An Eexperiment on the Steady Wake behind a Sphere

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Abstract

The characteristics of the steady wake behind a sphere were studied experimentally using dyed water for visualization. Throughout the detailed photographic observations this work infers the strong possibility of the existence of a closed recirculating eddy behind a sphere even at R = 10 or lower and shows that the eddy can preserve its steady state up to the case for R = 190 (R represents the Reynolds number based on the diameter of the sphere). In addition, new profiles for the variations of wake length and wake separation angle against the steady state Reynolds numbers are presented. The significance of the study on the sphere wake in relation to the cloud physics is also pointed out.

1. Introduction

It is a well known phenomenon that a sphere moving in a viscous fluid with a constant velocity is accompanied by a closed recirculating eddy behind it when the Reynolds number is moderate. Rather surprisingly, however, only a few detailed experimental investigations of the characteristics of the wake behind a sphere have been reported. In 1923, Nisi and Porter[1] determined the critical air velocity at which a recirculating eddy was initially created at the rear of a sphere in an air channel. The air flow was visualized by tobacco smoke and observed through a camera furnished with a microscopic objective lens. They obtained an empirical formula for the critical air velocity as a function of air density, air viscosity, the diameter of a sphere, and the diameter of an air channel. According to their formula, the critical Reynolds number for the initial appearance of a recirculating eddy can be interpreted to be about 8.2. Taneda[2] showed the photographs of the streamline of the fluid around a moving sphere in the Reynolds number ranging from 9.15 to 133. He succeeded in obtaining a toroidial eddy of the fluid behind a sphere for $R>30$ by using aluminum dust as a visualization technique. For $R<30$, however, the technique was not so effective in observing whether the attached eddy exists or not since the aluminum dust are not fine enough to drift along with the slow recirculating flow when the Reynolds number was decreased. Nevertheless, he concluded by extrapolating the observed values of the wake length and wake separation angle that the critical Reynolds number for the onset of a wake eddy

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was about 24. He also reported that a wake began to oscillate at about R=130. Magarvey and Bishop[3] made a classification of the shape of the sphere wake by observing liguid spheres of various velocities in water. Although the critical value of the Reynolds number for the wake separation was not given, the trail left by a sphere for the Reynolds number less than 210 was named "single-thread" regardless of separation. Garner and Grafton[4] and Masliyah[5] also carried out the observations on the wake separation angle. The former reported the existence of some transition of the separation angle at R=40~60, and the latter showed almost the same results as those of Taneda.[2]

Many theoretical attempts have been made to determine the flow field of a viscous fluid streaming past a sphere at low Reynolds numbers (Kawaguti,[6] Jenson,[7] Hamielec et al.,[8] Rimon and Cheng,[9] Le Clair et al.,[10] Pruppacher et al.,[11] Masliyah and Epstein,[12] Dennis and Walker,[13] Lin and Lee[14]). Most of the theoretical workers have refferred to the experimental work of Taneda[2] for the verification of the results concerning the wake characteristics (Hamielec et al.,[8] Pruppacher et al.,[11] Masliyah and Epstein,[12][15] Lin and Lee[14]). There seems to exist a tacit agreement among these threoretical authors that the critical Reynolds number for the initial wake separation at the rear of a sphere is somewhat about 20. Rimon and Cheng,[9] on the other hand, showed in their numerical work that recirculating eddy behind a sphere did exist even at R=10. They found their result to be comparable to the experimental result of Nisi and Porter.[1] Masliyah and Epstein,[15] however, raised some objections to it by mentioning that Rimon and Cheng had misinterpreted the paper by Nisi and Porter. As long as the range of observation lay in that of Taneda's work, close agreement was found between the values of wake size in experiment and those of theoretical calculations by Hamielec et al.,[8] Pruppacher et al.,[11] Van Dyke,[16] and Dennis and Walker,[13] respectively, though Rimon and Cheng[9] estimated the value of wake size for R<40 somewhat larger than Taneda's resulting in a finite dimension of a ring eddy at R=10. None of the theoretical workers, however, predicts the critical Reynolds number at which the wake behind a sphere becomes unstable; nevertheless, there are experimental reports that the wake begins to oscillate at about R=130 (Taneda[2]) and that the wake makes a transition from the single-thread wake to the double-thread one at about R=210 (Magarvey and Bishop[3]). Judging from the review just outlined, it seems that more experimental evidence for the steady wake behind a sphere is needed for further confirmation.

