The Low-Temperature Specific Heat of' the Charged-Boson and –Fermion System at the High-Density Limit

The Low-Temperature Specific Heat of the Charged-Boson and -Fermion System at the High-Density Limit

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Abstract

The contribution of Coulomb interaction between boson and fermion to the low-temperature specific heat of this system which is closely related to the individual excitation in the system is calculated exactly in the density parameter expansion at the high-density limit.

The specific heat of the system at the constant volume in the unit of the Sommerfeld value is given by

$$C_v/C_s = [1+(G-M)+F(\gamma,\Delta)+o(\gamma^2)]^{-1}$$

where (G−M) is the Gell-Mann's term, the function $F(\gamma,\Delta)$ is the contribution of the interaction between components and has the limiting behavior as follows:

$F(\gamma,\Delta) \to \frac{\gamma}{18}[\log\gamma\Delta + \frac{\pi(\Delta-1)}{2}\cdot(\frac{\gamma}{\Delta})^{1/2} + o(\gamma)]$ as γ approches zero for fixed Δ, where the density parameter $\gamma = 4m_1 e_1^2/3^{1/3}\pi^{5/3}n_1^{1/3}$, the parameter Δ is independent of the density, and m_1 and e_1 are mass and charge of fermion, respectively. At this limit, $F(\gamma,\Delta)$ is the same order as (G-M) in the density parameter.

The sharpness of the Fermi surface is also justified at this limit.

1. Introduction

In general, in the Coulomb many body system, the difficulty of the well-known divergency appears in the low momentum transfer region in the calculation of the energy, for example, by the conventional perturbation method. This divergency is caused by the long range interaction of the Coulomb field and also by the statistical nature in the case of the boson system. One of the interests attracted to many body theoretical physicists by electron gas and charged boson gas lies right on this point. Especially, at the high-density limit many studies have been carried out on the correlation energy of these system[1), 2)].

In relation to the problem of divergency described above, it is very interesting theoretically and pedagogically to investigate the problem : what effect the interaction between components has for the properties of the system. Bassichis[3)] studied the neutral system consisting of two species of bosons. As the elementary excitation in this system he obtained the charge density wave and mass density wave by diagonalizing the Hamiltonian in the Bogoliubov approximation. These excitations are quite different, especially in the low momentum region, from the bogolons belonging to each component in the case when the interaction between components does not exist. Investigating the

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two-component system of the charged boson and fermion, Ginoza and Kanazawa[4] calculated the correlation energy and the collective excitations in this system exactly at the high-density limit (their paper will be referred to as paper II hereafter), but they did not give explicit calculation of individual excitation.

The object of this paper is to calculate the contribution of the interaction between components to the low-temperature specific heat. The specific heat is the quantity that is closely related to the individual excitation in the system and that can be calculated easily in the form of density parameter expansion.

Incidentally, the coefficient of the first order term in temperature of the specific heat of the high density electron gas has been computed by Gell-Mann[5], Quinn and Ferrel[6], and Dubois[7] in terms of the density parameter expansion. According to their calculation, the ratio of the specific heat at the constant volume to Sommerfeld value is as follows:

$$C_v/C_s = [1-(\gamma/4)\left\{1+\tfrac{1}{2}\log(\gamma/4)\right\} - (\gamma/4)^2\left\{0.251(\log(\gamma/4))^2 + 1.49\log(\gamma/4)+4.10+O(\gamma)\right\}]^{-1},$$

where γ is the density parameter which will be defined later. The terms of the first and the second order of γ were obtained by Gell-Mann and Dubois, respectively.

In our work, we will first replace the uniformly distributed and fixed background charge in this idealized one-component system by the charged boson gas of equal charge, and then take account of the dynamics of this boson gas. The collective excitation of the charged-boson and -fermion system at the high density limit are, according to paper II, the optical and acoustic phonons. Therefore, as the main contribution of the collective excitations to the specific heat is the order of T^3 resulting from the acoustic phonon, the main contribution comes from the individual excitation. Now, our concern is to see, by performing exact calculation, what part in the expansion above will be affected by the interaction between components.

For the case when the properties of the system can be calculated by expanding in terms of the strength of interaction, Luttinger[8] proved that the sharp Fermi surface exists in the many body fermion system at absolute zero temperature since the imaginary part of the proper self-energy in many body fermion system is proportional to the square of the energy measured from chemical potential and therefore that the coefficient of the term of the first order in temperature in the specific heat at constant volume can be calculated from the self-energy at the absolute zero temperature.

