E_3 and contains the cochain α . $h_0(1)^2$, h_4 and $b_{2,2}^2$ converge to d_0, h_4 and g_2, respectively. For dimensional reasons, we have $h_4 d_0 = 0$ and $h_4 g_2 = 0$ in $H^*(A)$. Then Massey product $\langle d_0, h_4, g_2 \rangle$ is defined in $H^*(A)$. Furthermore it is easy to see that these Massey products satisfy the condition (*) of Therem 1.4. Then we have the result.

PROPOSITION 2.3. (a) $d_8(h_0 h_2 h_4 b_{0,4}^2) = h_0^2 h_5^2 h_0(1) b_{0,2}$;

(b) $d_8(h_1 h_4 b_{0,4}^2) = h_1 h_5^2 b_{0,2}^2$.

PROOF. We make use of manipulative methods. By Theorm 1.3, $d_2(h_4 b_{0,2} b_{0,4}^2) = h_0^2 h_2 h_4 b_{0,4}^2 + h_1^3 h_4 b_{0,4}^2$, $d_2(h_5^2 b_{0,2}^3) = h_0^3 h_5^2 h_0(1) b_{0,2} + h_1^3 h_5^2 b_{0,2}^2$ and $d_8(h_0^2 h_4 b_{0,4}^2) = h_0^2 h_5^2 b_{0,2}^2$. By Proposition 2.2, the cochain $h_0^3 h_5^2 h_0(1) b_{0,2}$ is non zero in E_8. Then $d_8(h_0^2 h_2 h_4 b_{0,4}^2) = d_8(h_1^3 h_4 b_{0,4}^2) = h_0^3 h_5^2 h_0(1) b_{0,2} = h_1^3 h_5^2 b_{0,2}^2$. Since the differential is a derivation, the results follow.

PROPOSITION 2.4. $d_6(h_1 b_{1,2} b_{0,3}^2(B)) = d_1 g^2 + h_0^2 P^1 A''$.

PROOF. There are two possible terms $d_1 g^2$ and $h_0^2 P^1 A''$. I know no proof of this proposition except by the imbedding method. Since the calculation of proof would be too long to write in detail, we will give only its sketch. It is a routine matter to verify that the dual elements of d_1g^2, $h_0^2P^1A''$ and $h_1b_{1,2}b_{0,3}^2(B)$ appear in the bar constuction as

$$\{P_2^1\}^{10}*\{P_2^2\}^2,$$
$$\{P_1^0\}^3*\{P_2^0\}^5*\{P_2^3\}*\{P_3^1\}^3+$$
$$\{P_1^0\}^2*\{P_2^0\}^6*\{P_2^2\}*\{P_2^3\}*\{P_3^1\}^2$$
$$+\{P_1^0\}*\{P_2^0\}^7*\{P_2^2\}^2*\{P_2^3\}*\{P_3^1\}$$
$$+\{P_2^0\}^8*\{P_2^2\}^3*\{P_2^3\}$$

and $\{P_1^2\}^3*\{P_3^0\}^6*\{P_4^0\}^2+$

$\{P_1^1\}*\{P_2^1\}^4*\{P_3^0\}^4*\{P_4^0\}^2$, respectively.

We must show that in bar construction $d(\{P_2^1\}^{10}*\{P_2^2\}^2)$ and

$$d(\{P_1^0\}^3*\{P_2^0\}^5*\{P_2^3\}*\{P_3^1\}^3+\{P_1^0\}^2*\{P_2^0\}^6*\{P_2^2\}*\{P_2^3\}*\{P_3^1\}^2$$
$$+\{P_1^0\}*\{P_2^0\}^7*\{P_2^2\}^2*\{P_2^3\}*\{P_3^1\}+\{P_2^0\}^8*\{P_2^2\}^3*\{P_2^3\})$$

are homologous to

$$\{P_1^2\}^3*\{P_3^0\}^6*\{P_4^0\}^2+\{P_1^1\}*\{P_2^1\}^4*\{P_3^0\}^4*\{P_4^0\}^2$$

modulo terms of weight greater than 18. We first consider the differential

$$d(\{P_2^1\}^{10}*\{P_2^2\}^2)=\{P_2^0P_4^0P_2^1\}*\{P_2^1\}^{10}+\{P_2^0P_4^0\}*\{P_2^1\}^9*\{P_2^2\}\ .$$

Already this differential contains no terms of weight less than 15. By adding to this the boundaries of two terms of weight 14,

$$\{P_2^0|P_4^0\}*\{P_2^1\}^9*\{P_2^2\}+\{P_1^1\}*\{P_1^2\}*\{P_4^0\}^2*\{P_2^1\}^8,$$

we eliminate all term of weight 15. By adding in the boundary of one term of weight 15,

$$\{P_1^2\}*\{P_4^0\}^2*\{P_1^0|P_3^0\}*\{P_2^1\}^7,$$

we obtain one term of weight 16,

$$\{P_1^2\}*\{P_4^0\}^2*\{P_3^0\}^2*\{P_2^1\}^6.$$

Now by Theorem 1.3, we have the differential $d_2(h_2b_{1,2}^3b_{0,3}b_{0,4})=h_2h_4b_{1,2}^3b_{0,3}^2+h_2^4b_{1,2}^2b_{0,3}b_{0,4}$. Then by adding in the aoundaries of one term of weighth 14,

