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## On Anomalous Green Function Method of the s-d Exchange Model

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### Summary

Making improvement on the Takano-Ogawa, Abrikosov, and Kurata theories, we investigated the behavior of the anomalous parameter  $\Delta$  by introducing anomalous Green functions. Because of the appearance of the factor  $(\ln D/T)^{-1}$ , the parameter  $\Delta$  is considerably suppressed compared with the one in their results.

### 1. Introduction

Since Kondo's discovery<sup>1)</sup> of the low-temperature anomaly in the model of dilute magnetic alloy, many non-perturbative approaches have developed toward a physical understanding of the low-temperature properties of these systems.

In these non-perturbative theories, there have been double-time Green function methods<sup>2)</sup>.

Nagaoka<sup>3)</sup> has derived a set of decoupled equations for the Green function. From Nagaoka's equations, a simple integral equation for the scattering  $t$  matrix which appears in the one-particle Green function has been constructed by Hamann<sup>4)</sup>.

This Nagaoka-Hamann equation has been investigated in detail and the various quantities such as resistivity, specific heat, magnetic susceptibility, and ground state energy have been calculated by Bloomfield-Hamann<sup>5)</sup> and by Zittartz-Müllerhartmann<sup>6)</sup>, respectively.

These quantities seem to contain, however, rather curious consequences: the resistivity shows the steepest rise from the Kondo's temperature  $T_K$  multiplied by one-tenth to zero temperature, while the plateau behavior is observed in this region. The magnetic susceptibility exhibits the over-cancellation of the localized spin by the induced polarization at zero temperature. The ground state is not the singlet state according to Yoshida Theory<sup>7)</sup>, and so on.

Though Nagaoka's approximation has been proved to succeed in including all the leading terms in a series expansion<sup>8)</sup>, we know nothing about the truncation of the hierarchy of lower divergences.

Another Green function approach is the method of the anomalous Green function which has first been introduced by Takano-Ogawa<sup>9)</sup>, and refined by Abrikosov<sup>10)</sup> and Kurata<sup>11)</sup>, respectively.

The characteristics of this method are to destroy the conservation of number as in the theory of the condensed system such as superconductivity and superfluidity, to be

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mathematically simple to give the singlet ground state, and to give the results which qualitatively agree with the experimental behaviors at finite temperature.

The anomalous Green function approach, which will be referred to as A-approach hereafter, seems, therefore, to be better than the Nagaoka's approach (which will be referred to as N-approach hereafter) which is not to destroy the conservation of number.

Would what have been mentioned above be true? For us, it seems doubtful, because an anomalous parameter defined by anomalous averages is contained in the A-approach and not in the N-approach. This anomalous parameter behaves in the same way as the order parameter of pure superconductors when temperature increases, and vanished at the same critical point.

The various physical quantities such as specific heat have, therefore, discontinuity at the critical point. Therefore, we must conclude, in the case of the A-approach, that "a kind of phase transition" appears in our s-d exchange model with single impurity. Can this be true?

We cannot believe and understand physically the appearance of the "phase transition" in our model.

The authors have been interested in the existence of this "curious" parameter in the A-approach and investigating its role in the theory.

In section 2, a few faults contained in the A-approach are pointed. We will propose an improvement of the A-approach and formulate integral equation of the scattering  $t$  matrix in section 3. In section 4, we will solve the integral equation exactly as the Riemann-Hilbert problem<sup>12)</sup>. Section 5 will cover the investigation of the self-consistent equation for the anomalous parameter and the final section will be devoted to discussion and conclusion.

## 2. On the A-approach.

In this section, we point out a few faults in the A-approach. For this purpose, it is advantageous to use an anisotropic s-d exchange Hamiltonian in the following calculation:

$$H_1 = \sum_{\ell\ell'} \left\{ \lambda (C_{\ell\uparrow}^+ C_{\ell'\uparrow} - C_{\ell\downarrow}^+ C_{\ell'\downarrow}) S_z + \mu C_{\ell\uparrow}^+ C_{\ell'\downarrow} S_- + \mu C_{\ell\downarrow}^+ C_{\ell'\uparrow} S_+ \right\}. \quad (2.1)$$

$C_{\ell\sigma}^+$  ( $C_{\ell\sigma}$ ) is the creation (annihilation) operator of a conduction electron with wave number  $\ell$  and spin  $\sigma$ , and  $\lambda$  and  $\mu$  are diagonal components of the anisotropic exchange tensor between conduction electrons and a localized impurity spin  $S$  (we will consider hereafter only the case  $S = 1/2$  for simplicity).

This model is a special case of the model treating in the most divergent accuracy by Miwa-Nagaoka<sup>13)</sup> and Shiba<sup>14)</sup>, respectively. We also investigated it with the Nagaoka-Hamann's approximate treatment<sup>15)</sup>.

For the discussion in this section, furthermore, we adopt a simple superconducting model as an unperturbed state; that is,

$$H_0 = \sum_{\ell\sigma} \epsilon_{\ell} C_{\ell\sigma}^+ C_{\ell\sigma} - \Delta_s \sum_{\ell} (C_{\ell\uparrow}^+ C_{-\ell\downarrow} + C_{-\ell\downarrow} C_{\ell\uparrow}), \quad (2.2)$$

$$\text{where} \quad \Delta_s \equiv g \sum_{\ell} \langle C_{\ell\uparrow}^+ C_{-\ell\downarrow} \rangle, \quad (2.3)$$

$g$  is the coupling constant in the B. C. S. model<sup>16)</sup> and the bracket denotes the ensemble average. This unperturbed "model Hamiltonian" is correct in the case when the quasi-particles (Bogolons)<sup>17)</sup> moves freely, and this condition would be satisfied in sufficiently low temperature region.

We also put, in  $H_1$ ,

$$S = \frac{1}{2} \sum_{\sigma\sigma'} d_{\sigma}^{+} \langle \sigma | \sigma' \rangle d_{\sigma'}$$

in accordance with Abrikosov<sup>18)</sup>.