In the sufficiently high density region we will assume that our problem is in similar situation with that of Luttinger.

In section 2, the contribution of the interaction between components to the specific heat and the imaginary part of the fermion proper self energy will be calculate din terms of density parameter expansion.

In section 3, we will discuss the criterion for the Bogoliubov prescription, give some conclusion obtained by our exact calculation, and discuss the points suggestive of (i) the breakdown of the sharpness of the Fermi surface and (ii) the concept of the background with a fixed and uniformly distributed charge which is introduced into an idealized one-component charged particle system.

2. Calculation

We consider the neutral system consisting of charged-bosons and -fermions whose mass, charge, spin, and number density are m_1, e_1, $\frac{1}{2}$ and n_1, and m_2, e_2, O, and n_2, respectively, with the first group referring to the charged-fermion. and the second group to charged-boson This system is specified by three independent parameter m_1/m_2, e_1/e_2, and $n_1=|e_2/e_1|n_2$. In this section, we will calculate the low-temperature specific heat of the system in the density parameter expansion for the fixed m_1/m_2 and e_1/e_2 at the high-density limit. The density parameter γ is defined as

$\gamma=(4m_1\ e_1^2/3^{1/3}\pi^{5/3}\ n_1^{1/3})$.

Let us define the fermion Green's function G_1 as follows:

$$iG_1(\bar{k}\sigma,\ t-t')=\langle\psi T\ \alpha_{k\sigma}(t)\alpha^{+}_{k\sigma}(t')\psi\rangle\ , \tag{2.1}$$

where ψ is the exact ground state of the system and $\alpha_{k\sigma}(t)$ and $\alpha^{+}_{k\sigma}(t)$ are the Heisenberg representations of destruction operator $\alpha_{k\sigma}$ and creation operator $\alpha^{+}_{k\sigma}$ of a fermion in the state specified with momentum $\bar{k}$ and spin σ, respectively. In this system, we can not apply directly the technique of the quantum field theory to the perturbation calculation of the function (2.1) because of the existence of the boson condensate in the noninteracting system. This difficulty can be avoided by the method of Hugenholtz-Pines. The discussion of this point is given in paper II in detail. We regard the definition (2.1) as the one obtained after such procedure. In addition to the definition of (2.1), the readers can refer to paper II about the Feynman diagram expansion in the perturbation calculation of (2.1), the rules for diagram, and the definitions of the other qantities related to the following discussion.

Now, let us denote the proper fermion self-energy as $\Sigma_1(\bar{k}\sigma,\epsilon)$. The Fourier transform of (2.1) is given by the following equation.

$$G_1(\bar{k}\sigma,\ \epsilon)=[\epsilon-\epsilon_1(k)+\mu_1-\Sigma_1(\bar{k}\sigma,\ \epsilon)]^{-1},$$

where $\epsilon_1(k)$ and μ_1 are the kinetic energy of fermion with momentum $\bar{k}$ and chemical potential, respectively. The rules given in paper II are for $iG_1(\bar{k}\sigma,\ \epsilon)$ and $-i\Sigma_1(\bar{k}\sigma,\ \epsilon)$. According to the work of Luttinger[8], the low-temperature specific heat at constant volume can be calculated by the proper fermion self-energy from the following equation:

$$\frac{C_v}{C_s}=\left[1+\frac{m_1}{k_f}\frac{\partial}{\partial k}\Sigma_1^R(\bar{k}\sigma,\ \epsilon(k))\Big|_{k=k_f}\right]^{-1}, \tag{2.2}$$

where $\epsilon(k)$ is the solution of

$$\epsilon-\epsilon_1(k)+\mu_1-\Sigma_1^R(\bar{k}\sigma,\ \epsilon)=0\ , \tag{2.3}$$

Σ_1^R is the real part of Σ_1, and $k_f=(3\pi^2 n_1)^{1/3}$. Therefore, the perturbation calculation of the specific heat is reduced to that of the proper fermion self-energy.

