Studies on Objective Weather-map Analysis

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Abstract

As one of the studies on the objective weather-map analysis, a polynomial was fitted to the 500-mb height field over North America. Wind data as well as height data were used on the geostrophic assumption.

The influence of data, the influence of the degree of the polynomial, and the influence of the weighting difference between the wind and the height, upon the analysis were all examined. The weighting of 0.7 to the 103 height observations against the 87 wind observations showed little influence on the analysis when the fifth-degree polynomial was used. As expected, both the increase of the degree of the polynomial and the consideration of some data from the surrounding of the analysis area led to a good analysis; however, the latter especially exerted a remarkable influence on the analysis.

1. Introduction

So far as objective weather-map analysis is concerned, fitting a polynomial to meteorologogical variables on a certain isobaric surface is one of the practical methods with the aid of a high-speed computer. But its success may depend on how to control the polynomial for the given analysis. Thus, it is very useful to examine some techniques of objective analysis of this kind. In this study, the contour analyais of the 500-mb height over North America is taken into consideration. Seven different types of fitting a polynomial are tested with the observed values of wind and height, and discussions on each type are made by comparing them with a subjective analysis drawn by an experienced analyst. In addition, summary and some conclusions are given.

2. Mathematical procedure

We assume that a 500-mb height field is expressed by a Nth-degree polynomial written in the form:

$$h = \sum_{i+j=o}^{i+j=N} A_{ij}\, x^i y^j \tag{1}$$

where x and y are cartesian coordinates. Then, the height at the station k is

$$h_k = \sum_{i+j=o}^{i+j=N} A_{ij}\, x_k^i\, y_k^j$$

and the geostrophic wind components at the station l are

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$$v_l = \frac{m_l g}{f_l}\frac{\partial h_l}{\partial x_l}$$
$$= \frac{m_l g}{f_l}\sum_{i+j=o}^{i+j=N} iA_{ij}\, x_l^{i-1}\, y_l^j$$
$$u_l = -\frac{m_l g}{f_l}\sum_{i+j=o}^{i+j=N} jA_{ij}\, x_l^i\, y_l^{j-1}$$

where

m_l = map scale factor

f_l = the coriolis parameter

g = the acceleration of gravity

When we have the observed values of height (h_{ok}) and wind (u_{ol}, v_{ol}), the least square method requires

$$R^2 = b\sum_{k=o}^{k=n}(R'_k)^2 + \sum_{l=o}^{l=m}(R''_l)^2 + \sum_{l=o}^{l=m}(R'''_l)^2 = \text{minimum}$$

where

$$R'_k = \sum_{i+j=o}^{i+j=N} A_{ij}\, x_k^i\, y_k^j - h_{ok}$$
$$R''_l = \frac{m_l g}{f_l}\sum_{i+j=o}^{i+j=N} iA_{ij}\, x_l^{i-1}\, y_l^j - v_{ol}$$
$$R'''_l = -\frac{m_l g}{f_l}\sum_{i+j=o}^{i+j=N} jA_{ij}\, x_l^i\, y_l^{j-1} - u_{ol}$$

b = weighting factor

n = the number of height observation

m = the number of wind observation

Since the best coefficient A_{PQ}, where $O \leqq p + Q \leqq N$, can be determined by equating

$$\frac{\partial R^2}{\partial A_{PQ}} = b\sum_{k=o}^{k=n} 2R'_k\frac{\partial R'_k}{\partial A_{PQ}} + \sum_{l=o}^{l=m} 2R''_l\frac{\partial R''_l}{\partial A_{PQ}} + \sum_{l=o}^{l=m} 2R'''_l\frac{\partial R'''_l}{\partial A_{PQ}} = O,$$

we can derive the following normal equation:

$$\left[b\sum_{k=o}^{k=n}\left(\sum_{i+j=o}^{i+j=N} A_{ij}\, x_k^{i+P}\, y_k^{j+Q}\right) + \sum_{l=o}^{l=m}\left\{\left(\frac{m_l g}{f_l}\right)^2\sum_{i+j=o}^{i+j=N} iA_{ij}\, Px_l^{i+P-2}\, y_l^{j+Q}\right\}\right.$$
$$\left.+\sum_{l=o}^{l=m}\left\{\left(\frac{m_l g}{f_l}\right)^2\sum_{i+j=o}^{i+j=N} jA_{ij}\, Qx_l^{i+P}\, y_l^{j+Q-2}\right\}\right] - \left[b\sum_{k=o}^{k=n} h_{ok}\, x_k^P\, y_k^Q\right.$$
$$\left.+\sum_{l=o}^{l=m}\left\{\frac{m_l g}{f_l}\left(v_{ol}\, Px_l^{P-1}\, y_l^Q - u_{ol}\, Qx_l^P\, y_l^{Q-1}\right)\right\}\right] = O$$