$$\{P_1^2\}^4*\{P_2^1\}^4*\{P_3^0\}^2*\{P_4^0\}^2,$$

and of one term of whight 15,

$$\{P_1^2\}^2*\{P_1^1|P_2^1\}*\{P_2^1\}^4*\{P_3^0\}^2*\{P_4^0\}^2,$$

we eliminate all terms of weight 16. By adding in the boundaries of six terms of weight 16,

$$\{P_2^0P_4^0|P_2^1\}*\{P_2^1\}^{10}+\{P_2^0|P_4^0|P_2^0|P_4^0\}*\{P_2^1\}^8$$
$$+\{P_1^2\}^3*\{P_1^1|P_1^0P_3^0\}*\{P_2^1\}^3*\{P_3^0\}^2*\{P_4^0\}^2$$
$$+\{P_1^1\}*\{P_4^0\}^2*\{P_1^1|P_1^0P_3^0\}*\{P_2^1\}^7$$
$$+\{P_4^0\}^2*\{P_1^1|P_2^0|P_3^0\}*\{P_2^1\}^7$$
$$+\{P_1^1|P_2^1|P_1^1|P_2^1\}*\{P_2^1\}^4*\{P_3^0\}^2*\{P_4^0\}^2,$$

we eliminate all terms of weight 17. By adding in the boundaries of twelve terms of weight 17,

$$\{P_1^2\}^3*\{P_1^0P_3^0|P_1^0|P_3^0\}*\{P_2^1\}^2*\{P_3^0\}^2*\{P_4^0\}^2$$
$$+\{P_1^2\}^3*\{P_1^0|P_3^0|P_3^0|P_3^0\}*\{P_2^1\}*\{P_3^0\}^2*\{P_4^0\}^2$$
$$+\{P_1^2\}^2*\{P_1^1|P_1^1|P_2^0P_3^0\}*\{P_2^1\}^3*\{P_3^0\}^2*\{P_4^0\}^2$$
$$+\{P_1^2\}*\{P_1^1|P_1^1|P_1^0P_3^0\}*\{P_2^1\}^4*\{P_3^0\}^2*\{P_4^0\}^2$$
$$+\{P_1^2\}*\{P_1^1|P_2^1\}*\{P_1^1|P_1^0P_3^0\}*\{P_2^1\}^3*\{P_3^0\}^2*\{P_4^0\}^2$$
$$+\{P_4^0\}^2*\{P_1^0|P_3^0\}*\{P_1^1|P_1^0P_3^0\}*\{P_2^1\}^6$$
$$+\{P_1^1\}*\{P_4^0\}^2*\{P_1^0|P_3^0|P_1^0P_3^0\{*\{P_2^1\}^6$$
$$+\{P_1^1\}*\{P_4^0\}^2*\{P_3^0|P_3^0|P_3^0|P_1^0\}*\{P_2^1\}^5$$
$$+\{P_4^0\}^2*\{P_2^0|P_1^0|P_3^0|P_3^0\}*\{P_2^1\}^6$$
$$+\{P_4^0\}^2*\{P_1^0|P_2^0|P_3^0|P_3^0\}*\{P_2^1\}^6$$
$$+\{P_4^0\}^2*\{P_1^0|P_3^0|P_2^0|P_3^0\}*\{P_2^1\}^6$$
$$+\{P_4^0\}^2*\{P_3^0\}^2*\{P_1^0\}*\{P_2^0\}*\{P_2^1\}^6,$$

we eliminate all terms of weight 18 except two terms,

$$\{P_1^2\}^3*\{P_3^0\}^6*\{P_4^0\}^2+\{P_1^1\}*\{P_2^1\}^4*\{P_3^0\}^4*\{P_4^0\}^2.$$

Next we consider the differential

$$d(\{P_1^0\}^3*\{P_2^0\}^5*\{P_2^3\}*\{P_3^1\}^3+\{P_1^0\}^2*\{P_2^0\}^6*\{P_2^2\}*\{P_2^3\}*\{P_3^1\}^2$$
$$+\{P_1^0\}*\{P_2^0\}^7*\{P_2^2\}^2*\{P_2^3\}*\{P_3^1\}+\{P_2^0\}^8*\{P_2^2\}^3*\{P_2^3\}).$$

Since this calculation for proof is very long, we only mention its steps. There are innumerable choices to be made in such a calculation and I do not claim that the proof indicated is the shortest possible. This differential contains no terms of weight less than 15. By adding to this the boundaries of 22 terms, we eliminate all terms of weight 15. By adding in the boundaries of 88 terms, we eliminate all terms of weight 16. By adding in the boundaries of 166 terms, we eliminate all terms of weight 17. And by adding in the boundaries of 501 terms, we eliminate all terms of weight 18 except two terms,

$$\{P_1^2\}^3*\{P_3^0\}^6*\{P_4^0\}^2+\{P_1^1\}*\{P_2^1\}^4*\{P_3^0\}^4*\{P_4^0\}^2.$$

Thus 2.4 is proved.

PROPOSITION. 2.5. $d_4(h_2h_1(1)^2b_{0,3}b_{0,4})=h_1(1)^4b_{1,2}$ $+h_2^2h_1(1)^2b_{1,2}b_{1,3}+h_1^2h_1(1)b_{2,2}^2b_{0,3}$.