Σ_1 can be expanded by the effective interaction(as for the definition, see Fig. 4 in paper II) as follows:

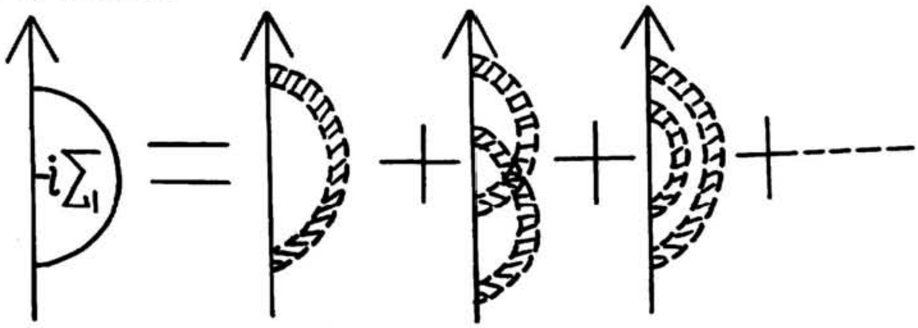
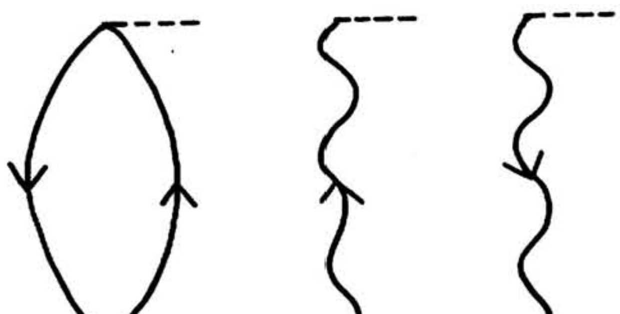


Fig. 1. The expansion of the proper fermion self-energy by the effective interaction.

In the electron gas, the diagram corresponding to the first term in Fig. 1 yields the Gell-Mann′s term in the specific heat and the type of diagrams such as the second and third terms in Fig. 1 produce the DuBois′. In our problem, the most important effect of the interaction between components, if it exists, should be contained in the first term in Fig. 1. Hence, using the rules given in paper II, we obtain

$$\Sigma_1(\vec{k}\sigma,\ \epsilon)=\sum_{\vec{p}}\int\frac{id\omega}{2\pi}G_1^{(0)}(\vec{k}+\vec{p}\sigma,\ \omega+\epsilon)\,v(p)e_1^2\,/\,K(\vec{p},\omega), \tag{2.4}$$

where $K(\vec{p},\omega)$ is the dynamical dielectric constant of this system and is given by (2.18) in paper II in terms of the proper polarization $\bar{D}(\vec{p},\omega)$. $\bar{D}(\vec{p},\omega)$ is also expanded by the effective interaction and the zero-order diagram is given by Fig. 2.



(a) (b) (c)

Fig. 2. The zero-order diagrams contributing to $i\bar{D}(\vec{p},\omega)$

Taking the contribution of only Fig. 2(a) to $i\bar{D}$ and calculating the specific heat by using (2.4), we can get the Gell-Mann′s term. The polarization given by Fig. 2 (b) and (c) represent the dynamics of the boson background in the fermion gas. Our interest lies right on the contribution of these diagrams.

From (2.4),

$$\frac{m_1}{k_f}\frac{\partial}{\partial k}\Sigma_1^R(\vec{k}\sigma,\ \epsilon(k))\Big|_{k=k_f}=R_e\sum_{\vec{p}}\int\frac{id\omega}{2\pi}\frac{m_1}{k_f}\frac{\partial}{\partial k}G_1^{(0)}(\vec{k}+\vec{p}\sigma,\omega+\epsilon(k))\Big|_{k=k_f}\frac{v(p)e_1^2}{K(\vec{p}\omega)}. \tag{2.5}$$

Calculating the solution of (2.3) by iteration consistently and substituting it into (2.5), we get (see Appendix)

$$(2.5)=[\,\mathrm{I}\,]+[\,\mathrm{II}\,],$$

where

$$[\,\mathrm{I}\,]=R_e\sum_{p}\int d\omega\ \frac{v(p)e_1^2}{K(p\,\omega)}\ \frac{m_1}{k_f}\ \delta(\omega)\,\delta(|\,k+p\,|-k_f)\ \frac{\partial}{\partial k}\,|\,k+p\,|\Big|_{k=k_f}$$
$$=-\frac{\gamma}{4}\left\{1+\frac{1}{2}\log\frac{\gamma}{4}\right\}+F(\gamma,\Delta)+O(\gamma^2), \tag{2.6a}$$

