Observation on the Vortex Rings

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by

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Abstract

Observation was made on the vortex rings produced by ejecting dyed water into transparent water from a circular orifice of a glass tube of 0.75 cm in diameter. The translational velocity of the ring at the eight-diameter distance (6.0 cm) was used for the calculation of the Reynolds number. First, the graphs of the translational velocity vs. distance traveled by the ring plotted on semi-log paper indicate that in the later stage of the motion there exists an exponential type of dependence of the velocity on the distance, $v^* \sim \exp(-ky^*)$, the value of k depending on the Reynolds number, with k decreasing as the Reynolds number increases. Next, from the relation between the velocity and time elapsed plotted on log-log paper, it is found that when the power law approximation, $v^* \sim t^{*-b}$, is adopted for the later part of the motion, the value of b approximately equals to 1 for each ring regardless the Reynolds number. Finally, our result suggeststs a linear relationship between the maximum distance traveled by the ring before its breakdown and the Reynolds number.

1. Introduction

The so-called vortex ring may be very familiar to cigarrette smokers and they may be already performing a sort of experiment when they puff successive rings of smoke out of their mouths. Another less familiar but easily producible phenomenon is the case when we push ink in a squirt very slowly into still water. As these examples suggest, vortex rings may be easy to reproduce in the laboratory and well fitted to the fundamental studies of fluid dynamics. Because of difficulties in exact theoretical solution and subtleness in observing and obtaining certain data in good accuracy, howevever, work from both ends, that is, from pure theoretical or numerical and pure experimental aspects have been going on yet even for these rather simple looking phenomena like vortex rings.

Our work here is concerned with production of the vortex ring in laboratory and observation of its motion. Basing on the data obtained through photographic measurements, we want to find or infer any correlationship among quantities obtained and compare the results with others'.

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2 . Apparatus

A glass vessel of 70cm in hight and 40cm × 40cm in cross-section was used as water tank which was filled with the ordinary municipal water. At the bottom of the tank was placed a mounting for a glass tube with two pipes attached, one for colored water and the other for transmitting impulse. The hypodermic syringe C_1 was used for supplying the dyed water and C_2 for transmitting inpulse. In the mounting, a rubber bellow was attched to the end of one of the pipes.

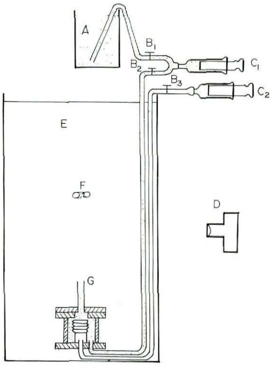


Fig. 1. Schematic diagram of the apparatus
A: dyed water. B_1, B_2, and B_3 : stop cocks.
C_1 and C_2 : hypodermic syringes. D: motor drive camera.
E: water. F: vortex ring. G: glass tube.

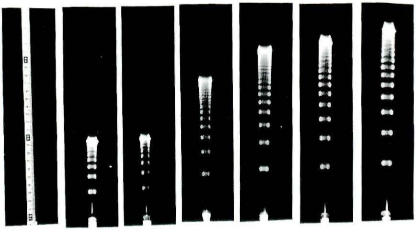
The glass walls of the tank were covered with black paper leaving space for three slits, two for lamps, and one for a camera. Two lamps of 250 watts each were used for illumination (not shown in the Figure). A motor-drive camera was electrically connected with a metronome which enabled the shuttering of the camera at desired interval of time.

3 . Experimental

Prior to the main experiment, a measuring steel tape was placed along the line, which is to be followed by the vortex rings in subsequent experiment, and photoaphed. This photgraph was used in later photographic measurement.

With the cock B_3 in Fig. 1. opened, the rubber bellow at the end of the pipe was pulled down by withdrawing the piston of the syringe C_2. This rubber bellow was kept at the withdrawn position by closing the cock B_3, and rubber bands were engaged to the end of the piston (not shown in the Figure) so the the piston could return to its original position on opening the cock B_3. Next, using the syringe C_1, the colored water (dotted part in Fig. 1.) was adjusted so that its level reached the end of the glass tube. After all part of the water came to stillness, by opening the cock B_3, the piston of C_2 was pulled back, causing a mass of dyed water to be ejected from the glass tube into still water. On leaving the tube, the ejected colored water would turn into a well-shapted toroidal form and proceeded along the axis of the glass tube with decreasing translational velocity.