PROOF. I know no proof of this proposition except by the imbedding method. We must show that in the bar construction $d(\{P_2^1\}^6*\{P_2^2\}^4)$ is homologous to

$$\{P_1^2\}*\{P_2^1\}^2*\{P_2^2\}^2*\{P_3^0\}^2*\{P_4^0\}^2$$

modulo terms of weight greater than 14. This differential contains no terms of weight less than 13. By adding to this the boundaries of three terms of weight 12,

$$\{P_2^0|P_4^0\} * \{P_2^1\}^5 * \{P_2^2\}^3 + \{P_1^1\} * \{P_1^2\} * \{P_2^1\}^4 * \{P_2^2\}^2 * \{P_4^0\}^2$$
$$+ \{P_1^2\} * \{P_1^3\} * \{P_2^1\}^5 * \{P_4^0\}^2,$$

we eliminate all terms of weight 13. By adding in the boundaries of three terms of weight 13,

$$\{P_1^2\} * \{P_2^1\}^3 * \{P_2^2\}^2 * \{P_1^0|P_3^0\} * \{P_4^0\}^2$$
$$+ \{P_1^1\} * \{P_2^1\}^4 * \{P_2^2\} * \{P_1^1|P_3^1\} * \{P_4^0\}^2$$
$$+ \{P_1^3\} * \{P_2^1\}^5 * \{P_1^1|P_3^1\} * \{P_4^0\}^2,$$

we eliminate all terms of weight 14 except two terms,

$$\{P_1^2\} * \{P_1^2\}^2 * \{P_2^2\}^2 * \{P_3^0\}^2 * \{P_4^0\}^2$$
$$+ \{P_1^1\} * \{P_2^1\}^4 * \{P_3^1\}^2 * \{P_4^0\}^2.$$

Since we have a differential $d_2(h_1 b_{1,2}^2 b_{1,3} b_{0,4}) = h_1 h_0(1) b_{1,2}^2 h_0(1,2) + h_1^2 b_{1,2}^2 b_{1,3}^2$ by Theorem 1.3, we can eliminate the term $\{P_1^1\} * \{P_2^1\}^4 * \{P_3^1\}^2 * \{P_4^0\}^2$ by adding the boundaries of the dual elements of $h_1 h_0(1) b_{1,2}^2 h_0(1,2)$ or $h_1^2 b_{1,2}^2 b_{1,3}^2$ which have the weight greater than 11. Thus proposition is proved.

PROPOSITION. 2.6. (a) $d_4(h_0 h_4 b_{0,5}) = h_0 h_5^2 b_{0,3} + h_0 h_3(1) b_{0,2}$;

(b) $d_6(h_3 b_{0,2}^4 b_{1,3}^2) = h_0^6 h_3^2 b_{2,2} b_{0,3} + h_0^7 h_4 h_0(1) b_{1,3}^2 + h_0^8 b_{2,2} b_{1,3}^2$.

PROOF. We make use of the Adams vanishing theorem. For dimensional and filtrational reasons there are no other ways to kill the cocycles $h_0^n(h_0 h_5^2 b_{0,3} + h_0 h_3(1) b_{0,2})$ and $h_0^n(h_0^6 h_3^2 b_{2,2} b_{0,3} + h_0^7 h_4 h_0(1) b_{1,3}^2 + h_0^8 b_{2,2} b_{1,3}^2)$, for large n.

PROPOSITION 2.7. $d_6(h_0 h_2 b_{0,3}^6) = h_0^2 d_0 B_4 + h_0 P^2 D_2$.

PROOF. There are two possible terms $h_0^2 d_0 B_4 = h_1 d_0 B_{21}$ and $h_0 P^2 D_2$ for dimensional and filtrational reasons. First we will prove that $d_6(h_0 h_2 b_{0,3}^6)$ contains the term $h_0^2 d_0 B_4 = h_1 d_0 B_{21}$. We must show that in the bar construction.

$$d(\{P_1^0\}^3 * \{P_2^0\}^2 * \{P_2^1\}^6 * \{P_3^0\}^2 * \{P_4^0\}^2$$
$$+ \{P_1^0\}^2 * \{P_1^2\} * \{P_2^0\}^3 * \{P_2^1\}^5 * \{P_3^0\}^2 * \{P_4^0\}^2$$
$$+ \{P_1^0\} * \{P_1^2\}^2 * \{P_2^0\}^4 * \{P_2^1\}^4 * \{P_3^0\}^2 * \{P_4^0\}^2$$
$$+ \{P_1^2\}^3 * \{P_2^0\}^5 * \{P_2^1\}^3 * \{P_3^0\}^2 * \{P_4^0\}^2 + \{P_1^1\} * \{P_2^0\}^5 * \{P_2^1\}^7 * \{P_4^0\}^2)$$

is homologous to

$$\{P_1^0\} * \{P_1^2\} * \{P_3^0\}^{12}$$

modulo terms of weight greater than 24, since we have a differential
$d_2(h_0(1)^5 b_{0,3} b_{0,4} + h_0(1)^3 b_{0,2} b_{0,3}^2 b_{1,3} + h_1^2 h_0(1)^3 b_{0,3}^2 b_{0,4}) = h_0^2 d_0 B_4 + h_1 d_0 B_{21}$ by Theorem 1.3.