$$[\,\mathrm{II}\,]=Re\sum_{p}\int\frac{id\omega}{2\pi}\frac{v(p)e_1^2}{K(p,\omega)}\frac{m_1}{k_f}\,G_1^{(0)^2}(k+p\,\sigma,\ \omega+\epsilon(k)-\mu_1^{(0)})\ \frac{\partial}{\partial k}$$
$$[\epsilon_1(k+p)-\epsilon_1(k)]\Big|_{k=k_f}\sim O(\gamma^2\log\gamma), \tag{2.6b}$$

$$F(\gamma,\Delta)=\frac{\gamma}{16}[\log\gamma\,\Delta+\gamma\frac{\Delta-1}{\Delta}\int_0^4 dZ\frac{1}{Z^2+\gamma Z+\gamma/\Delta} \tag{2.7}$$

$$=\frac{\gamma}{16}[\log\gamma\,\Delta+\gamma\frac{\Delta-1}{\Delta}\times\begin{cases}\dfrac{2}{\sqrt{\gamma(4/\Delta-\gamma)}}\tan^{-1}\dfrac{2Z}{\sqrt{\gamma(4/\Delta-\gamma)}}\Bigg|_{\frac{\gamma}{2}}^{4+\frac{\gamma}{2}}, & \dfrac{4}{\Delta}>\gamma\\[2ex] \dfrac{1}{\sqrt{\gamma(\gamma-4/\Delta)}}\log\left|\dfrac{Z-2^{-1}\sqrt{\gamma(\gamma-4/\Delta)}}{Z+2^{-1}\sqrt{\gamma(\gamma-4/\Delta)}}\right|_{\frac{\gamma}{2}}^{4+\frac{\gamma}{2}}, & \dfrac{4}{\Delta}<\gamma\\[2ex] \dfrac{16}{\gamma(8+\gamma)}, & \dfrac{4}{\Delta}=\gamma\end{cases}$$

$$\Delta=3m_1e_1/4m_2e_2) \tag{2.8}$$

[I] is the contribution from the transition mechanism on the Fermi surface as is obvious from its definition. The other mechanism contributes to [II]. From (2.2), (2.6a), and (2.6b) we get

$$\frac{C_v}{C_s}=\left[\,1-\frac{\gamma}{4}\left\{1+\frac{1}{2}\log\frac{\gamma}{4}\right\}+F(\gamma,\Delta)+O(\gamma^2)\,\right]^{-1}, \tag{2.9}$$

In this result, the second and third terms in the square braket are the Gell-Mann's term and the contribution of the interaction between components which will be discussed in the following section in detail, respectively.

We can calculate the imaginary part of the proper fermion self-energy from (2.4) and obtain (see Appendixb)

$$\mathrm{Im}\,\Sigma(k\sigma,\epsilon)=-\frac{k_f\,\epsilon_f}{k}\gamma\left[\frac{\gamma}{32}\left(\frac{\epsilon}{\epsilon_f}\right)^2\int_0^\infty dx\,\frac{x^4}{(x^4+\gamma x^2+\gamma/\Delta)^2}\left[\theta\left(\frac{\epsilon}{\epsilon_f}\right)\right.\right.$$

$$- \theta \left(\frac{\epsilon}{\epsilon_f}\right) \Big] + \frac{\pi \gamma^2}{6} \left(\frac{9\eta}{4\Delta}\right)^{1/2} \int_0^\infty dx \frac{x^5}{(x^4 + \gamma x^2 + \gamma/\Delta)^2}$$

$$\left\{ \theta \left(\sqrt{-\left(\frac{1}{4\eta\Delta}\right)^{1/2} \frac{\epsilon}{\epsilon_f}} - x \right) \theta \left(-\frac{\epsilon}{\epsilon_f} \right) - \theta \left(\sqrt{\left(\frac{1}{4\eta\Delta}\right)^{1/2} \frac{\epsilon}{\epsilon_f}} - x \right) \right.$$

$$\left. \theta \left(\frac{\epsilon}{\epsilon_f}\right) \right\} \Big].$$

We will also discuss this result in detail in the following section.