The importance of a study on the fluid flow around a sphere has been stressed in the field of cloud physics (Le Clair et al.,[10] Pruppacher and Beard,[17] Pitter et al.[18]). The better understanding both theoretically and experimentally of the flow features around a sphere is related to the bases for computing accurate collision efficiencies of hydrodynamically interacting cloud drops. Lin and Lee,[19] for example, caluculated the collision efficiency of water drops in the atmosphere using a method of superposition of flow fields around a sphere obtained by the numerical work which they had verified with the experimental data of Taneda.[2] In addition, the wake of a water drop in air has been concidered to play an important role in the onset of the deformation of the drop (Pruppacher and Beard[17]). Therefore, the investigation of the sphere wake will be of significance from the standpoints of not only pure hydrodynamics but also meteorology.

In this paper, the configurations of the steady wake behind a sphere falling in water with its terminal velocity have been examined. Dye is used for visualization. A careful observation of various eddy-shapes with their corresponding Reynolds numbers has been made. In addition, the variations of wake length and separation angle with Reynolds numbers have been presented. Although this work has not been able to predict accurately the critical value of the Reynolds number for the initiation of the eddy, it does suggest the strong possibility of the existence of an eddy even at R=10 or lower. The upper limit of the Reynolds number for the stable wake is also presented.

2. Apparatus and Method

Fig. 1 shows the side and top views of the apparatus. A glass water tank of 46 cm × 46 cm in cross-section and 100 cm in height was used. In order to keep the water still, careful attention was paid to the isolation of the water from the disturbances especially due to the variation of the external temperature and air movement in its surround-

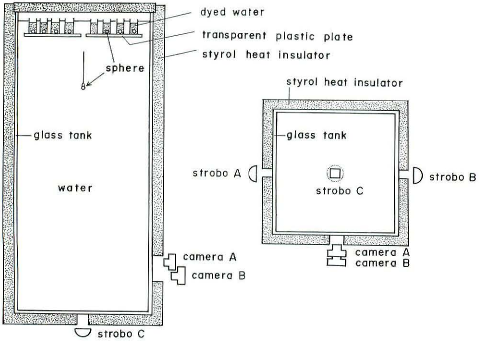

Fig. 1 Apparatus : side view (left) and top view (right).

ings. Every plastic sphere used was carefully polished since the sharp axi-symmetrical curl of eddy fluid behind a sphere depended much on the smoothness of the spherical surface. The sphere which was released from a pair of tweezers through the hole in the plate near the free surface of the water traveled 75 cm before it came into the view of the camera. This distance corresponded to that ranging from 79.36 to 60.58 in units of the diameters of the spheres which ranged from 0.945 cm to 1.238 cm. With the narrow range of the dimension of the geometrical factors, the diameters of the spheres, relatively wide range of the Reynolds number was obtained by producing large variations in the terminal velocities which in turn were accomplished simply by adjusting the mass of the contents in the plastic spherical shells. The fall velocity of the sphere was carefully examined in preliminary experiment. Four examples of the profile of the distance traveled by the falling spheres vs. the time elapsed are illustrated in Fig. 2. The fall distance was measured by the photograph of the measuring stick which had been placed in water in the preliminary experiment. The unit of the abscissa in the figure is 1.74 sec, the interval of the light flash given by strobe C. Although the distance traveled by the sphere before it reached its terminal velocity was affected by the releasing conditions, all the spheres attained their terminal velocities within the traveling

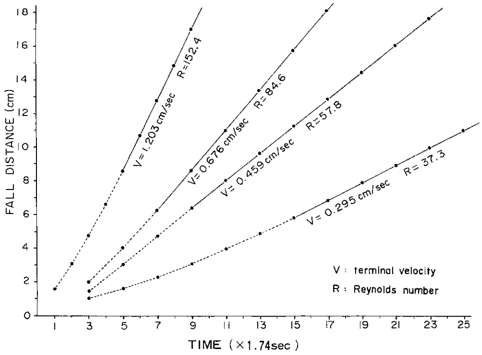

Fig. 2. Variation of fall distance with time for the spheres of different terminal velocities.

distance of 10 cm.

Having been dipped in the dyed water, a dilute mixture of water and fluorescein, the sphere was released, illuminated with periodic flashes given by strobe C, and the photograph of its successive positions was taken by camera A with its shutter left open. On closing the shutter of camera A, a full picture of the wake behind the sphere was taken by another camera B using strobes A and B for illumination.