We introduce the Green functions in the matrix form

$$\hat{G}_{kk'}(z) = \begin{pmatrix} \langle\langle C_{k\uparrow} | C_{k'\uparrow}^{+} \rangle\rangle & \langle\langle C_{k\uparrow} | C_{-k'\downarrow} \rangle\rangle \\ \langle\langle C_{-k\downarrow}^{+} | C_{k'\uparrow} \rangle\rangle & \langle\langle C_{-k\downarrow}^{+} | C_{-k'\downarrow} \rangle\rangle \end{pmatrix}, \quad (2.4)$$

and

$$\hat{X}_{kk'}(z) = \begin{pmatrix} \langle\langle d_{\uparrow} | C_{k'\uparrow}^{+} \rangle\rangle & \langle\langle d_{\uparrow} | C_{-k'\downarrow} \rangle\rangle \\ \langle\langle d_{\downarrow}^{+} | C_{k'\uparrow} \rangle\rangle & \langle\langle d_{-k'\downarrow} | C_{-k'\downarrow} \rangle\rangle \end{pmatrix}, \quad (2.5)$$

where  $\langle\langle AB \rangle\rangle$  denotes the time Fourier component of the retarded or advanced Green function which is defined by  $-i\theta(t)\langle\langle A(t), B \rangle\rangle$  or  $i\theta(-t)\langle\langle A(t), B \rangle\rangle$ , respectively. Here,  $\theta(t)$  is the step function and  $A(t)$  shows an operator in the Heisenberg representation of the operator  $A$  in the Schrödinger representation, and the correlation function  $\langle BA \rangle$  and the definition of the symbol  $\mathfrak{F}_{\omega}$  are given by

$$\langle BA \rangle = \mathfrak{F}_{\omega} \langle\langle A | B \rangle\rangle = -\frac{i}{2\pi} \int_{-\infty}^{\infty} d\omega f(\omega) \{ \langle\langle A | B \rangle\rangle_{\omega+i\delta} - \langle\langle A | B \rangle\rangle_{\omega-i\delta} \},$$

$f(\omega) = (1 + e^{\beta\omega})^{-1}$  being the Fermi function.

Extending the approximation by Takano Ogawa and Abrikosov (or Kurata) to the superconducting state, we can construct equations for the Green functions introduced above and solve them easily to give the solution:

$$\hat{G}_{kk'}(z) = \delta_{kk'} \hat{\Omega}_k^{-1}(z) + \hat{\Omega}_k^{-1}(z) \hat{t}_A(z) \hat{\Omega}_{k'}^{-1}(z), \quad (2.6)$$

where  $\hat{\Omega}_k^{-1}(z)$  denotes the free Green function in the superconducting state. Matrix

$\hat{t}_A(z)$  is defined as follows:

$$\hat{t}_A(z) = \frac{\Delta}{\pi\rho} \left[ z - \frac{\Delta}{\pi\rho} \hat{F}(z) \right]^{-1}, \quad (2.7)$$

where "anomalous parameter"  $\Delta$  is given by

$$\Delta = \pi(\lambda + \mu)^2 \rho \left\{ \left| \sum_l \langle C_{l\uparrow}^{+} | d_{\uparrow} \rangle \right|^2 + \left| \sum_l \langle C_{l\uparrow}^{+} | d_{\downarrow}^{+} \rangle \right|^2 \right\}, \quad (2.8)$$

$$\text{and } \hat{F}(z) = \sum_l \hat{\Omega}_l^{-1}(z). \quad (2.9)$$

Constant  $\rho$  denotes the state density of the conduction electrons at the Fermi surface.

We have, also, the self-consistent equation for the parameter  $\Delta$ ,

$$\frac{1}{\lambda + \mu} = \mathfrak{F}_z \left[ \frac{F(z)}{z - \Delta F(z) / \pi \rho} \right], \text{ or } \Delta = 0, \quad (2.10)$$

where  $F(z) = F_1(z) - F_2(z)$  and  $F_1$  ( $F_2$ ) is the diagonal (off-diagonal) element of the matrix  $\hat{F}$ . Our solution and the self-consistent equation for the  $\Delta$  reduce to those of Machida-Inoue-Shibata<sup>19)</sup>, if we put  $\lambda = \mu = -J/(2N)$

Though the calculation mentioned above have been done with the A-approach, we can obtain simply the exact solution<sup>20)</sup> for the case  $\lambda = J/(2N)$  and  $\mu = 0$ :

$$\hat{t}_E(z) = \frac{J^2}{16N^2} \left[ 1 - \frac{J^2}{16N^2} \hat{F}^2(z) \right]^{-1} \hat{F}(z). \quad (2.11)$$

The scattering matrix  $\hat{t}_A(z)$  for  $\lambda = J/(2N)$  and  $\mu = 0$  is very different from  $\hat{t}_E(z)$ , especially at Fermi surface. This fact brings about large difference in various physical properties at low temperature, too. Examples mentioned above are shown in Fig. I for the normal state and in Fig. II for the superconducting state.

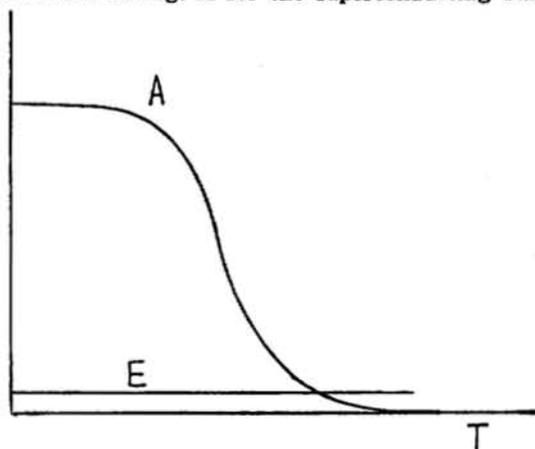


Fig. I. Temperature dependence of resistivities. Curve A is given by the approximate scattering matrix  $t_A$  in the A-approach and curve E is given by the exact scattering matrix  $t_E$  in the non-decoupling scheme for the case of  $\lambda = J/2N$  and  $\mu = 0$ , respectively.

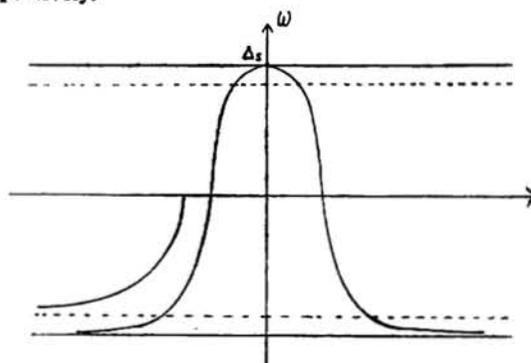


Fig. II. Position of excited bound states in the energy gap of the superconducting state. Curve A is obtained by using the approximate result of the scattering  $t_A$  matrix in the A-approach and curve E is obtained by using the exact result of the scattering  $t_E$  matrix in the non-decoupling scheme, for the case of  $\lambda = J/2N$  and  $\mu = 0$ , respectively.

The differences between the exact results and the results of the A-approach could not be neglected as shown in the Fig.'s, so we attempt to improve the A-approach in the next section.

### 3. An improved A-approach and Formulation in the normal state.

As discussed in section 2, the scattering matrix  $\hat{t}_A$  does not coincide with the exact scattering of  $\hat{t}_E$  in the "limiting case" of  $\lambda = J/(2N)$  and  $\mu = 0$ . To treat the A-approach more accurately, we attempt to introduce the anomalous Green functions into our scheme, using approximate procedure, at the decoupling stage in the N-approach. Hereafter, we treat only the case of normal state  $\Delta_s = 0$  in order to grasp more clearly the role anomalous functions in our scheme.