3. Discussions and Conclusions

In case when the properties of the system can be calculated by expanding in terms of the strength of the interaction, the low-temperature specific heat is calculated by (2.2) which is justified on the basis of the existence of the sharp Fermi surface and is given by (2.9). This result is closely related to the individual excitation in the system. Also, the imaginary part of the proper fermion self-energy is calculated and is given by (2.10). The calculations performed above are based on the Bogoliubov prescription of the boson condensate whose criterion is satisfied when there exist many bosons in the Bohr sphere whose radius is given by $(m_2 e_2^2)^{-1}$. Therefore, the parameter Δ, η, and γ must satisfy the following inequality:

$$\gamma \eta^{2/3} \Delta^{-5/3} < 1,$$

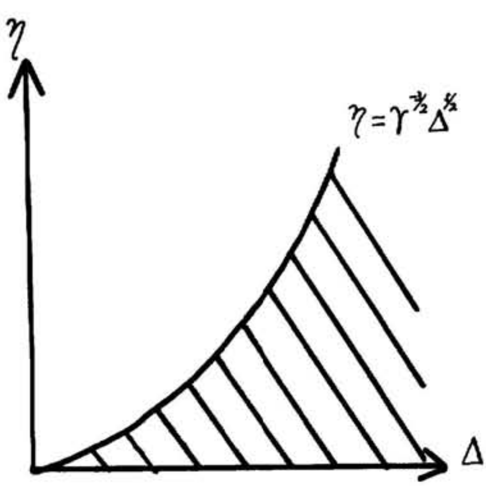


Fig. 3. Bogoliubov's criterion is satisfied in the shaded region.

where $\eta = (m_1 e_2 / 3 m_2 e_1)$. The above inequality is shown in Fig. 3. Let us fix η and Δ in the region shown in Fig. 3. At the limit of $\gamma \longrightarrow 0$, (2.9) and (2.10) become as follows:

$$\frac{C_v}{C_s} = \left[1 - \frac{\gamma}{4} \left\{ 1 + \frac{1}{2} \log \frac{\gamma}{4} \right\} + \frac{\gamma}{16} \left\{ \log \gamma \Delta + \frac{\pi(\Delta - 1)}{2} \left(\frac{\gamma}{4}\right)^{1/2} + O(\gamma) \right\} \right]^{-1}, \quad (3\ 1)$$

$$\mathrm{Im}\,\Sigma_1(k\sigma,\epsilon)=-\frac{k_f\,\epsilon_f}{k}\left\{\frac{\pi^2}{256\sqrt{2}}\Delta^{3/4}\gamma^{5/4}\left(\frac{\epsilon}{\epsilon_f}\right)^2-\frac{\pi}{192\,\eta}\left|\frac{\epsilon}{\epsilon_f}\right|^3+\cdots\right\}$$
$$\times\left[\theta\left(\frac{\epsilon}{\epsilon_f}\right)-\theta\left(-\frac{\epsilon}{\epsilon_f}\right)\right]. \tag{3.2}$$

At this limit, the imaginary part of the proper fermion self-energy is proportional to $(\epsilon/\epsilon_f)^2$ and, according to Luttinger's discussion, this means the existence of the sharp Fermi surface in this system at the absolute zero temperature. Therefore, it is justified that the formula (2.2) can yield the specific heat at this limit exactly. We conclude that the low-temperature specific heat is given exactly by (3.1). This exact result, which is the same order as the Gell-Mann's term in the density parameter, shows that the effect of the interaction between components to the specific heat and hence to the individual excitation in the system can not always be neglected. The imaginary part of the proper fermion self-energy in case when we neglect the interaction between components is[6), 7)]

$$-\frac{k_f\,\epsilon_f}{k}\left[\frac{\pi\gamma^{1/2}}{96}\left(\frac{\epsilon}{\epsilon_f}\right)^2+\cdots\right]\left[\theta\left(\frac{\epsilon}{\epsilon_f}\right)-\theta\left(-\frac{\epsilon}{\epsilon_f}\right)\right]. \tag{3.3}$$

The comparison of (3.2) with (3.3) indicates that the sharpness of the Fermi surface tends to be enhanced by the interaction between components as the density becomes higher and higher.

Let us consider the case for $\Delta\to\infty$ with γ fixed in the high density region. Then (2.9) and (2.10) become as follows:

$$\frac{C_v}{C_0}=\left[1-\frac{\gamma}{4}\left\{1+\frac{1}{2}\log\frac{\gamma}{4}\right\}+O(\gamma^2)\right]. \tag{3.4}$$

$$\mathrm{I_m}\,\Sigma_1(k\sigma,\epsilon)=-\frac{k_f\,\epsilon_f}{k}\left[\frac{\pi}{16\Delta}\frac{k_f}{k}\left|\frac{\epsilon}{\epsilon_f}\right|+\frac{\pi\gamma^{1/2}}{96}\left(\frac{\epsilon}{\epsilon_f}\right)^2+\cdots\cdots\right]$$
$$\times\left[\theta\left(\frac{\epsilon}{\epsilon_f}\right)-\theta\left(-\frac{\epsilon}{\epsilon_f}\right)\right]. \tag{3.5}$$