3. Wake features

Numerous pictures of the steady wake behind a sphere for the Reynolds numbers between 7.3 and 186.7 were taken. Typical samples of them are shown in Fig. 3 and Fig. 4. The wakes were always axi-symmetrical and their trails were straight. Throughout their considerable fall distances, no change in their shapes was observed as illustrated in Fig. 5, assuring that all the wakes dealt with in this experiment were fully developed. It may be natural to say that there is a ring eddy behind a sphere as long as the sphere is accompanied by an everlasting stream of dyed water in a steady cup-shape as seen in the figures. The pictures of wake for $R>70$ in Fig. 4 are self-explanatory, showing the clear curls of fluid behind spheres. In the pictures for $R<70$ in Fig. 3, on the other hand, the inner recirculating fluid can not be distinguished. However, the existence of the eddy at the rear of the sphere for $R<70$ can be shown by the fact that the color of dyed water fades at the conic portion of fluid near each separation point, where the recirculating inner fluid is supposedly to meet the outer flow. No dyed water may be contained in the fluid which sweeps over the surface of the sphere. The inner recirculatory fluid with dyed water only makes the closed wake rigion visible. Due to the predominat stagnation of the fluid induced by the reverse flow in a wake of low Reynolds number the color of dyed water fades at the vicinity of the separation point. As the Reynolds number is decreased further, the fluid of this portion becomes almost transparent. The author considers this experimental evidence as proof of the existence of a recirculating eddy behind a sphere. As seen in the blown-up pictures in Fig. 6, the discontinuity of dye contour of this sort is cleary observed in the cases down to for $R=10.6$. The picture for $R=7.3$, however, is not so clear to distinguish such discontinuity at its separation point, though this could be accouted for by asserting that the accumulation of the dyed water behind this sphere is due to a ring eddy behind it since the sphere was kept attached by the dyed water of the shape as seen in Fig. 3(a) even after traveling as long as 68.18 sphere diameters in distance or 18 minutes in time (which might cause the contour of dye stream to become fuzzy). From the experimental facts cited above it could be suggested that the standing eddy appears at the rear of a moving sphere in a viscous fluid even at $R=10$ or lower. The present result fairly agrees with the theoretical work

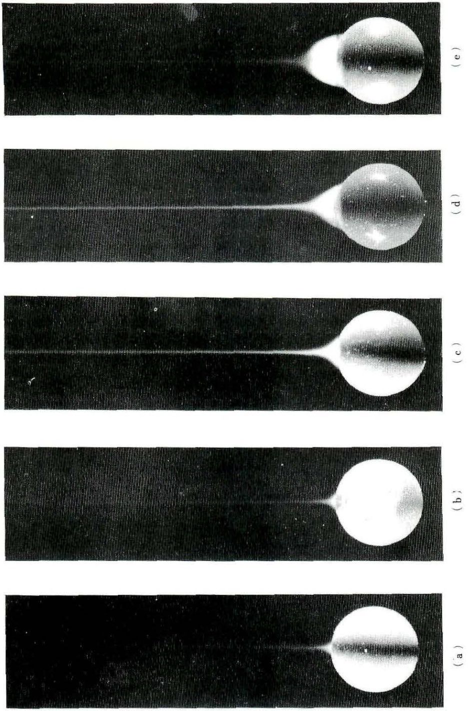

Fig. 3. Visualized wakes behind a sphere for different Reynolds numbers; (a) R=7.3, (b) R=10.6, (c) R=25.2, (d) R=33.2, (e) R=47.7.

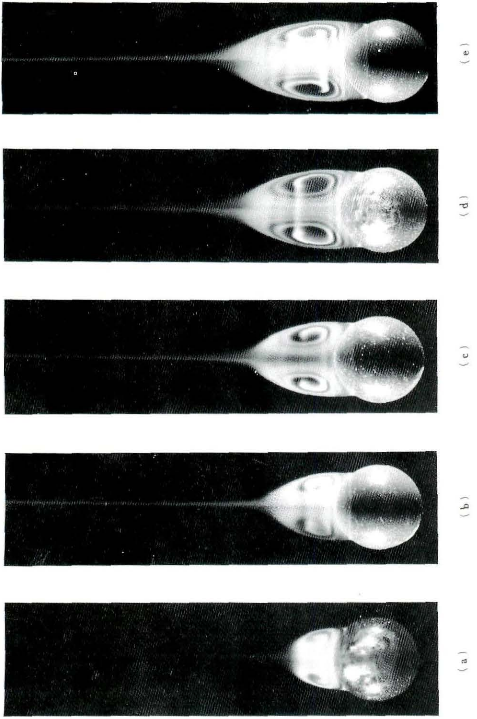

Fig. 4. Visualized wakes behind a sphere for different Reynolds numbers; (a) R=72.5, (b) R=93.7, (c) R=119.7, (d) R=151.9, (e) R=184.9.