We need to define other new Green functions:

$$\Gamma_{hh'}(z) = \begin{pmatrix} \langle\langle C_{k\uparrow} S_z | C_{k'\uparrow}^+ \rangle\rangle & \langle\langle C_{k\uparrow} S_z | C_{-k'} \rangle\rangle \\ \langle\langle C_{-k\downarrow}^+ S_z | C_{k'\uparrow}^+ \rangle\rangle & \langle\langle C_{-k\downarrow}^+ S_z | C_{-k'} \rangle\rangle \end{pmatrix}, \quad (3.1)$$

and

$$\hat{\Lambda}_{hh'}(z) = \begin{pmatrix} \langle\langle C_{-k\downarrow} S_- | C_{k'\uparrow}^+ \rangle\rangle & \langle\langle C_{-k\downarrow} S_- | C_{-k'} \rangle\rangle \\ \langle\langle -C_{k\uparrow}^+ S_- | C_{k'\uparrow}^+ \rangle\rangle & \langle\langle -C_{k\uparrow}^+ S_- | C_{-k'} \rangle\rangle \end{pmatrix}. \quad (3.2)$$

Using the usual technique, we can construct the following set of the equations of motion for the Green functions:

$$\hat{\Omega}_k(z) \hat{G}_{hh'}(z) + \lambda \sum_l \hat{F}_{lh'}(z) + \mu \sum_l \hat{\Lambda}_{lh'}(z) = \delta_{hh'}, \quad (3.3)$$

$$\hat{\Omega}_k(z) \hat{F}_{hh'}(z) + \frac{\mu}{2} \sum_l \hat{\Lambda}_{lh'}(z) + \frac{\lambda}{4} \sum_l \hat{G}_{lh'}(z) = \hat{A}_k \hat{X}_{hh'}(z) \quad (3.4)$$

$$\hat{\Omega}_k(z) \hat{\Lambda}_{hh'}(z) + \mu \sum_l \hat{F}_{lh'}(z) + \frac{\lambda}{2} \sum_l \hat{\Lambda}_{lh'}(z) + \frac{\mu}{2} \sum_l \hat{G}_{lh'}(z) = \hat{B}_k \hat{X}_{hh'}(z), \quad (3.5)$$

and

$$z \hat{X}_{hh'}(z) - (\lambda + \mu) \hat{C} \sum_l \hat{G}_{lh'}(z) = 0, \quad (3.6)$$

where

$$\hat{A}_k = \mu \sum_{ll'} \begin{pmatrix} \langle C_{k\uparrow} C_{i\uparrow}^+ C_{l'\downarrow} d_i^+ \rangle & -\langle C_{k\uparrow} C_{i\uparrow}^+ C_{l'\downarrow} d_i \rangle \\ \langle C_{-k\downarrow}^+ C_{i\uparrow}^+ C_{l'\downarrow} d_i^+ \rangle & -\langle C_{-k\downarrow}^+ C_{i\uparrow}^+ C_{l'\downarrow} d_i \rangle \end{pmatrix}, \quad (3.7)$$

$$\hat{B}_k = -\lambda \sum_{ll'} \begin{pmatrix} \langle C_{-k\downarrow} C_{i\uparrow}^+ C_{l'\downarrow} d_i^+ - C_{-k\downarrow} C_{i\downarrow}^+ C_{l'\downarrow} d_i^+ \rangle \\ -\langle C_{k\uparrow} C_{i\uparrow}^+ C_{l'\downarrow} d_i^+ - C_{k\uparrow} C_{i\downarrow}^+ C_{l'\downarrow} d_i^+ \rangle \end{pmatrix}$$

$$-\mu \Sigma_{II'} \left( \begin{array}{cc} -\langle C_{-k_1} C_{i_1}^+ C_{i'_1} d_1 - C_{-k_1} C_{i_1}^+ C_{i'_1} d_1 \rangle & \\ \langle C_{k_1}^+ C_{i_1}^+ C_{i'_1} d_1 - C_{k_1}^+ C_{i_1}^+ C_{i'_1} d_1 \rangle & \end{array} \right) \left( \begin{array}{cc} -\langle C_{k_1} C_{i_1}^+ C_{i'_1} d_1^+ \rangle & -\langle C_{-k_1} C_{i_1}^+ C_{i'_1} d_1 \rangle \\ \langle C_{k_1}^+ C_{i_1}^+ C_{i'_1} d_1^+ \rangle & \langle C_{k_1}^+ C_{i_1}^+ C_{i'_1} d_1 \rangle \end{array} \right), \quad (3.8)$$

and

$$\hat{C} = \Sigma_I \left( \begin{array}{cc} \langle C_{i_1}^+ d_1 \rangle & \langle C_{i_1} d_1 \rangle \\ \langle C_{i_1}^+ d_1^+ \rangle & \langle C_{i_1} d_1^+ \rangle \end{array} \right). \quad (3.9)$$

In order to close the above equations, we have adopted the usual approximation method of decoupling for the Green functions of higher order than those introduced before. For example, they are given by

$$\begin{aligned} \ll C_{k_1} C_{i_1}^+ C_{i'_1} S_- | C_{k'_1}^+ \gg &= \ll C_{k_1} C_{i_1}^+ C_{i'_1} d_1^+ d_1 | C_{k'_1}^+ \gg \\ &\simeq \langle C_{k_1} C_{i_1}^+ C_{i'_1} d_1^+ \rangle \ll d_1 | C_{k'_1}^+ \gg - \langle C_{k_1} C_{i_1}^+ C_{i'_1} d_1 \rangle \ll d_1^+ | C_{k'_1}^+ \gg, \end{aligned} \quad (3.10)$$

$$\begin{aligned} \ll 2 C_{k_1} C_{i_1}^+ C_{i'_1} S_x | C_{k'_1}^+ \gg &= \ll C_{k_1} C_{i_1}^+ C_{i'_1} (d_1^+ d_1 - d_1 d_1^+) | C_{k'_1}^+ \gg \\ &\simeq \langle C_{k_1} C_{i_1}^+ C_{i'_1} d_1^+ \rangle \ll d_1 | C_{k'_1}^+ \gg + \langle C_{k_1} C_{i_1}^+ C_{i'_1} d_1 \rangle \ll d_1^+ | C_{k'_1}^+ \gg \end{aligned} \quad (3.11)$$

$$\begin{aligned} \ll C_{i_1}^+ C_{i'_1} d_1 | C_{k'_1}^+ \gg &\simeq -\langle C_{i_1}^+ d_1 \rangle \ll C_{i'_1} | C_{k'_1}^+ \gg + \langle C_{i'_1} d_1 \rangle \ll C_{-i_1}^+ | C_{k'_1}^+ \gg, \\ & \quad (3.12) \end{aligned}$$

and

$$\begin{aligned} \ll (C_{i_1}^+ C_{i'_1} - C_{i_1}^+ C_{i'_1}) d_1 | C_{k'_1}^+ \gg \\ \simeq -\langle C_{i_1}^+ d_1 \rangle \ll C_{i'_1} | C_{k'_1}^+ \gg - \langle C_{i'_1} d_1 \rangle \ll C_{-i_1}^+ | C_{k'_1}^+ \gg. \end{aligned} \quad (3.13)$$

This decoupling scheme would be discussed in section 6. The expressions (3.10), (3.11), and others have been used to construct the equations (3.4) and (3.5), and (3.12), (3.13), and others have been used to construct the equation (3.6).