For each pair of P and Q between P + Q = O to P + Q = N, i and j vary from $i + j = O$ to $i + j = N$. Then the order of the normal equation is the number of the combinations of P and Q or that of i and j (e.g. when N = 3, the order of the normal equation is 10). Once A_{ij} is determined from the above normal equation, the 500-mb height field is analysed by equation (1) by giving various values of x and y which cover the whole area being analysed. Since the method contains tremendous calculations, the complete arithmetical process was performed by an electronic computer.

3. Types of polynomial

It is obvious that the possible factors exerting influences upon the analysis are the degree of the polynomial, the weighting factor, and the number of stations. Then, the following types of fitting the polynomial are tested using the wind and height data over North America. The analysis area and grid points for x and y coordinates are shown in Fig. 1.

Type A; The fifth-degree polynomial with the weighting factor of 1.0 was fitted to the 87 wind and the 103 height observations.

Type B; The same type as the type of A with the weighting factor of 0.7 was tested.

Type C; In addition to Type B, the 21 height data from the one-grid-length outer area as shown in Fig. 1 were introduced in order to determine the coefficient A_{ij}, and the height only inside the boundary was analysed.

Type D; The seventh-degree polynomial with the weighting factor of 0.7 was used for the data inside the boundary.

Type E; In addition to Type D, the 21 height data from the surrounding of the boundary were added again.

Type F; The analysis area was divided into the four small areas as shown in Fig. 2. The fifth-degree polynomial with weighting factor of 1.0 was used for each part, and the four independent analysis were combined in order to get the complete analysis of the whole area. The resultant heights at the boundaries of the areas were smoothed with arithmetic mean values.

Type G; The data from the one-grid-length outer area of each small area were considered when Type F determined the coefficient A_{ij}.

4. Data and subjective analysis

The 500-mb wind and height observed over North America at 1500 GMT, November 21, 1956 were used in this study. The subjective analysis of these data was drawn by an experienced meteorological analyst at the Department of Geosciences, University of Hawaii, and it (shown in Fig. 3) was used as the standard analysis with which the above seven types of objective analysis were compared.

5. Error distribution chart and root-mean-square error

By the graphical subtraction between the objective and the subjective contour lines, one can obtain an error distribution chart of the objective analysis. The resultant charts for the above seven types of analysis are shown in the latter pages. The root-mean-square error (RMS) was calculated from the height differences at all the grid points.[3]

6. Map scale factor

Throughout the study the maps of polar stereographic projections were used.

The values of m_l ranged from 0.8 to 1.4 according to the different latitudes of the stations in the analysis area.[4]

7. Discussion of the result

Fig. 4 shows an example of the resultant objective analysis drawn by IBM 7040. And for the convenience of the comparison between the objective analyses and the subjective analysis, the subsequent contour lines on the computer charts were traced on ordinary plain papers.

Type A (shown in Fig. 5) ; This resultant analysis and the subjective analysis are quite alike in contour pattern. Very pronounced distortions are found near the corners. But, with all the sufficient data, the anlysis does not show proper kinks at the ridge and the trough on the contour lines. These are shown by the $\pm$ 60-m lines on the error distribution chart. The RMS of this analysis is 125.5 m.

Type B (Fig. 6) ; No marked difference from Type A is observed. The RMS of this analysis is 122.3 m, which is very close to that of Type A. It seems that the effect of the weighting factor difference of 0.3 to the analysis is very small. It is rather negligible.

Type C (Fig. 7) ; Since the 21 height data from the one-grid-length outer area were added, the big distortions near the corners have vanished. Its RMS, which is 70.8 m, shows about 55 m improvement from Type A and Type B. But Fig. 7-b shows that the error of the analysis at the ridge and the trough became worse than those of the previous two.