As soon as the colored water left the orifice, the electric circuit of the metronome and motor-drive camera was closed to take successive pictures of the traveling vortex rings at equal interval of time. This way a series of photographs were taken for each rising ring.



SCALE R=55 R=65 R=130 R=180 R=220 R=240

Plate. Representative series of successive pictures of the rising vortex rings. The diameter of the glass tube is 0.75 cm. The time interval is 0.5 seconds.

The translational velocity of the ring could be changed by varying the impulse sent, that is, by varying the tension of the rubber bands and the withdrawn distance of the piston of C_2. These variations also changed the maximum distances the rings traveled before breaking down. A series of photographs for different initial translational velocities were shown on the plate. With the photographed measuring tape, the positions of the rising rings at equal time intervals were read out.

The way of finding the translational velocity at a specified position seems to depend on workers. For instance, Oshima[1] utilized the excess pressure of air reservoir for trasmitting impulse and showed good agreement with experiment where the mean velocity was obtained over certain range near the orifice. In our experiment, we used the simple and straight-forward way as descrided below, though we never mean it to be superior to others' methods. First the positions of the rising ring at regular interval of time (1.5 seconds) were plotted and on the same sheet the average velocities for each interval were plotted (See Fig. 2.). From this sheet of graph the velocity at an arbitrary distance from the orifice can be read out.

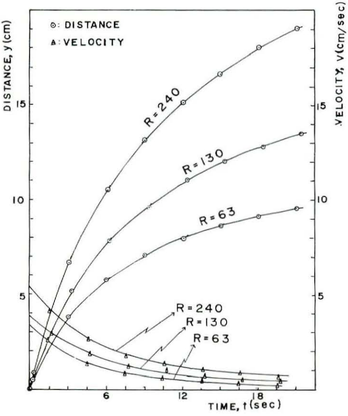


Fig. 2. Graphs of distance traveled vs. time elapsed (curves in upper group) and translational velocity vs. time (curves in lower group).

For the calculation of the Reynolds number and nondimensionalization of other quantities, we used the translational velocity, U, at the eight-diameter distance from the orifice. Because of the size of our water tank, only one kind of glass tube with 0.75 cm in diameter was used. Therefore, the value of U which could be changed by the amount of the impulse given was the main factor for changing the value of the Reynolds number. Once the value of U for each ring read out from the graphs, all the quantities were nondimensionalized as follows:

Reynolds number, $R = Ud/\nu = \rho Ud/\eta$

Distance traveled by the ring, $y^* = y/d$

Time elapsed, $t^* = tU/d$

Translational velocity, $v^* = v/U$

with U : translational velocity of the ring at the eight-diameter distance from the orifice

d : diameter of the cross-section of the glass tube

ρ, η : density and viscosity of the water

Except for the Reynolds number, all quantities above without star (*) denote physical quantities with conventional dimensions.

4. Result and discussion

The Reynolds number obtained basing on the way described in preceding section ranged from 240 to 55. With the value of U determined, all other quantities were nondimensionalized according to the expressions listed in preceding section. Incidentally, the graph of the distance traveled by the ring, y, vs. the time elapsed, t, had a slight y-intercept due to experimental difficulty in synchronizing the shuttering of camera with the moment the mass of dyed water left the orifice. In such case, the y-axis was translated to the t-intersection of the extended graph. Fig. 2. was drawn after such minor translation of the y-axis.