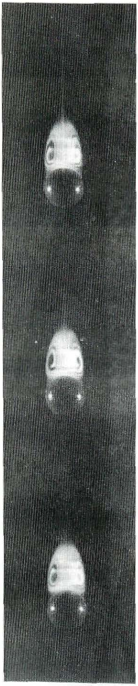

Fig. 5. Three successive pictures of a wake behind a sphere with three successive exposures.

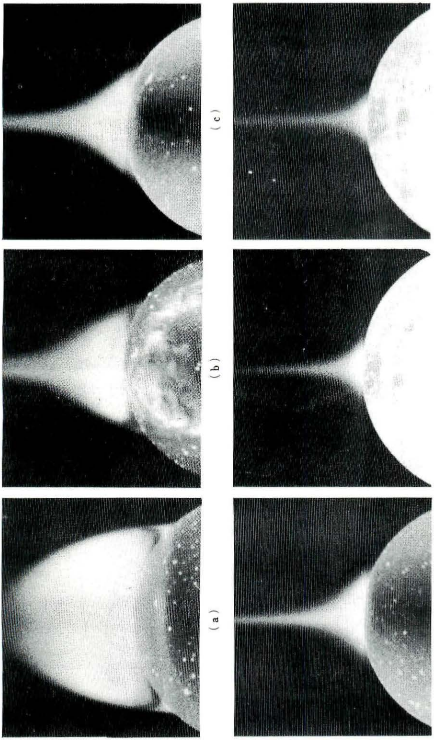

Fig. 6. Illustration of the color-fading phenomena found at the separation point of wakes for various low Reynolds numbers; (a) R=66.5, (b) R=37.3, (c) R=31.5, (d) R=22.8, (e) R=15.1, (f) R=10.6.

by Rimon and Cheng.[9]

As seen in Fig. 3 and Fig. 4, the standing eddy changes in shape according to the Reynolds number. The inclination angle, β in Fig. 7, between the line of separation and the line perpendicular to the direction of motion of the sphere varies according to the Reynolds number, and the cross-section of the envelope separating the fluid of eddy from the outer uniform fluid takes various shapes as the Reynolds number increases. When R is less than about 35, β is larger than 90 degrees and the cross-section of the envelope takes a concave shape. When R is about 35, it becomes a triangular form. As R increases from 35, it assumes a convex shape and enlarges in size. The β is still larger than 90 degrees at this stage. At the stage wher R is about 90, β becomes 90 degrees and the dividing line near the separating point is parallel to the direction of motion of the sphere. When R is increased further above 90, β becomes less than 90 degrees and decreases, and the subsequent tube of the wake begins to swell. The upper limit of the Reynolds number for the stable and axi-symmetrical ring eddy is found to be about 190, where β reaches its minimum value and the width of the central portion of the tube of wake becomes equal to the diameter of the sphere. No oscillation of the downstream part of the wake was observed until this stage, while Taneda[2] reported that oscillation occured and the wake became unsteady at about $R=130$.

When the Reynolds number was larger than about 190, the wake behind the sphere showed deformation as seen in Fig. 8. The sphere for such large Reynolds number always fell in water with its direction changing. Sometimes, the double-thread wake, which was first found by Magarvey and Bishop[3] for $R>210$, was also

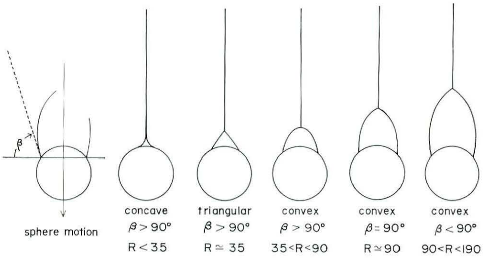

Fig. 7. Schematic diagrams of wake-shape developed behind a sphere for different range of Reynolds numbers.