In(3.5), the first term is proportional to (ϵ/ϵ_f), but this term vanishes at $\Delta\to\infty$. Therefore, the results (3.4) and (3.5) at this limit accords with the ones in the case when the interaction between components is neglected. Incidentally, if

$$(m_1/m_2)-0\ \textit{and}\ (e_1/e_2)\to\infty \tag{3.6}$$

at the stage of our Hamiltonian we can obtain the Hamiltonian which describes the idealized charged fermion ststem with the neutralizing, fixed and uniformly distributed background charge. However, (3.6) formally becomes, in our parameters, as follows:

$$\eta\to 0\ \textit{and}\ \Delta\to 0\times\infty. \tag{3.7}$$

For the indefiniteness of the latter, our result obtained above seems to imply that $\Delta\to\infty$. The parameter Δ has the physical meaning which is proportional to the fourth power of the ratio of the static screening length in the charged-boson gas to the one in the charged-fermion gas, while η is proportional to the square of the ratio of the classical plasma frequency of the charged-boson gas to the one of the charged-fermion gas.

Finally, in (3.5), the condition which the first term is much smaller than the second is as follows:

$$\frac{\epsilon}{\epsilon_f} \gg \frac{6}{\Delta\,\gamma^{1/2}} \to 0, as\ \Delta \to \infty for\ fixed\ \gamma. \tag{3.9}$$

The condition (3.9) seems to suggest that the sharpness of the Fermi surface is guaranteed only at $\Delta \to \infty$ and breaks down as Δ becomes finite.

Appendix A. Calculations of (2.6a) and (2.6b)

From Fig. 2, the dynamical dielectric constant is obtained by the rules given in paper II as follows:

$$K(\vec{k},\ \omega) = 1 + \frac{k_S^2}{p^2} Q_0\ (\vec{p}, \omega), \tag{A.1}$$

$$\text{where}\quad Q_0\ (\vec{p}, \omega) = -\frac{4\pi}{k_S^2}\left[D_{11}^{(0)}(\vec{p}, \omega) + D_{22}^{(0)}(\vec{p}, \omega) \right], \tag{A.2}$$

$$D_{11}^{(0)}(\vec{p}, \omega) = e_1^2 \sum_{k\sigma} n_{k\sigma}(1 - n_{k+p\sigma}) \left[\frac{1}{\omega - [\epsilon_1(\vec{k}+\vec{p}) - \epsilon_1(k)] + i\delta} - \frac{1}{\omega + [-\epsilon_1(k) + \epsilon_1(\vec{k}+\vec{p})] - i\delta} \right], \tag{A.3}$$

$$D_{22}^{(0)}(\vec{p}, \omega) = n_2\ e_2^2 \left[\frac{1}{\omega - \epsilon_2(p) + i\delta} - \frac{1}{\omega + \epsilon_2(p) - i\delta} \right], \tag{A.4}$$

$$k_S = [4 m_1 e_1^2\ k_f / \pi]^{1/2}$$

$$k_f = (3\pi^2 n_1)^{1/3}$$

$\epsilon_2(\vec{p})$ is the kinetic energy of a boson with momentum $\vec{p}$, and $n_{k\sigma}$ is the occupation number in the state specified with momentum $\vec{k}$ and spin σ, respectively.

From (A.2), (A.3), and (A.4) we obtain, by expanding in terms of p/k_f,

$$Q_0\ (x, 0) = \Delta^{-1} x^{-2} + 1 - \frac{1}{12}\ x^2 + \cdots\cdots\cdots \tag{A.5}$$

$$Q_0\ (x, iux) = \Delta^{-1}\left[x^2 + (2m_2 u/m_1)^2 \right]^{-1} + R(u) + R_1(u)x^2 + \cdots\cdots\cdots\cdots \tag{A.6}$$

where $x = p/k_f$, $R(u) = 1 - u\tan^{-1}(1/u)$, and $R_1(u) = -1/[2(1+u)^2]$,

The integration of [I] with respect to ω yields

$$[\mathrm{I}] = (\gamma/4)\int_0^2 dx\ \mathrm{Re}\ \frac{1}{K(x,o)}\left[\frac{1}{x} - \frac{x}{2}\right]$$

$$= (\gamma/4)\left\{ 1 + \tfrac{1}{2}\log(\gamma/4) + F(\gamma, \Delta) + o(\gamma^2) \right\}$$

where we used (A.1) and (A.5) and

$$F(\gamma, \Delta) = (\gamma/16)\left[\log(\gamma\Delta) + \gamma.\frac{\Delta - 1}{\Delta}\int_0^4 dZ \frac{1}{Z^2 + \gamma Z + \gamma/\Delta}\right].$$