Furthermore, using these expressions (3.12), (3.13), and others,  $\hat{A}_k$  and  $\hat{B}_k$  are rewritten down as follows:

$$\hat{A}_k = \mu \hat{n}_k \hat{C}, \quad (3.14)$$

and

$$\hat{B}_k = (\lambda + \mu) \hat{n}_k \hat{C}, \quad (3.15)$$

where

$$\hat{n}_h = \sum_l \begin{pmatrix} \langle C_{l\uparrow}^+ C_{h\uparrow} \rangle & \langle C_{-l\downarrow} C_{h\uparrow} \rangle \\ \langle C_{l\uparrow}^+ C_{-h\downarrow}^+ \rangle & \langle C_{-l\downarrow} C_{-h\downarrow}^+ \rangle \end{pmatrix} - I. \quad (3.16)$$

Now we have obtained a set of simultaneous equations for Green functions and it is easy to solve them, the formal solutions being found to be as

$$\hat{G}_{hh'}(z) = \delta_{hh'} \hat{\Omega}_h^{-1}(z) + \hat{\Omega}_h^{-1}(z) \hat{t}_o(z) \hat{\Omega}_{h'}^{-1}(z), \quad (3.17)$$

$$\hat{X}_h(z) = \frac{(\lambda + \mu)}{z} \hat{C} \hat{\Omega}_h^{-1}(z) \hat{t}(z), \quad (3.18)$$

and  $\hat{t}_o(z) + I = \hat{t}(z)$

$$= \left\{ \hat{A}(z) + \Delta \hat{B}(z) \mathfrak{F}_\eta \left[ \frac{\hat{F}(\eta) - \hat{F}(z)}{z - \eta} \hat{t}(\eta) \right] \right\}^{-1} \hat{C}(z), \quad (3.19)$$

where

$$\hat{A}(z) = \left[ 1 + \frac{\lambda}{2} \hat{F}(z) - \frac{\lambda^2 + 4\mu^2}{4} \hat{F}^2(z) - \frac{\lambda^3 - 4\lambda\mu^2}{8} \hat{F}^3(z) \right] z - \frac{\mu}{\pi \rho (\lambda + \mu)} \left[ 2\lambda + \mu - \frac{\lambda\mu + 2\mu^2}{2} \hat{F}(z) \right] \hat{F}^2(z) \Delta, \quad (3.20)$$

$$\hat{B}(z) = \mu \left[ 2\lambda + \mu - \frac{\lambda\mu + 2\mu^2}{2} \hat{F}(z) \right] / (\lambda + \mu), \quad (3.21)$$

and

$$\hat{C}(z) = \left[ 1 + \frac{\lambda}{2} \hat{F}(z) - \frac{\mu^2}{2} \hat{F}^2(z) \right] z. \quad (3.22)$$

The anomalous parameter  $\Delta$  is given by

$$\Delta = \pi \rho (\lambda + \mu)^2 \hat{C}^2 = \pi \rho (\lambda + \mu)^2 \hat{C} \mathfrak{F}_\eta \left[ \sum_l \hat{X}_l(z) \right] \\ = (\lambda + \mu) \Delta \mathfrak{F}_\eta \left[ \frac{\hat{F}(\eta) \hat{t}(\eta)}{\eta} \right],$$

thus,

$$\frac{1}{\lambda + \mu} = \mathfrak{F}_\eta \left[ \frac{\hat{F}(\eta) \hat{t}(\eta)}{\eta} \right], \text{ or } \Delta = 0. \quad (3.23)$$

Here, assuming that the state density is an even function of the conduction electron energy, we can obtain

$$\hat{F}(z) = F(z) = \sum_l \frac{1}{z - \epsilon_l} = \sum_l \frac{1}{z + \epsilon_l}. \quad (3.24)$$

Therefore, we can put

$$\hat{t}(z) = t(z). \quad (3.25)$$

We will drop the symbol " $\wedge$ " in the equations (3.19) through (3.23) and consider them as the scalar equations hereafter.

Finally, we must refer to the limiting case of  $\lambda = J/(2N)$  and  $\mu = 0$ . Using

Eqs. (3.17) through (3.20), the following result is obtained:

$$\hat{t}_0(z) = \frac{J^2}{16N^2} \frac{F(z)}{1 - \frac{J^2}{16N^2} F^2(z)} \quad \mu \rightarrow 0,$$

which coincides with Eq. (2.11) in the normal state  $\Delta_S = 0$ , so our approach in this section is a better approximation the original A-approach.

**4. Solution of the integral equation.**

In this section, we derive a solution for the t-matrix. We assume, for simplicity, that the density of state is constant, and that  $F(\omega)$  is given by

$$F(\omega \pm i\delta) = \begin{cases} \mp i\pi\rho & \text{for } |\omega| < D \\ 0 & \text{otherwise,} \end{cases} \quad (4.1)$$

where  $D$  is the half band width of conduction state.

In this case, the equation of  $t(\omega)$  can be written as

$$t(\omega + i\delta) = \frac{C_1 \omega}{A_1(\omega) + \Delta B_1 \int_{-D}^D d\eta \frac{f(\eta) - \frac{1}{2}}{\omega - \eta + i\delta} t(\eta - i\delta)}, \quad (4.2)$$

$$t(\omega - i\delta) = \frac{C_2 \omega}{A_2(\omega) + \Delta B_2 \int_{-D}^D d\eta \frac{f(\eta) - \frac{1}{2}}{\omega - \eta - i\delta} t(\eta + i\delta)}, \quad (4.3)$$

where

$$\begin{aligned} A_1(\omega) &= \left(1 - i\frac{\lambda}{2}\pi\rho + \frac{\lambda^2 + 4\mu^2}{4}\pi^2\rho^2 + i\frac{\lambda^3 - 4\lambda\mu^2}{8}\pi^3\rho^3\right)\omega \\ &\quad + \pi\rho\frac{\mu}{\lambda + \mu}(2\lambda + \mu + i\frac{\lambda\mu + 2\mu^2}{2}\pi\rho)\Delta \\ &= a_1\omega + b_1\Delta, \end{aligned} \quad (4.4)$$

$$A_2(\omega) = (\text{Complex conjugate of } A_1) = a_2\omega + b_2\Delta \quad (4.5)$$

$$B_1 = -ib_1\rho, \quad (4.6)$$

$$B_2 = (\text{Complex conjugate of } B_1), \quad (4.7)$$

$$C_1 = 1 - i(\lambda\pi\rho)/2 + (\mu^2\pi^2\rho^2)/2, \quad (4.8)$$

$$C_2 = (\text{Complex conjugate of } C_1), \quad (4.9)$$

and  $f(\omega)$  is the Fermi distribution function.