This differential contains no terms of weight less than 20. By adding to this the boundaries of five terms of weight 19,

$$\begin{aligned}&\{P_2^0\}^5*\{P_2^1\}^6*\{P_1^0|P_3^0\}*\{P_4^0\}^2\\
&+\{P_1^0\}*\{P_2^0|P_1^1\}*\{P_2^0\}^3*\{P_2^1\}^5*\{P_3^0\}^2*\{P_4^0\}^2\\
&+\{P_1^2\}*\{P_2^0|P_1^1\}*\{P_2^0\}^4*\{P_2^1\}^4*\{P_3^0\}^2*\{P_4^0\}^2\\
&+\{P_1^0\}*\{P_2^1|P_1^1\}*\{P_2^0\}^4*\{P_2^1\}^4*\{P_3^0\}^2*\{P_4^0\}^2\\
&+\{P_1^2\}*\{P_2^1|P_1^1\}*\{P_2^0\}^5*\{P_2^1\}^3*\{P_3^0\}^2*\{P_4^0\}^2,\end{aligned}$$

we eliminate all terms of weight 20. By adding in the boundaries of five terms of weight 20,

$$\begin{aligned}&\{P_1^0\}^2*\{P_2^0\}^3*\{P_2^1\}^4*\{P_1^1|P_1^0P_3^0\}*\{P_3^0\}^2*\{P_4^0\}^2\\
&+\{P_1^0\}*\{P_1^2\}*\{P_2^0\}^4*\{P_2^1\}^3*\{P_1^1|P_1^0P_3^0\}*\{P_3^0\}^2*\{P_4^0\}^2\\
&+\{P_1^2\}^2*\{P_2^0\}^5*\{P_2^1\}^2*\{P_1^1|P_1^0P_3^0\}*\{P_3^0\}^2*\{P_4^0\}^2\\
&+\{P_1^0\}*\{P_1^0|P_3^0\}*\{P_2^0\}^4*\{P_2^1\}^4*\{P_3^0\}^2*\{P_4^0\}^2\\
&+\{P_1^2\}*\{P_1^0|P_3^0\}*\{P_2^0\}^5*\{P_2^1\}^3*\{P_3^0\}^2*\{P_4^0\}^2,\end{aligned}$$

we eliminate all terms of weight 21. In our calculation all terms of weight 22 are eliminated by adding in the boundaries of 20 terms and all terms of weight 23 are eliminated by adding in the boundaries of 32 terms. There now remain 61 terms of weight 24 which are transformed, by the addition of the boundaries of 47 elements of weight 23, into the single term $\{P_1^0\}*\{P_1^2\}*\{P_3^0\}^{12}$. Next we will prove that $d_6(h_0h_2b_{0,3}^{6})$ contains the term $h_0P^2D_2$. We must show that in the bar construction

$$\begin{aligned}&d(\{P_1^0\}^2*\{P_2^0\}^8*\{P_2^1\}*\{P_2^2\}*\{P_3^0\}^2*\{P_5^0\}\\
&+\{P_1^0\}*\{P_2^0\}^8*\{P_2^2\}*\{P_2^3\}*\{P_3^0\}^4\\
&+\{P_1^0\}^2*\{P_2^0\}^7*\{P_2^3\}*\{P_3^0\}^4*\{P_3^1\}\\
&+\{P_1^0\}^3*\{P_2^0\}^7*\{P_2^1\}*\{P_3^0\}^2*\{P_4^0\}*\{P_5^0\})\end{aligned}$$

is homologous to

$$\{P_1^0\}*\{P_1^2\}*\{P_3^0\}^{12}$$

modulo terms of weight greater than 24. This differential contains no terms of weight less than 21. In our calculation all terms of weight 21, 22 and 23 are eliminated by adding in the boundaries of 11, 13 and 19 terms, respectively. There now remain 31 terms of weight 24 which are transformed, by the addition of the boundaries of 51 elements of weight 23, into the single term $\{P_1^0\}*\{P_1^2\}*\{P_3^0\}^{12}$. Thus proposition is proved.

PROPOSITION. **2.8.** $d_6(h_3^3b_{1,3}b_{0,4}+h_1b_{2,2}b_{1,3}^{2})=0$.

PROOF. There is only one possible term $h_0^2h_2h_6b_{0,2}^2=h_1^3h_6b_{0,2}^2$ for dimensional and filtrational reasons. We must show that in bar construction

$$d(\{P_1^1\}^3*\{P_1^6\}*\{P_2^0\}^4+\{P_1^0\}^2*\{P_1^2\}*\{P_1^6\}*\{P_2^0\}^4$$
$$+\{P_1^0\}^3*\{P_1^6\}*\{P_2^0\}^3*\{P_2^1\})$$

is homologous to zero modulo terms of weight greater than 10, since we have a differential $d_2(h_6b_{0,2}^3)=h_1^3h_6b_{0,2}^2+h_0^3h_6h_0(1)b_{0,2}$ by Theorem 1.3. This differential contains no terms of weight less than 6. By adding to this the boundaries of two terms of weight 5,

$$\{P_1^0|P_2^0\}*\{P_1^1\}*\{P_1^6\}*\{P_2^0\}^4+\{P_2^0|P_1^1\}*\{P_1^0\}*\{P_1^6\}*\{P_2^0\}^4,$$

we eliminate all terms of weight 6 and 7. By adding in the boundaries of two terms of weight 7,