First, the relationship between v^* and y^* was investigatigated by plotting v^* vs. y^* on semi-log paper as shown in Fig. 3. For the straight part of the graph, that is, for later part of the motion of the ring, the result shows the existence of an exponential type of relation between v^* and y^*, that is, $v^* \sim \exp(-ky^*)$, with the value of k decreasing as the Reynlods number increases. The value of k ranged from ~ 0.64 for $R = 240$ to ~ 1.5 for $R = 55$. Though there was a slight difference in apparatus used, Maxworthy[2] reported from his experiment that the translational velocity of the ring depends exponentially on the distance traveled. He used a hole drilled in brass as an orifice instead of the end of a glass tube as was in our case. The $v^* \sim \exp(-ky^*)$ relation gives $v^* \sim t^{*-1}$ as the relation between v^* and t^*. Some graphs in Fig. 3 were made parallel translation for easier comparison.

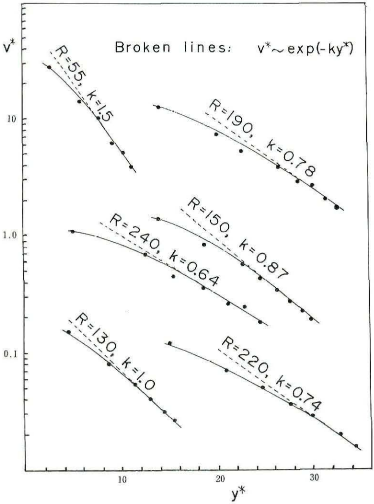


Fig. 3 Relation between the translational velocity, v^*, and the distance, y^*. Some graphs were made parallel translation to avoid proximity of points belonging to different graphs : thus, 10 should be substracted from y^* for R = 190, 150, and 220, and v^* should be multiplied by 0.1 and 10 for R = 55 and 190, and for R = 130 and 220, respectively.

Next, the relationships between v^* and t^* plotted on log-log paper are shown in Fig. 4. If they are approximated by a power law, $v^* \sim t^{*-b}$, for the later stage of the motion, the value of b for each ring regardless the value of Reynolds number is approximately equal to 1. As far as the present range of the Reynolds number in our experiment is concerned, the result, $b = 1$, is in accord with the result in preceding paragraph, $v^* \sim \exp(-ky^*)$, which results in $v^* \sim t^{*-1}$ after slight mathematical manipulation. Except for a little difference in apparatus, this result is the same with that of Maxworthy as mentioned in preceding paragraph. This could be due to the fact that the physical condition in our experiment in terms of the Reynolds number might have been closer to that of Maxworthy. Using asymptotic expansion improved by applying the method of matched asymptotic expansion, Kambe and Oshima[3] showed that the value of b is 1.5 for the final stage of low Reynolds number.

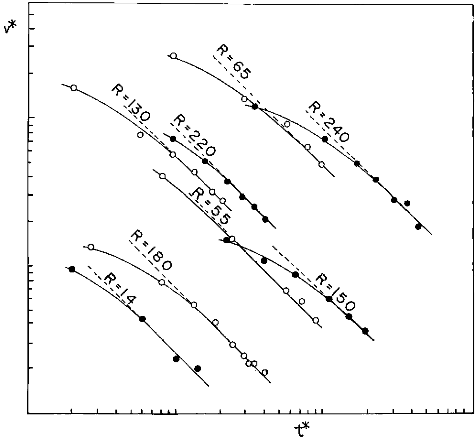


Fig. 4. Relation between translational velocity, v*, and time, t*. In order to avoid the proximity of the points belonging to different curve, the scale of the axes are multiplied by the power of 10 for the cases needed.

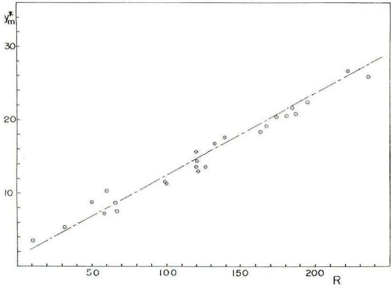


Fig. 5. Relation between the maximum distance traveled by the ring before its break-down, y_m^*, and the Reynolds number, R.

Finally, the relationship between the maximum distance traveled by the ring before its breakdown, y_m^*, and the Reynolds number, R, as shown in Fig. 5. suggests a linear relation between y_m^* and R.

Acknowledgments

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