Furthermore, through Eqs.( 3.23) and (4.1), we have

$$\frac{1}{\lambda + \mu} = \frac{\rho}{2} \int_{-D}^D d\eta \frac{f(\eta) - \frac{1}{2}}{\eta} [t(\eta + i\delta) + t(\eta - i\delta)]. \quad (4.10)$$

If we put  $\Delta = 0$  in the integral equations (4.2) and(4.3), we will have (4.3)

$$t(\omega + i\delta) = C_1 / a_1, \quad (4.11)$$

and

$$t(\omega - i\delta) = C_2 / a_2. \quad (4.12)$$

In this case. Eq (4.10) is written as follows:

$$\frac{1}{\lambda + \mu} = \frac{\rho}{2} \left( \frac{C_1}{a_1} + \frac{C_2}{a_2} \right) \int d\eta \frac{f(\eta) - \frac{1}{2}}{\eta}, \quad (4.13)$$

from which we can calculate the critical point  $T_c$ ; that is,

$$T_c = 1.14 D \exp \left[ \frac{1}{\alpha (\lambda + \mu) \rho} \right] \quad (4.14)$$

where

$$\alpha = \frac{C_1}{a_1} + \frac{C_2}{a_2} \simeq 1 - \frac{\lambda^2}{4} \pi^2 \rho^2 - \frac{\mu^2}{2} \pi^2 \rho^2. \quad (4.15)$$

The critical temperature  $T_c$  calculated above is slightly lower than that in the original A-approach.

In order to solve the integral equations, we define several functions on the complex plane:

$$\Phi_r(z) = A_1(z) + \Delta B_1 \varphi_r(z). \quad (4.16)$$

$$\Phi_a(z) = A_2(z) + \Delta B_2 \varphi_a(z), \quad (4.17)$$

$$\varphi_r(z) = \int_{-D}^D d\eta \frac{f(\eta) - \frac{1}{2}}{z - \eta} t(\eta - i\delta), \quad (4.18)$$

$$\varphi_a(z) = \int_{-D}^D d\eta \frac{f(\eta) - \frac{1}{2}}{z - \eta} t(\eta + i\delta) \quad (4.19)$$

and

$$g(z) = \int_{-D}^D d\eta \frac{f(\eta) - \frac{1}{2}}{z - \eta} \quad (4.20)$$

In terms of these new functions, our integral equations are rewritten as follows:

$$t^+(\omega) = t(\omega + i\delta) = \frac{C_1 \omega}{\Phi_r^+(\omega)} \quad (4.21)$$

$$\text{and } t^-(\omega) = t(\omega - i\delta) = \frac{C_2 \omega}{\Phi_a^-(\omega)}. \quad (4.22)$$

If  $t^\pm(\omega)$  has not poles on the cut which is along the real axis for  $|\omega| < D$ ,  $\varphi_r(z)$  and  $\varphi_a(z)$  are sectionally holomorphic on the complex plane.

Therefore,  $\Phi_r(z)$ ,  $\Phi_a(z)$ , and  $g(z)$  are also sectionally holomorphic on the same plane.

Now we must examine the properties of these functions on the cut. The discontinuities of these functions across the cut are expressed as

$$g^+(\omega) - g^-(\omega) = -2\pi i [f(\omega) - \frac{1}{2}] \quad (4.23)$$

$$\begin{aligned} \Phi_r^+(\omega) - \Phi_r^-(\omega) &= \Delta B_1 [\varphi_r^+(\omega) - \varphi_r^-(\omega)] \\ &= -2\pi i \Delta B_1 [f(\omega) - \frac{1}{2}] t^-(\omega), \end{aligned} \quad (4.24)$$

$$\begin{aligned} \Phi_a^+(\omega) - \Phi_a^-(\omega) &= \Delta B_2 [\varphi_a^+(\omega) - \varphi_a^-(\omega)] \\ &= -2\pi i \Delta B_2 [f(\omega) - \frac{1}{2}] t^+(\omega). \end{aligned} \quad (4.25)$$

Using Eqs. (4.21) through (4.24), we have

$$\Phi_r^+(\omega) - \Phi_r^-(\omega) = \Delta B_1 C_2 \frac{g^+(\omega) - g^-(\omega)}{\Phi_a^-(\omega)}, \quad (4.26)$$

$$\text{and } \Phi_a^+(\omega) - \Phi_a^-(\omega) = \Delta B_2 C_1 \frac{g^+(\omega) - g^-(\omega)}{\Phi_r^+(\omega)} \quad (4.27)$$

From these equations, we can obtain

$$\left[ \Phi_r(\omega) \Phi_a(\omega) - \Delta \omega (B_1 C_2 + B_2 C_1) g(\omega) - A_1(\omega) A_2(\omega) \right]^+ = \left[ \begin{matrix} \text{''} & \text{''} & \text{''} \end{matrix} \right]^- \quad (4.28)$$

which means that the function

$$\Phi_r(z) \Phi_a(z) - \Delta z (B_1 C_2 + B_2 C_1) g(z) - A_1(z) A_2(z)$$

is regular anywhere on the entire complex plane. Therefore, this regular function should be constant. The constant number is determined by using the boundary condition at infinity:

$$\Phi_r(z) \Phi_a(z) - \Delta z (B_1 C_2 + B_2 C_1) g(z) - A_1(z) A_2(z) = C, \quad (4.29)$$

where

$$C = \Delta a_2 B_1 \int_{-D}^D d\eta [f(\eta) - \frac{1}{2}] t^-(\eta) + \Delta a_1 B_2 \int_{-D}^D d\eta [f(\eta) - \frac{1}{2}] t^+(\eta). \quad (4.30)$$

Equations (4.21) and (4.26) or (4.27) lead to

$$\Phi_r^+(\omega) \Phi_a^-(\omega) = A_1(\omega) A_2(\omega) + \omega \Delta (B_1 C_2 g^+(\omega) + B_2 C_1 g^-(\omega)) + C.$$

We can solve this equation. If we put

$$\Phi_r^+(\omega) = A_1(\omega) e^{-Q^+(\omega)}, \quad (4.31)$$

$$\text{and } \Phi_a^-(\omega) = A_2(\omega) e^{-Q^-(\omega)}, \quad (4.32)$$

we will easily get

$$Q(z) = \frac{1}{2\pi i} \int_{-D}^D d\eta \frac{\ln K(\eta)}{z - \eta} \quad (4.33)$$

where

$$K(\omega) = \frac{A_1(\omega) A_2(\omega) + \Delta \omega [B_1 C_2 g^+(\omega) + B_2 C_1 g^-(\omega)] + C}{A_1(\omega) A_2(\omega)} \quad (4.34)$$