$$\{P_1^0\}^2*\{P_2^2|P_1^3P_1^4P_1^5\}*\{P_2^0\}^4+\{P_1^0\}^3*\{P_2^0\}^3*\{P_3^1|P_1^3P_1^4P_1^5\},$$

we eliminate all terms of weight 8. By adding in the boundaries of eight terms of weight 8,

$$\{P_1^1\}^3*\{P_2^0\}^3*\{P_3^0P_1^2|P_1^3P_1^4P_1^5\}$$
$$+\{P_1^1\}^2*\{P_2^0\}^4*\{P_2^1P_1^2|P_1^3P_1^4P_1^5\}$$
$$+\{P_1^0\}^3*\{P_2^0\}^2*\{P_2^1\}*\{P_3^0P_1^2|P_1^3P_1^4P_1^5\}$$
$$+\{P_1^0\}^2*\{P_1^2\}*\{P_2^0\}^3*\{P_3^0P_1^2|P_1^3P_1^4P_1^5\}$$
$$+\{P_1^0\}^2*\{P_2^0\}^3*\{P_2^2|P_3^0P_1^2|P_1^4P_1^5\}$$
$$+\{P_1^0\}^3*\{P_2^0\}^2*\{P_3^1|P_3^0P_1^2|P_1^4P_1^5\}$$
$$+\{P_1^0\}^2*\{P_2^0\}^3*\{P_3^0P_1^2|P_2^2|P_1^4P_1^5\}$$
$$+\{P_1^0\}^3*\{P_2^0\}^2*\{P_3^0P_1^2|P_3^1|P_1^4P_1^5\},$$

we eliminate all terms of weight 9. In our calculation there now remain 34 terms of weight 10 which are eliminated by the addition of the boundaries of 24 terms of weight 9. Thus proposition is proved.

PROPOSITION 2.9. $d_4(h_4b_{0,2}b_{1,4}+h_1^2h_4b_{0,5}+h_2b_{3,2}b_{0,4})=h_5^2h_0(1)^2$.

PROOF. We make use of manipulative methods. We have

$h_0^2(h_4b_{0,2}b_{1,4}+h_1^2h_4b_{0,5}+h_2b_{3,2}b_{0,4})$

$=h_4b_{0,2}^2b_{2,3}+h_1J$

$+d_2(b_{0,2}b_{3,2}b_{0,4}+h_3h_5b_{0,3}b_{0,4}+h_1^2h_1(1)b_{0,5})$, where

$J=h_3h_5b_{0,3}b_{1,3}+h_1^2h_1(1)b_{1,4}+b_{0,2}b_{3,2}b_{1,3}+h_1^2b_{3,2}b_{0,4}+h_1^2h_3^2b_{0,5}$ is a permanent cocycle, and $d_4(h_4b_{0,2}^2b_{2,3})=h_0^2h_5^2h_0(1)^2$ by Theorem 1.3, from which the result follows.

PROPOSITION 2.10. $d_8(h_0h_2b_{1,2}b_{0,3}^2(B))=0$.

PROOF. We make use of Theorem 1.4. Since we have a differential $d_4(h_2b_{0,3})=h_0^2h_3^2$, Massey product $\langle h_0^2h_3, h_3, B_4\rangle$ is defined in E_5 and is equal to $h_0h_2b_{1,2}b_{0,3}^2(B)$, where $B_4=h_0b_{1,2}b_{0,3}(B)$ is a survivor in the 60-sterm. Next we will verify that this Massey product is defined in $H^*(A)$. We have a relation $h_0^2h_3^2=0$ in $H^*(A)$ for dimensional and filtrational reasons. If the class h_3B_4, which is zero in $E^0H^*(A)$, is non zero in $H^*(A)$, then it must be equal to a class h_0X_3 and therefore we have two relations $h_1h_3B_{21}=h_0^2h_3B_4=h_0^3X_3\neq 0$. But $h_3B_{21}=0$ in $H^*(A)$. This is a contradiction. Then we have a relation $h_3B_4=0$. Therefore, the Massey product $\langle h_0^2h_3, h_3, B_4\rangle$ is well-defined in $H^*(A)$. Furthermore it is easy to see that this Massey product satisfies condition (*) of Theorem 1.4. Thus 2.10 is proved.

PROPOSITION 2.11. $d_4(h_3b_{0,2}^3b_{1,4}+h_5b_{0,2}^2b_{0,3}b_{1,3}$
$+h_1h_0(1)^2b_{02}b_{1,4}+h_1^3b_{0,2}b_{0,3}b_{1,4}+h_0^3h_0(1)b_{0,3}b_{1,4}$
$+h_0^3h_2b_{0,4}h_0(1,2))=h_0^4h_3h_5b_{0,3}b_{1,3}+h_0^4b_{0,2}b_{3,2}b_{1,3}$.

PROOF. We make use of the Adams vanishing theorem. For dimensional and filtrational reasons there are no other ways to kill the cocycles
$h_0^n(h_0^4h_3h_5b_{0,3}b_{1,3}+h_0^4b_{0,2}b_{3,2}b_{1,3})$, for large n.

PROPOSITION 2.12 $d_4(h_2h_4b_{0,5})=h_2h_5^2b_{0,3}$.