Type D (Fig. 8) ; The contour pattern, especially near the ridge and the trough, has improved. But quite large distortions in the middle on the left and in the lower left hand-side corner are still remarkable. Since its RMS, which is 99.2 m, is less than those of Type A and Type B, it can be said that in general increasing the degree of the polynomial leads to a good analysis. And if we compare this analysis with Type C it can be also said that increasing some data from the surrounding of the analysis area instead of increasing the degree of the polynomial leads to a better analysis as a whole, but the former lacks accuracy in the central area while the latter does near the boundary areas.

Type E (Fig. 9) ; Although the three $\pm$ 60-m error lines exist in the central part of Fig. 9-b, Type E is a very good analysis because its RMS, which is 48.4 m, is very small. As it is seen in the comparison between Type C and Type A, here, the same phenomenon is observed that the large distortions near the boundary in Type D have disappeared in this analysis. It is also found that the error in the central portion has slightly increased as the same effect was observed when Type C was compared with Type A.

Type F (Fig. 10) ; The contour pattern at the trough seems to be very similar to that of the subjective analysis. The lack of observational data over the left hand-side area caused a large distortion again in this area-dividing technique. The

RMS of this analysis is 68.9 m, which is larger than that of Type E. Since we applied a relatively high degree polynomial to a relatively small area in this analysis, it may be said that the consideration of the data from the surrounding of the analysis area will contribute more to a good analysis than the increase of the degree of the polynomial does. Although its RMS is almost the same value as that of Type C, there are such differences between the two that the former is good in the central area and the latter is good near the boundary.

Type G (Fig. 11) ; From the both viewpoints in contour pattern and RMS, this analysis is the best one of all the types tested here. Its RMS is 46.3 m. Comparing this Type G with Type F, the distortions near the boundary are less, and comparing it with Type E whose RMS is 48.4 m, it is better in contour pattern.

Table 1. Types of polynomial and their root-mean-square errors (RMS)

Type	degree of polynomial	weighting factor	data	RMS (m)
A	5	1.0	87-W 103-H	125.5
B	5	0.7	87-W 103-H	122.3
C	5	0.7	87-W 103-H 21-HO	70.8
D	7	0.7	87-W 102-H	99.2
E	7	0.7	87-W 103-H 21-HO	48.4
F	5	1.0	87-W 103-H	68.9
G	5	1.0	87-W 103-H 21-HO	46.3

W = wind data
H = height data
HO = the height data from the one-grid-length outer area

8. Summary and conclusion

On the objective 500-mb height analysis by the use of a fifth-degree polynomial, the influence of the weighting factor difference of 0.3 was negligible when the 87 wind and the 103 height data over North America were used.

About 20 per cent height data (21 height data) addition from the surrounding of the boundary to the analysis produced about the 55 m improvement of RMS in the case of the fifth-degree polynomial and about the 20 m improvement in the case of the seventh-degree polynomial. The pronounced distortions near the boundary before the addition of the data were almost eliminated after the addition.

Increasing the degree of the polynomial from the fifth to the seventh improved RMS about 23 m both before and after the addition of 20 per cent height data.

As it is frequently seen in the previous discussions, the increase of the degree of the polynomial and the consideration of some data from the surrounding of the analysis area lead to a good objective analysis of 500-mb height by the use of a polynomial. And it is also found that the contribution of the data to the analysis was very noticeable. Thus it can be suggested that this kind of method should be used to such area where sufficient data as well as surrounding data are available for the elimination of the distortions near the boundary.

References

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3) Panofsky, H. A. and Brier, G. W., "Some application of Statistics to Meteorology," The Pennsylvania State University, 223 pp, (1958)

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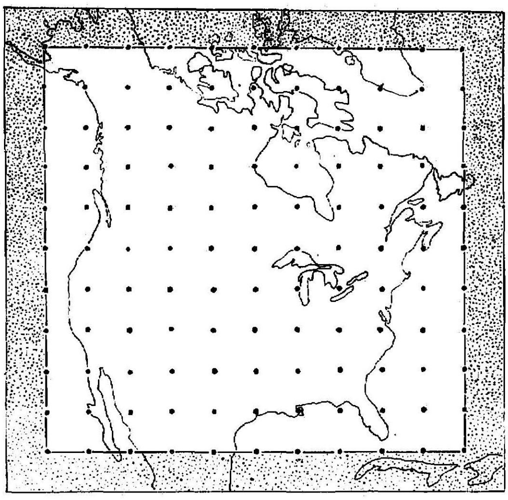


Fif. 1 Analysis area and grid points for x and y coordinates. Shaded part is one-grid-length outer area.