Then, from (4.20), (4.21), (4.29), and (4.30) we can obtain

$$t_r^+(\omega) = -\frac{C_1 \omega}{A_1(\omega)} e^{Q^+(\omega)} \quad (4.35)$$

$$\text{and } t_a^-(\omega) = -\frac{C_2 \omega}{A_2(\omega)} e^{-Q^-(\omega)}. \quad (4.36)$$

Useful relations can be obtained from the moment expansions of Eqs. (4.29) and (4.30) for sufficiently large  $|\omega|$ ; that is,

$$\Delta B_1 \int_{-D}^D d\eta [f(\eta) - \frac{1}{2}] t^-(\eta) = -\frac{a_1}{2\pi i} \int_{-D}^D d\eta \eta \ln K(\eta), \quad (4.37)$$

and

$$\Delta B_2 \int_{-D}^D d\eta [f(\eta) - \frac{1}{2}] t^+(\eta) = \frac{a_2}{2\pi i} \int_{-D}^D d\eta \eta \ln K(\eta) \quad (4.38)$$

from the first order term in moment, and furthermore,

$$\int_{-D}^D d\eta \ln K(\eta) = 0$$

from the zeroth-order term in moment. Finally, using Eqs. (4.30), (4.37), and (4.38), it is shown that

$$C=0.$$

### 5. The self-consistent equation for $\Delta$ .

We can investigate the self-consistent equation for the anomalous parameter  $\Delta$  using the solution of the scattering t-matrix.

Since it is advantageous for the calculation below, we will adopt a few approximations in the integral  $Q^\pm(\omega)$ :

$$\begin{aligned} K(\omega) &= I + \frac{\omega \Delta [B_1 C_2 g^+(\omega) + B_2 C_1 g^-(\omega)]}{(a_1 \omega + b_1 \Delta)(a_2 \omega + b_2 \Delta)} \\ &\simeq I + \frac{\omega \Delta (B_1 C_2 + B_2 C_1) g(0)}{(a_1 \omega + b_1 \Delta)(a_2 \omega + b_2 \Delta)} \\ &= \frac{(\omega - z_1)(\omega - z_2)}{(\omega - z_1)(\omega - z_2)}, \end{aligned}$$

because the main contribution to the behavior of the function  $K(\omega)$  comes from  $\omega / (a_1 \omega + b_1 \Delta)(a_2 \omega + b_2 \Delta)$ , but not from  $g^\pm(\omega)$ . Now, since the function  $K(\omega)$  can be analytically continued into the complex plane, the integral in the  $Q^\pm(\omega)$  can be carried out simply, result being as follows:

$$\begin{aligned} Q^+(\omega) &= \frac{1}{2\pi i} \int_{-D}^D d\eta \frac{\ln K(\eta)}{\omega - \eta + i\delta} \simeq \frac{1}{2\pi i} \int_{-\infty}^{\infty} d\eta \frac{\ln K(\eta)}{\omega - \eta + i\delta} \frac{D^2}{\eta^2 + D^2} \\ &= -\ln K(\omega) \frac{z_2'}{z_2} \simeq -\ln K(\omega) \end{aligned}$$

The path of integration is shown in Fig. III. Therefore, we have

$$\begin{aligned} I &\equiv \int_{-D}^D d\eta \frac{f(\eta) - \frac{1}{2}}{\eta} t^+(\eta) = C_1 \int_{-D}^D d\eta \frac{f(\eta) - \frac{1}{2}}{A_1(\eta)} e^{Q^+(\eta)} \\ &\simeq C_1 \int_{-D}^D d\eta \frac{A_2(\eta) [f(\eta) - \frac{1}{2}]}{A_1(\eta) A_2(\eta) + (B_1 C_2 + B_2 C_1) \Delta \eta g(0)} \\ &\simeq \frac{C_1}{a_1} \int_{-D}^D d\eta \frac{f(\eta) - \frac{1}{2}}{\eta + [\frac{b_1}{a_1} + \frac{B_1 C_2 + B_2 C_1}{a_1 a_2} g(0)] \Delta} \\ &\simeq \frac{C_1}{a_1} \int_{-D}^D d\eta \frac{f(\eta) - \frac{1}{2}}{\eta + \gamma g(0) \Delta + i\delta} = \frac{-C_1}{a_1} g^- [-\gamma g(0) \Delta], \end{aligned} \quad (5.1)$$

where

$$\gamma = \frac{B_1 C_2 + B_2 C_1}{a_1 a_2} \quad (5.2)$$

From Eqs. (4.10) and (5.1), we have

$$\frac{\Delta}{\Delta_0} \simeq \frac{\sqrt{1 - \frac{T^2}{T_c^2}}}{\ln \frac{D}{T}} \quad (5.3)$$

as the final result, where  $T_c$  is given by (4.14), and the maximum  $\Delta_0$  of  $\Delta$  which appears at  $T \simeq T_c / 3$  is given by  $D/1\gamma l$ . This result  $\Delta_0 \gg T_c$ , seems to be a fault in our approach. It is, however, due to our rough calculation of (5.1), not to our approach.

More accurate calculation of (5.1) should give

$$\Delta_0 \simeq T_c \quad (5.4)$$

as in the original A-approach.

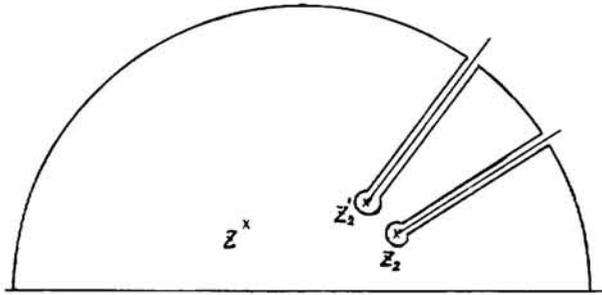


Fig. III. Integral contour in the function  $Q(z)$ . There are logarithmic cuts along the halfstraight lines, respectively.

## 6. Discussions

The appearance of the curious parameter  $\Delta$  in the Takano-Ogawa, Abrikosov, and Kurata theorems seems to result from the introduction of the anomalous Green functions and should cause unlike phase transition, and for this reason, we have been mostly interested in its appearance and have examined the behavior of the anomalous parameter  $\Delta$  in our improved A-approach.

In our approach the critical temperature  $T_c$  has come out considerably lower than that calculated by the original A-approach. Further-more, in our approach, the behavior of the anomalous parameter  $\Delta$  is different from that in the original A-approach: that is, the  $\Delta - T$  curve in our result seems to be suppressed more than the one in the original A-approach treatment by the factor of  $(\log D/T)^{-1}$ .