PROOF. We make use of manipulative methods. We have
$h_0^2h_2h_4b_{0,5}=h_1h_4b_{0,2}b_{1,4}+h_1^3h_4b_{0,5}+d_2(h_4b_{0,2}b_{0,5})$,
$h_0^2h_2h_5^2b_{0,3}=h_1h_5^2h_0(1)^2+d_2(h_5^2b_{0,2}b_{0,3}+h_3(1)b_{0,2}^2)$, and
$d_4(h_1h_4b_{0,2}b_{1,4}+h_1^3h_4b_{0,5})=h_1h_5^2h_0(1)^2$ by Proposition 2.9, from which the result follows.

PROPOSITION 2.13. $d_6(h_0h_3h_5b_{0,2}b_{0,3}^3+h_0b_{0,2}^3b_{0,3}b_{2,3}+h_0^3b_{0,2}^2b_{0,3}b_{1,4})=h_0^5h_4b_{0,2}^2b_{2,3}$.

PROOF. We make use of the Adams' vanishing theorem. For dimensional and filtrational reasons there are no other ways to kill $h_0^nh_4b_{0,2}^2b_{2,3}$, for large n.

The differentials which we have proved in thirteen propositions above and those of Theorem 1.3 give all the essential informations about the May spectral sequence in the range $t-s\leqq 77$. Countless other differentials must be craked out, but they all follow from those given above by more or less elementary arguments or routine calculations. For example, we prove that the cochain $h_2h_1(1)b_{0,3}h_0(1,2)$ is a permanent cocycle. $h_1(1)h_0(1,2)=H_1$ is a permanent cocycle and $d_4(h_2b_{0,3})=h_0^2h_3^2$ by Theorem 1.3. Then $d_4(h_2h_1(1)b_{0,3}h_0(1,2))=0$ by Table I.B. For dimensional and filtrational reasons $d_r(h_2h_1(1)b_{0,3}h_0(1,2))=0$ for $r\geqq 5$.

We collect the results into the following theorem. Relations which are derived from the May relation in E_2 are indicated by equality signs; relations which come out of differentials in the May spectral sequence are indicated by the homology sign $\sim$.

THEOREM. 2.14. *Table II lists all generators of the Z_2-module $E_\infty = E^0H^*(A)$ in the range $71 \leqq t-s \leqq 77$. These generators are subject to the relations indicated.*

Table II

$t-s$	s	Generators
71	3	$h_1h_3h_6$
	4	h_6c_0
	5	$h_1p_1 \sim h_2d_2$
	6	$h_1h_5^2c_0$
	6— 7	$A_2, h_0A_2 = h_3A'' \sim Sq^0(l)$
	9—10	$h_3^2Q_2,\ h_0h_3^2Q_2$
	11	P^1X_2
	12	h_5d_0j
	12	$Q_4 = \langle q_1, h_0, h_2^2 \rangle$
	13	g^2n
	13—15	$P^1h_2B_4, h_0P^1h_2B_4, h_0^2P^1h_2B_4 \sim h_1P^1B_{22}$
	13—14	Q_5, h_0Q_5
	15	h_1P^2G
	16	$d_0e_0g^2$
	19	P^1d_0gk
	20	$h_1P^2S_1 \sim h_1P^3B_1$
	22	P^2d_0z
	25	P^4u
	28—30	$P^5d_0e_0, h_0P^5d_0e_0 \sim P^7h_4, h_0^2P^5d_0e_0$
	31—36	$P^6i,\ \cdots\cdots\cdots, h_0^5P^6i$
	33	$P^7h_1d_0$
72	4	$h_1^2h_3h_6 \sim h_2^3h_6$
	5	$h_1h_6c_0$
	6	$P^1h_1h_6$
	8	B_6
	8— 9	$h_3^2D_2, h_0h_3^2D_2 \sim gD_1$
	10—12	$P^1A'', h_0P^1A'', h_0^2P^1A'' \sim d_1g^2$

Table II (continued)

$t-s$	s	Generators
	13	h_1Q_4
	15	g^2l
	18	P^1d_0gr
	18	P^2Q_1
	19—21	$P^3B_2, h_0P^3B_2, h_0^2P^3B_2 \sim h_1^2P^3B_1$
	21	P^2d_0v
	24	$P^2d_0^4$
	26	h_1P^4u
	27	$P^5l, h_0P^5l, h_0^2P^5l \sim h_1P^5d_0e_0$
	34	$h_1^2P^7d_0$
	35	P^8c_0
73	7	$P^1h_2h_5^2$
	7	$h_1P^1h_1h_6$
	7— 9	$h_4D_2, h_0h_4D_2, h_0^2h_4D_2 \sim h_1B_6$
	14—16	$d_0B_{21}, h_0d_0B_{21}, h_0^2d_0B_{21}$
	17	d_0kr
	17—19	$P^1R_2, h_0P^1R_2, h_0^2P^1R_2 \sim h_1P^2Q_1$
	20	$d_0^4e_0$
	23	P^3d_0m
	26—27	$P^4z, h_0P^4z \sim h_1^2P^4u$
	32—35	$P^7e_0, h_0P^7e_0, h_0^2P^7e_0, h_0^3P^7e_0 \sim h_1^3P^7d_0$
	36	$h_1P^8c_0$
	37	P^9h_1
74	6	h_3n_1
	6— 8	$P^1h_2h_6, h_0P^1h_2h_6, h_0^2P^1h_1h_6$
	8—13	$F_1 = <d_0, h_4, g_2>, \cdots\cdots, h_0^5F_1$
	13—15	$d_0B_4, h_0d_0B_4, h_0^2d_0B_4 \sim h_1d_0B_{21}$
	14	$D_2'' = P^2D_2 + h_0d_0B_4$
	16	d_0g^3
	19	P^1d_0gl
	22	$P^2d_0^2r$
	25	P^4v