Next, we will calculate (2.6b). (2.6b) is equal to

$$[\mathrm{II}] = \mathrm{Re}\sum_{p}\int \frac{id\omega}{2\pi}\left[\frac{1}{\omega + \epsilon_1(k) - \epsilon_1(\vec{k}+\vec{p}) + i\delta\,\mathrm{sgn}(|\vec{k}+\vec{p}| - k_f)}\right]^2 \times \frac{1 - K(\vec{p}, \omega)}{K(\vec{p}, \omega.)}\, v(p) e_1^2\, \frac{\vec{k}.\vec{p}}{k_f^2} \tag{A.7}$$

Instead of carrying out the integration with respect to ω along the real axis, we obtained the integration along the imaginary axis by performing analytical continuation. Hence we transform ω into a pure imaginary number iux.

$$[\mathrm{II}]=-\frac{\gamma^2}{8\pi}\,\mathrm{Re}\int_{-\infty}^{\infty}du\int_{-1}^{1}d\mu\frac{\mu}{[iu-\mu]^3}\int_0^{\infty}xdx\frac{\mathrm{R}(u)x^2+[\mathrm{R}(u)2m_2u/m_1)^2+\Delta^{-1}]}{x^4+(2m_2u/m_1)^2x^2+\gamma(\Delta^{-1}+\mathrm{R}(u))}$$

$$\simeq \gamma^2\log\gamma.$$

Appendix B Calculation of (2.10)

From(2.4)

$$\mathrm{I}_m\Sigma_1(\vec{k}\sigma,\epsilon)=\mathrm{I}_m\sum_{\vec{p}}\int\frac{id\omega}{2\pi}\frac{1}{\epsilon-\omega-\epsilon_1(\vec{k}-\vec{p})+\mu_1^{(0)}+i\delta\,sgn(\vec{k}-\vec{p}\,|-k_f)}$$
$$\times v(p)\frac{1-\mathrm{K}(\vec{p},\omega_0)}{\mathrm{K}(\vec{p},\omega_0)}$$

where we used the following identity:

$$\mathrm{I}_m\sum_{\vec{p}}\int\frac{id\omega}{2\pi}\frac{v(p)}{\epsilon-\omega-\epsilon_1(\vec{k}-\vec{p})+\mu_1^{(0)}+i\delta\,sgn(|\vec{k}-\vec{p}|-k_f)}=0$$

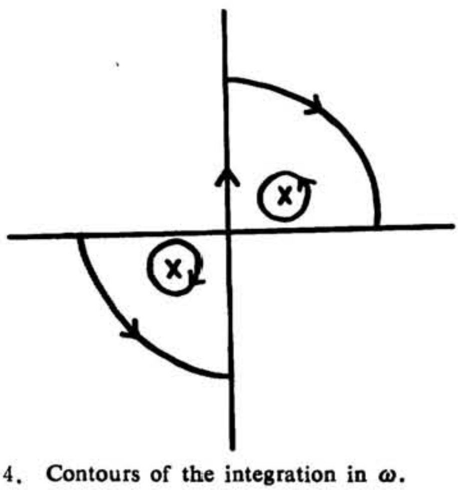


Fig. 4. Contours of the integration in ω.

Changing the path of integration from the real axis to the path shown in Fig. 4 and dropping the integration on the imaginary axis because one can show by using the property of $\mathrm{K}(\vec{p},\ \omega)$ that it is real, we obtain

$$\mathrm{I}_m\Sigma_1(\vec{k}\sigma,\epsilon)=\mathrm{I}_m\sum_{\vec{p}}v(p)\frac{1-\mathrm{K}(\vec{p},\omega_0)}{\mathrm{K}(\vec{p},\omega_0)}\Big\{\theta(\omega_0)\theta(|\vec{k}-\vec{p}|-k_f)-\theta(-\omega_0)\theta$$
$$(k_f-|\vec{k}-\vec{p}|)\Big\},\qquad \text{(B.1)}$$