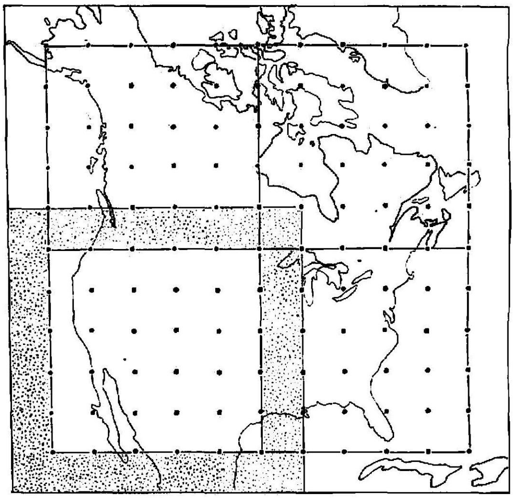


Fig. 2 Illustration of the area-dividing technique adopted by Type F and G. Shaded part is one-grid-length outer area of the small area.

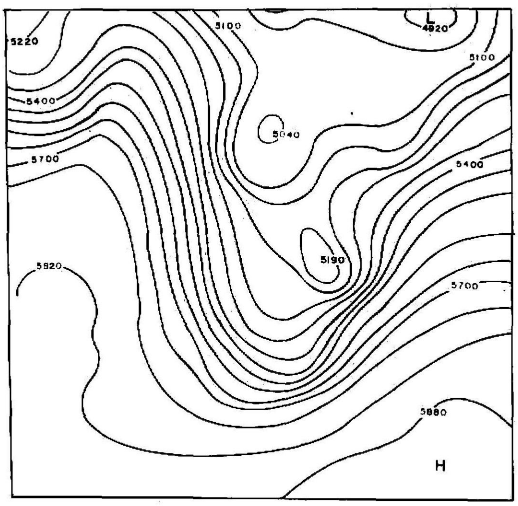


Fig. 3 Subjective analysis of 500-mb height at 1500 GMT, Nov. 21, 1956. Contour interval is 60 m.