Table II (continued)

$t-s$	s	Generators
	28—30	$P^4d_0^3, h_0P^4d_0^3, h_0^2P^4d_0^3$
	31—33	$P^6j, h_0P^6j, h_0^2P^6j \sim h_1P^7e_0$
	38	$h_1P^9h_1$
75	5— 6	$h_3d_2 = h_5g_2, h_0h_3d_2$
	7— 9	$A_1, h_0A_1 \sim h_3^2A', h_0^2A_1 \sim h_1F_1$
	8	h_4B_3
	9—11	$P^1Q_3 \sim h_3G_{21}, h_0P^1Q_3, h_0^2P^1Q_3$
	12	$B_7 = \langle h_0^2h_3, h_3, B_4 \rangle$
	15	g^2m
	18	d_0gz
	18—20	$P^2B_{21}, h_0P^2B_{21}, h_0^2P^2B_{21}$
	21	P^2d_0w
	24	$P^2e_0d_0^3$
	27—29	$P^5m, h_0P^5m, h_0^2P^5m \sim h_1P^4d_0^3$
	37—39	$P^9h_2, h_0P^9h_2, h_0^2P^9h_2 \sim h_1^2P^9h_1$
76	5	h_4D_3
	6— 9	$J, h_0J, h_0^2J \sim P^2d_2, h_0^3J$
	7	$h_3^2H_1$
	8	d_1g_2
	9	D_4
	10	$h_1P^1Q_3$
	14	g^2t
	14	d_0B_{22}
	16—18	$D_{22}, h_0D_{22} = P^1G_{11}, h_0^2D_{22}$
	17	d_0gv
	17—19	$P^2B_4, h_0P^2B_4, h_0^2P^2B_4 \sim h_1P^2B_{21}$
	19—22	$P^2g_2', h_0P_2g_2', h_0^2P^2g_2', h_0^3P^2g_2'$
	20	$d_0^4g,$
	23	P^2d_0gj
	26	P^4d_0r
	32—34	$P^6d_0^2, h_0P^6d_0^2, h_0^2P^6d_0^2$
77	3— 4	$h_3^2h_6, h_0h_3^2h_6$

Table II (continued)

$t-s$	s	Generators
	5— 7	$h_6d_0, h_0h_6d_0, h_0^2h_6d_0$
	6	$h_1h_4D_3$
	7	$Y_1 = \langle h_3, y, h_5 \rangle$
	7	h_1J
	7— 9	$m_1, h_0m_1 = pg_2, h_0^2m_1 \sim h_1d_1g_2$
	8	H_2
	8—11	$P^1p', h_0P^1p', h_0^2P^1p', h_0^3P^1p'$
	11	C_2
	12	P^2D_3
	13—15	$e_0B_4, h_0e_0B_4 \sim P^2A, h_0^2e_0B_4 \sim h_1d_0B_{22}$
	16	e_0g^3
	16—21	$B_8, h_0B_8 \sim P^2X_1, h_0^2B_8, \cdots\cdots, h_0^5B_8$
	19	P^1gmd_0
	22	$P^2d_0e_0r$
	25	P^4w
	28	$P^4d_0^2e_0$
	31—33	$P^6k, h_0P^6k, h_0^2P^6k \sim h_1P^6d_0^2$