where $\omega_0=\epsilon-\epsilon_1(\vec{k}-\vec{p})+\mu_1^{(0)}$, and $\theta(x)=1$ for $x>0$, and 0 for $x<0$. From (A.1), and (A.2), (A.3), and (A.4)

$$I_m\frac{1-K(p,\omega_0)}{K(p,\omega_0)}=-\frac{1}{|K(p,\omega_0)|^2}\ \frac{k_S^2}{p^2}\left[\frac{2m_1^2\ e_1^2}{k_S^2\ p}\ \omega_0+\frac{4\pi^2 n_2 e_2^2}{k_S^2}\right]$$

$$\left\{\delta(\omega_0-\epsilon_2(p))+\delta(\omega_0+\epsilon_2(p)\right\}. \qquad (B.2)$$

The substitution of (B. 2)into (B.1) yields

$$I_m\Sigma_1(k\sigma,\epsilon)=-\frac{m_1^3\ e_1^4\ \epsilon^2}{\pi k}\int_0^\infty dp\ \frac{p^4}{(p^4+p^2k_S^2+p_0^4\ )^2}\ [\theta(\epsilon)-\theta(-\epsilon)]$$

$$+\frac{4\pi n_1 e_1^3 e_2 m_2}{k}\int_0^\infty dp\frac{p^5}{(p^4+p^2k_S^2+p_0^4)^2}\left[\theta(\sqrt{-2m_2\ \epsilon}-p)\,\theta(-\epsilon)-\theta\right.$$

$$\left.(\sqrt{2m_2\ \epsilon}-p)\ \theta\ (\epsilon)\right]$$

where we approximated K (p,ω_0) in (B.2) by K$(p,0)$.

Nondimensionalizing the above equation with use of momentum in the unit of k_f and energy in the unit of the Fermi energy ϵ_f, we obtain

$$I_m\Sigma_1(k\sigma,\epsilon)=-\frac{k_f\ \epsilon_f}{k}\ \left[\frac{\gamma}{32}\ (\frac{\epsilon}{\epsilon_f})^2\int_0^\infty dx\frac{x^4}{(x^4+\gamma x^2+\gamma/\Delta)^2}\left[\theta\ (\frac{\epsilon}{\epsilon_f})\right.\right.$$

$$\left.-\theta\ (-\frac{\epsilon}{\epsilon_f})\ \right]+\frac{\pi\gamma^2}{6}\left[\frac{9\eta}{4\Delta}\right]^{1/2}\int_0^\infty dx\frac{x^5}{(x^4+\gamma x^2+\gamma/\Delta)^2}$$

$$\left.\left\{\theta\ (\sqrt{-(\frac{1}{4\eta\Delta})^{1/2}\frac{\epsilon}{\epsilon_f}}-x)\ \theta(-\frac{\epsilon}{\epsilon_f})-\theta(\sqrt{(\frac{1}{4\eta\Delta})^{1/2}\frac{\epsilon}{\epsilon_f}}-x)\theta(\frac{\epsilon}{\epsilon_f})\right\}\ \right] \qquad (B.3)$$

The results for two cases described below are shown as follows:

(i) $\gamma\cdot O$ for fixed Δ.

$$I_m\Sigma_1(k\sigma,\ \epsilon)=-\frac{k_f\ \epsilon_f}{k}\ \left\{\frac{\pi^2}{256\sqrt{2}}\Delta^{3/4}\gamma^{5/4}\ (\frac{\epsilon}{\epsilon_f})^2-\frac{\pi}{192\eta}\left|\frac{\epsilon}{\epsilon_f}\right|^3+\cdots\right\}$$

$$\left[\theta\ (\frac{\epsilon}{\epsilon_f})\ -\theta\ (-\frac{\epsilon}{\epsilon_f})\ \right].$$

(ii) $\Delta\to\infty$ for γ which is fixed with sufficient high-density region.

$$I_m\Sigma_1(k\sigma,\epsilon)\ =-\frac{k_f\ \epsilon_f}{k}\ \left\{\frac{\pi}{16\Delta}\ \frac{k_f}{k}\left|\frac{\epsilon}{\epsilon_f}\right|+\frac{\pi\gamma^{1/2}}{96}\ (\frac{\epsilon}{\epsilon_f})^2+\cdots\right\}\ \left[\theta\ (\frac{\epsilon}{\epsilon_f})\right.$$

$$\left.-\theta\ (-\frac{\epsilon}{\epsilon_f})\ \right].$$

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