What will be the physical significance of the result concerning this  $\Delta$ ? In principle, our treatment is still in the realm of the A-approach, and by introducing anomalous Green functions at the stage of decoupling in the N-approach we tried to clarify the behavior of the anomalous parameter  $\Delta$ , but as the result we have come out with the suppressed  $\Delta - T$  curve.

Could this result mean that the phase transition-like anomaly would fade out more and more as we improve the approximation even in the realm of the A-approach?

Next, we will refer to the decoupling scheme in section 3. For example, the Green function  $\ll C_{k\uparrow} C_{i\uparrow}^+ C_{i'\downarrow} S_- | C_{k'\uparrow}^+ \gg$  must be decoupled in our approach. If we take up, according to the criterion that is only to conserve the total spin, all possible types of terms introduced by the decoupling, we have the following result:

$$\begin{aligned}
 \ll C_{k\uparrow} C_{i\uparrow}^+ C_{i'\downarrow} S_- | C_{k'\uparrow}^+ \gg &= \ll C_{k\uparrow} C_{i\uparrow}^+ C_{i'\downarrow} d_{i\downarrow}^+ d_{i\uparrow} | C_{k'\uparrow}^+ \gg \\
 &= \left\{ \langle C_{k\uparrow} C_{i\uparrow}^+ \rangle \ll C_{i'\downarrow} S_- | C_{k'\uparrow}^+ \gg + \langle C_{i\uparrow}^+ C_{i'\downarrow} S_- \rangle \ll C_{k\uparrow} | C_{k'\uparrow}^+ \gg \right. \\
 &\quad \left. - \langle C_{k\uparrow} C_{-i'\downarrow} \rangle \ll C_{i\uparrow}^+ S_- | C_{k'\uparrow}^+ \gg \right\} \quad (\text{N-term}) \\
 &+ \left\{ \langle C_{k\uparrow} C_{i\uparrow}^+ C_{-i'\downarrow} d_{i\downarrow}^+ \rangle \ll d_{i\uparrow} | C_{k'\uparrow}^+ \gg \right. \\
 &\quad \left. - \langle C_{k\uparrow} C_{i\uparrow}^+ C_{i'\downarrow} d_{i\uparrow} \rangle \ll d_{i\downarrow}^+ | C_{k'\uparrow}^+ \gg \right\} \quad (\text{A-term}) \\
 &+ \left\{ - \langle C_{i\uparrow}^+ d_{i\downarrow}^+ \rangle \ll C_{k\uparrow} C_{i'\downarrow} d_{i\uparrow} | C_{k'\uparrow}^+ \gg + \langle C_{i\uparrow}^+ d_{i\uparrow} \rangle \ll C_{k\uparrow} C_{i'\downarrow} d_{i\downarrow}^+ | C_{k'\uparrow}^+ \gg \right. \\
 &\quad + \langle C_{i'\downarrow} d_{i\downarrow}^+ \rangle \ll C_{k\uparrow} C_{i\uparrow}^+ d_{i\uparrow} | C_{k'\uparrow}^+ \gg \\
 &\quad \left. - \langle C_{i'\downarrow} d_{i\uparrow} \rangle \ll C_{k\uparrow} C_{i\uparrow}^+ d_{i\downarrow}^+ | C_{k'\uparrow}^+ \gg \right\} \quad (\text{A'-term}), \quad (6.1)
 \end{aligned}$$

From now on, all Green functions are assumed to be those in the time representation. Then, we obtain

$$\ll A | B \gg \Leftrightarrow \mp i \langle [A, B] \rangle$$

at the limit

$$t \rightarrow \pm 0,$$

where the upper sign corresponds to the retarded Green function and the lower to the advanced one, respectively.

Using above procedure for Eq. (6.1), the following is obtained:

$$\begin{aligned}
 \langle C_{i\uparrow}^+ C_{i'\downarrow} S_- \rangle &= \langle C_{i\uparrow}^+ C_{i'\downarrow} d_{i\downarrow}^+ d_{i\uparrow} \rangle \\
 &= \langle C_{i\uparrow}^+ C_{i'\downarrow} S_- \rangle + \left\{ - \langle C_{i\uparrow}^+ d_{i\downarrow}^+ \rangle \langle C_{i'\downarrow} d_{i\uparrow} \rangle \right. \\
 &\quad + \langle C_{i\uparrow}^+ d_{i\uparrow} \rangle \langle C_{i'\downarrow} d_{i\downarrow}^+ \rangle + \langle C_{i'\downarrow} d_{i\downarrow}^+ \rangle \langle C_{i\uparrow}^+ d_{i\uparrow} \rangle \\
 &\quad \left. - \langle C_{i'\downarrow} d_{i\uparrow} \rangle \langle C_{i\uparrow}^+ d_{i\downarrow}^+ \rangle \right\},
 \end{aligned}$$

$$\text{or } \langle C_{i\uparrow}^+ d_{i\uparrow} \rangle \langle d_{i\downarrow}^+ C_{i'\downarrow} \rangle + \langle C_{i\uparrow}^+ d_{i\downarrow}^+ \rangle \langle d_{i\downarrow} C_{i'\downarrow} \rangle = 0, \quad (6.2)$$

where we have used the symmetry relations

$$\langle C_{i\downarrow} d_{i\uparrow} \rangle = - \langle C_{i\uparrow} d_{i\downarrow} \rangle$$

and

$$\langle C_{i\downarrow} d_{i\downarrow}^+ \rangle = \langle C_{i\uparrow} d_{i\uparrow}^+ \rangle.$$

These symmetry relations have also been used to formulate the simultaneous equations (3.3) through (3.6). From Eq. (6.2), we can get

$$\langle C_{i\uparrow}^+ d_{i\uparrow} \rangle = \langle C_{i\uparrow}^+ d_{i\downarrow}^+ \rangle \equiv 0, \tag{6.3}$$

and  $\Delta \equiv 0.$  (6.4)

These results, (6.3) and (6.4), should mean, to be surprised that the anomalous Green function  $\hat{X}_{k'}(z)$  does not exist in the whole temperature region ! It is very strange in the realm of the A-approach. How can we understand this curious property ?

If attention is paid to the fact that Eqs. (6.3) and (6.4) arise from the existence of the A'-term in Eq. (6.1), can it not be considered that, from the outset, only the N-term and the A-term are contained but not the A'-term ? If so, the queer Eqs. (6.3) and (6.4) do not appear.