Appendix 1

Dictionary of New Indecomposable Elements of E_∞

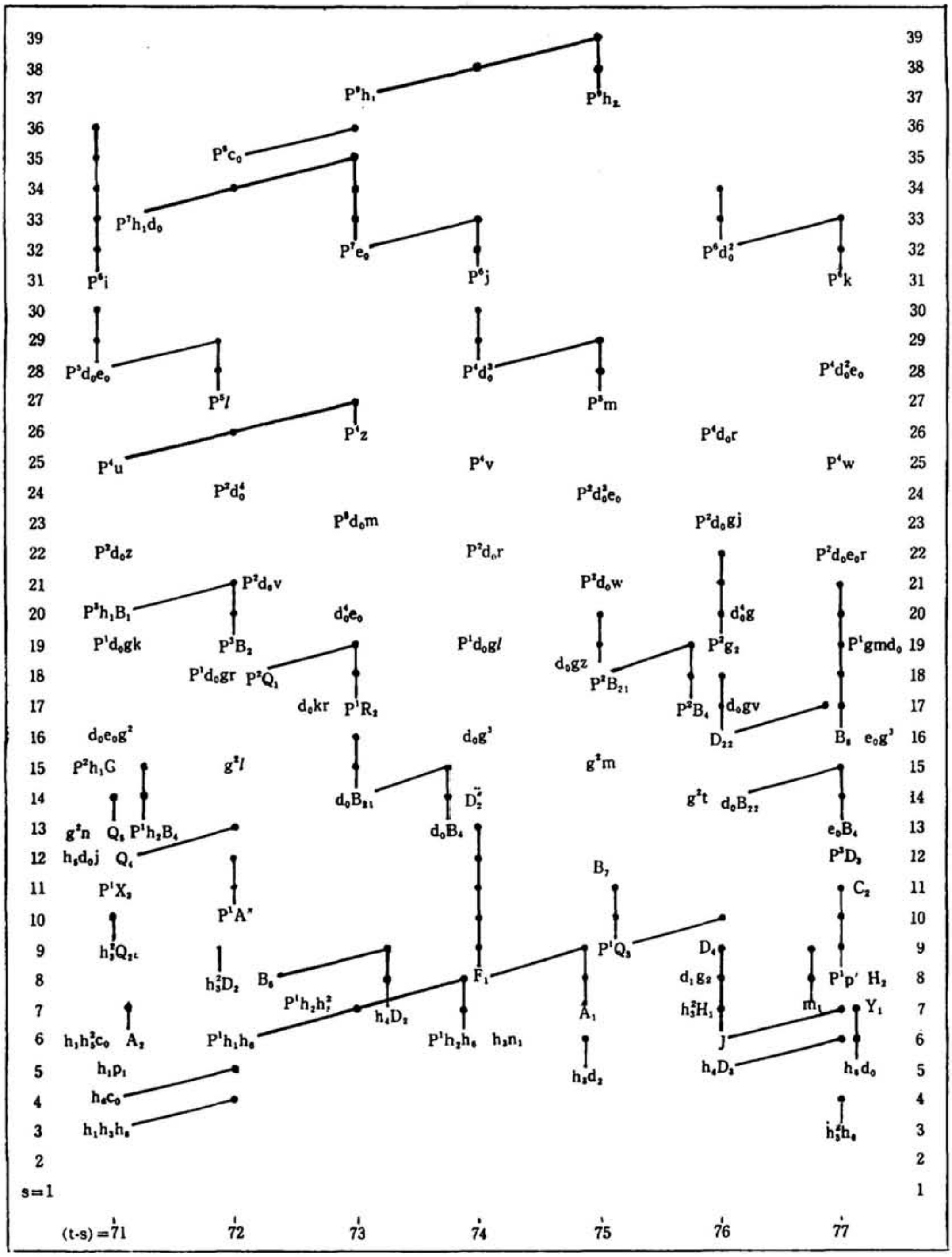
Name	$t-s$	s	Description
A_2	71	6	$h_3b_{13}h_0(1,3) \sim h_4b_{12}h_0(1,2) + h_2b_{22}h_0(1,2)$
Q_4	71	12	$h_1h_4h_0(1)b_{03}^4$
Q_6	71	13	$h_2b_{12}^2b_{03}^4 \sim h_4b_{02}^2b_{03}^4 \sim h_3b_{02}^3b_{03}^2b_{04} +$
			$h_1b_{02}^2b_{03}^3b_{13} + h_1h_0(1)^2b_{02}b_{03}^3b_{04}$
B_6	72	8	$h_4^2b_{02}(B) + h_1^2b_{02}b_{03}b_{23} + h_1h_5h_1(1)b_{03}^2$
			$\sim h_1h_5h_1(1)b_{03}^2 + h_1h_4b_{02}b_{13}^2 + h_1^2b_{02}b_{03}b_{23} + h_1^3h_4b_{13}b_{04}$
F_1	74	8	$h_3^2b_{22}b_{03}b_{13} + h_1^2b_{22}^2b_{04} + h_0h_4h_0(1)b_{13}^2 + h_0^2b_{22}b_{13}^2$
D_2''	74	14	$h_0b_{02}^4b_{03}h_0(1,2) + h_0^2h_0(1)^2b_{12}b_{03}(B) = P^2D_2 + h_0d_0B_4$
A_1	75	7	$h_3^3b_{13}b_{04} + h_1b_{22}b_{13}^2 \sim h_2b_{22}^2b_{04} +$
			$h_2^2h_4b_{13}b_{04} + h_0h_2b_{13}h_0(1,2)$

Dictionary of New Indecomposable Elements of E_∞ (continued)

Name	$t-s$	s	Description
B_7	75	12	$h_0h_2b_{12}b_{03}^2(B)$
J	76	6	$h_3h_5b_{03}b_{13}+b_{02}b_{32}b_{13}+h_1^2h_1(1)b_{14}+h_1^2b_{32}b_{04}+h_1^2h_3^2b_{05}$
D_4	76	9	$h_2^2b_{03}^2h_0(1,2)$
D_{22}	76	16	$h_0(1)^4b_{03}^4$
m_1	77	7	$h_3b_{22}^2b_{13}$
Y_1	77	7	$h_3^3b_{04}^2\sim h_2^2h_4b_{04}^2$
H_2	77	8	$h_2h_1(1)b_{03}h_0(1,2)$
C_2	77	11	$h_2h_1(1)b_{22}b_{03}^3\sim h_1h_1(1)b_{12}b_{03}^2b_{13}\sim h_2^3h_1(1)b_{03}^2b_{04}$
B_8	77	16	$h_0^2h_0(1)b_{02}^2b_{03}^2(B)$
B_3	60	7	$h_4h_0(1)(B)=d_0h_0(1,2)\sim h_1h_0(1)b_{13}^2$
B_5	66	10	$h_2^2b_{03}b_{12}(B)+h_1(1)^2b_{12}b_{03}^2+h_1h_3b_{22}b_{03}^3$
G_0	66	7	$b_{12}^2h_0(1,2)+h_1h_3b_{03}h_0(1,2)$

Appendix 2

Display chart of E_∞ for $71 \leqq t-s \leqq 77$.



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