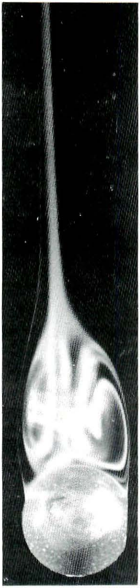

Fig. 8. An example of deformed wake (R=223.0).

found for the range of Reynolds number above 190.

The value of the wake length (S) and the separation angle (θ) were directly measured on the photographs and the value of the nondimensional wake length S/D (D is the diameter of sphere) and θ against Reynolds number were plotted in Fig.9 and Fig. 10, respectively. The representative prfiles of the two variables are expressed by the solid lines in the figures. For comparison, Taneda's[2] result are also shown by the broken lines on the same figures. As for the separation angle which was observed in the range of the Reynolds number above 35, a close agreement was found between Taneda's result (as well as Masliyah's[5]) and the present result ; while as for the wake length for the same range Taneda estimated its value a little larger than the present result except at R = 35 and R = 150, where both agreed exactly. The whole trends of the two variables found in this study agrees fairly well with the theoretical results of Rimon and Cheng[9]. Garner and Grafton[4] reported that there was a sudden change in the variation of the separation angle at R = 40～60. No significant transition of the angle was found within the range of Reynolds number in question. The wake length and the separation angle which gave the stable wake for the highest Reynolds number (R = 190) were found to be about 1.4 and 64 degrees, respectively.

4. Summary and conclusion

The following features of the steady wake behind a Sphere were demonstrated through the carefull photographic observations. The recirculating eddy seemed to be initiated at the rear of a sphere even at such low Reynolds numbers as 10 or lower. The highest Reynolds number for a stable and axi-symmetrical eddy was found to be about 190. Within the range of the Reynolds number which corresponded to such a stable state as above, the wake assumed the shapes changing from concave to convex form as the Reynolds number increased, as shown in Fig. 7. As for the variations of wake length and wake separation angle, some agreements were found between the present results and the previous ones (Taneda,[2] Masliyah[5]) as long as for the higher range of the Reynolds number was concerned, but this report is the first to reveal the profiles for the whole range of steady state Reynolds number including for the extended lower portion as seen in Fig. 9 and Fig. 10.

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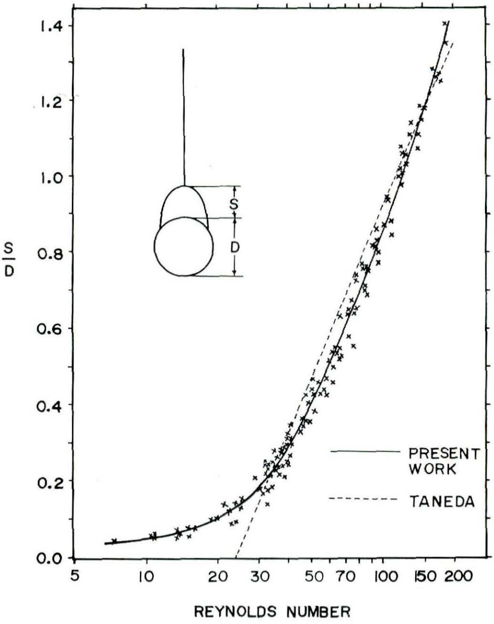

Fig. 9. Variation of the wake length with Reynolds numbers.

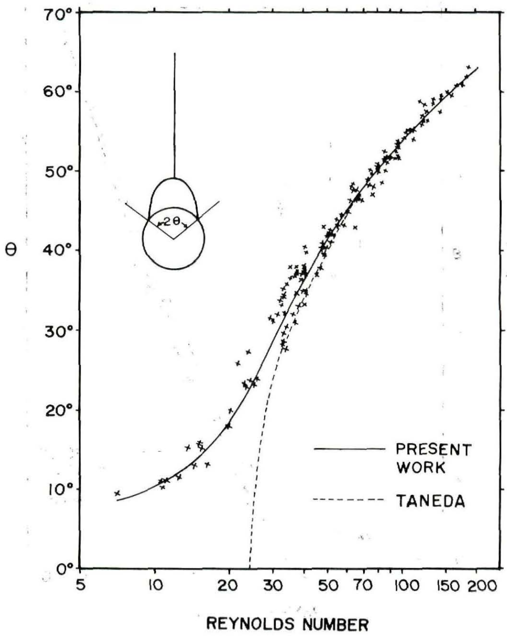

Fig. 10. Variation of the wake separation angle with Reynolds numbers.

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