Therefore, according to the discussion given above, we put as follows:

$$\begin{aligned} \ll C_{k\uparrow} C_{i\uparrow}^+ C_{i'\downarrow} S_- | C_{k'\uparrow}^+ \gg &= \ll C_{k\uparrow} C_{i\uparrow}^+ C_{i'\downarrow} d_{i\downarrow}^+ d_{i\uparrow} | C_{k'\uparrow}^+ \gg \\ &\simeq \left\{ \langle C_{k\uparrow} C_{i\uparrow}^+ \rangle \ll C_{i'\downarrow} S_- | C_{k'\uparrow}^+ \gg + \langle C_{i\uparrow}^+ C_{i'\downarrow} S_- \rangle \ll C_{k\uparrow} | C_{k'\uparrow}^+ \gg \right. \\ &\quad \left. - \langle C_{k\uparrow} C_{i'\downarrow} \rangle \ll C_{i\uparrow}^+ S_- | C_{k'\uparrow}^+ \gg \right\} \text{ (N-term)} \\ &+ \left\{ \langle C_{k\uparrow} C_{i\uparrow}^+ C_{i'\downarrow} d_{i\downarrow}^+ \rangle \ll d_{i\uparrow} | C_{k'\uparrow}^+ \gg \right. \\ &\quad \left. - \langle C_{k\uparrow} C_{i\uparrow}^+ C_{i'\downarrow} d_{i\uparrow} \rangle \ll d_{i\downarrow}^+ | C_{k'\uparrow}^+ \gg \right\} \text{ (A-term),} \end{aligned} \tag{6.5}$$

instead of Eq. (6.1). Furthermore, being interested in the behavior of the anomalous parameter dependence on temperature, we have dropped the N-term from Eq. (6.5) to pursue anomalous properties with the emphasis on the roles of the anomalous Green functions: that is,

$$\begin{aligned} \ll C_{k\uparrow} C_{i\uparrow}^+ C_{i'\downarrow} S_- | C_{k'\uparrow}^+ \gg &= \ll C_{k\uparrow} C_{i\uparrow}^+ C_{i'\downarrow} d_{i\downarrow}^+ d_{i\uparrow} | C_{k'\uparrow}^+ \gg \\ &\simeq \langle C_{k\uparrow} C_{i\uparrow}^+ C_{i'\downarrow} d_{i\downarrow}^+ \rangle \ll d_{i\uparrow} | C_{k'\uparrow}^+ \gg - \langle C_{k\uparrow} C_{i\uparrow}^+ C_{i'\downarrow} d_{i\uparrow} \rangle \ll d_{i\downarrow}^+ | C_{k'\uparrow}^+ \gg, \end{aligned} \tag{6.6}$$

which coincides with Eq. (3.10) in section 3.

As mentioned in the first paragraph of current section, there is much difference in that the anomalous parameter  $\Delta$  in our A-approach contains the factor  $(\ln D/T)^{-1}$  but not in the original one. The appearance of this factor  $(\ln D/T)^{-1}$  or the product  $(\ln D/T) \Delta$  is due to the thermal average such as  $\langle C_{k\uparrow}^+ C_{i\uparrow} C_{i'\downarrow} d_{i\uparrow} \rangle$ . The following symbolical illustration may help make clear what is being said here:

$$\langle \{ C^+ C \} \{ C d \} \rangle \Rightarrow (\ln \frac{D}{T}) \Delta, \tag{6.7}$$

with  $\{ C^+ C \}$  producing  $(\ln D/T)$   
and  $\{ C d \}$  producing  $\Delta$ .

Furthermore, if a higher order correction is taken into consideration, the thermal average such as

$$\langle C_{k_1\uparrow}^+ C_{k_1'\uparrow} C_{k_2\uparrow}^+ C_{k_2'\uparrow} \dots C_{k_m\uparrow}^+ C_{k_m'\uparrow} C_{i\downarrow} d_{i\uparrow} \rangle \tag{6.8}$$

will appear as "A-term". Expressing symbolically the roles of this average, (6.8), corresponding to (6.7), we would have as follows:

$$\langle (C^+C)^m (Cd) \rangle \Rightarrow (\ln \frac{D}{T})^m \Delta. \quad (6.9)$$

This might give, from the self-consistent equation, the following:

$$\Delta \propto \frac{1}{(\ln \frac{D}{T})^m} \quad (6.10)$$

and then

$$\Delta \rightarrow 0 \quad \text{as } m \rightarrow \infty. \quad (6.11)$$

Although our discussions on the higher order correction might be over-simplified, nevertheless (6.11) might be reasonable in accordance with physical insight.

As for  $T_c$ , nothing much can be said, but we conject that  $T_c \rightarrow 0$  as  $m \rightarrow \infty$  like  $\Delta$  in (6.11).

May we consider then that we have to develop further the theory in line with the N-approach in order to reach the essence of the s-d exchange model ?

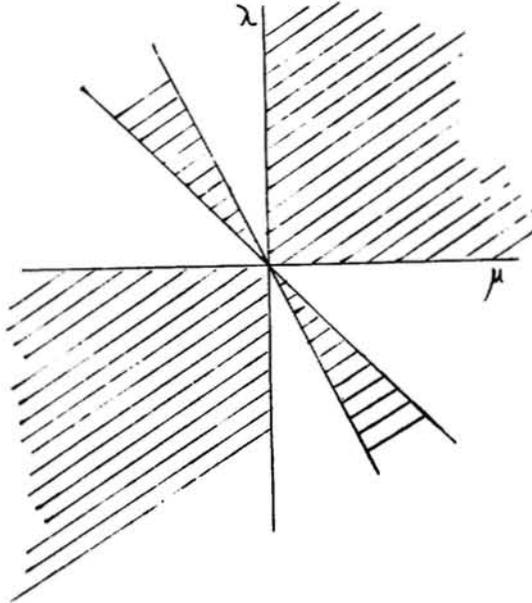


Fig. IV. Condition for the causality of the retarded or the advanced Green function. The shaded region is fit to the condition, but the un-shaded region not.

Finally, we would like to point out interesting properties of Green function relating to poles. The pole is given by  $A_1(\omega) = 0$  in Eq. (4.35); that is,

$$\omega \equiv z_1 = -\frac{b_1}{a_1} \Delta \approx \pi \rho \frac{\mu(2\lambda + \mu)}{\lambda + \mu} \Delta - i \frac{\pi \rho}{2} \frac{\lambda \mu(2\lambda + \mu)}{\lambda + \mu} \Delta. \quad (6.12)$$

In order for the causality of the retarded Green function to hold, the following relation must be satisfied:

$$\lambda \mu (\lambda + \mu) (2\lambda + \mu) > 0. \quad (6.13)$$

The (6.13) is not, however, satisfied for arbitrary values of  $\lambda$  and  $\mu$ . These circumstances are shown on the  $\lambda$ - $\mu$  plane in the Fig. IV. The shaded region in the Fig. IV fits the causality. Now, where will the unshaded region unfit to the condition come from? Will it be due to the choice of the model or to the decoupling scheme?

Incidentally, the abnormal behaviors regarding resistivity in approximately the same region were reported in our previous paper<sup>16)</sup>.

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