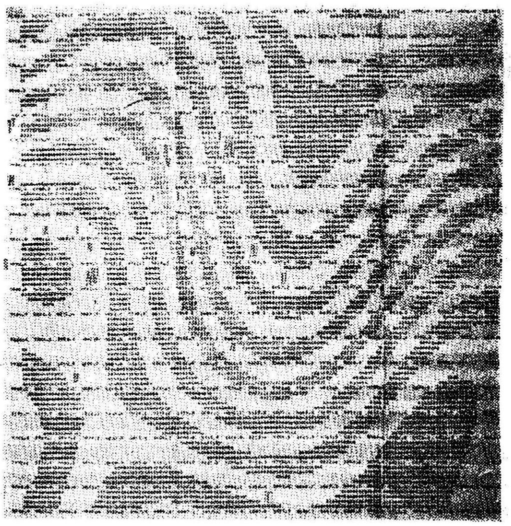
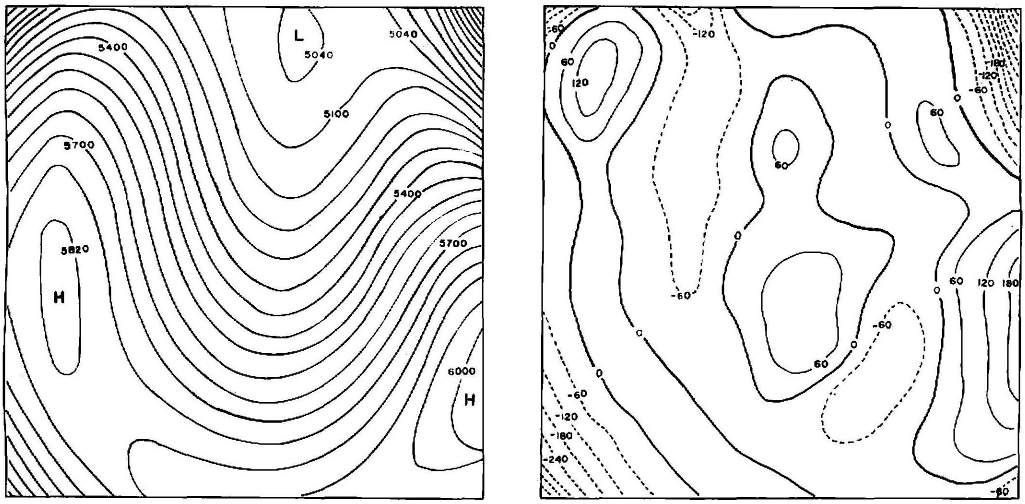
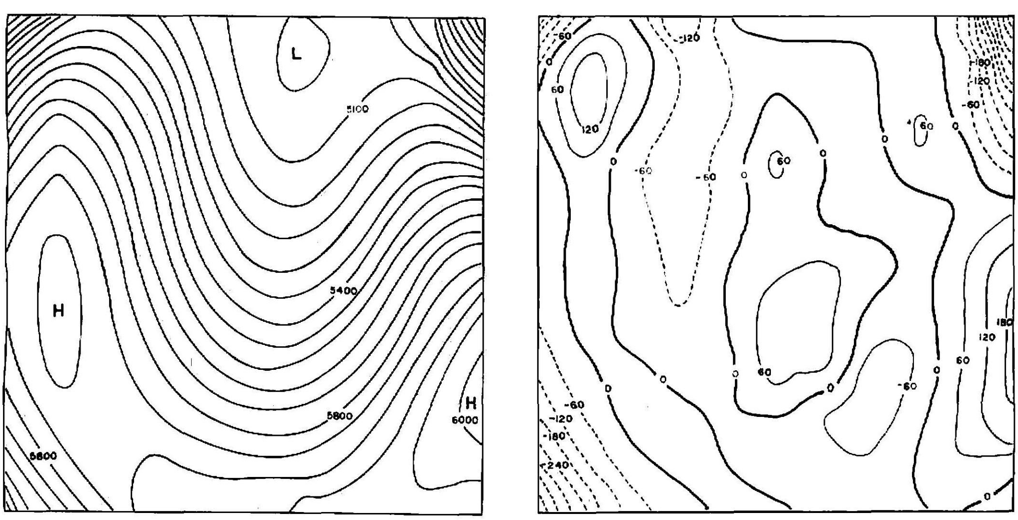


Fig. 4 Resultant objective analysis drawn by IBM 7040



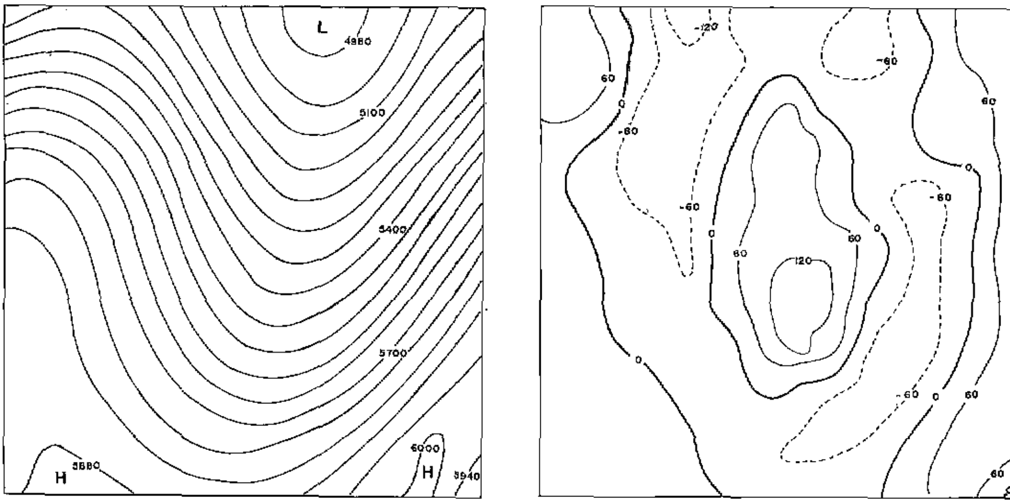
(a) (b)

Fig. 5 Objective 500-mb height analysis by Type A (a), and its error distribution (b). Contour interval is 60 m. RMS = 125.5 m.



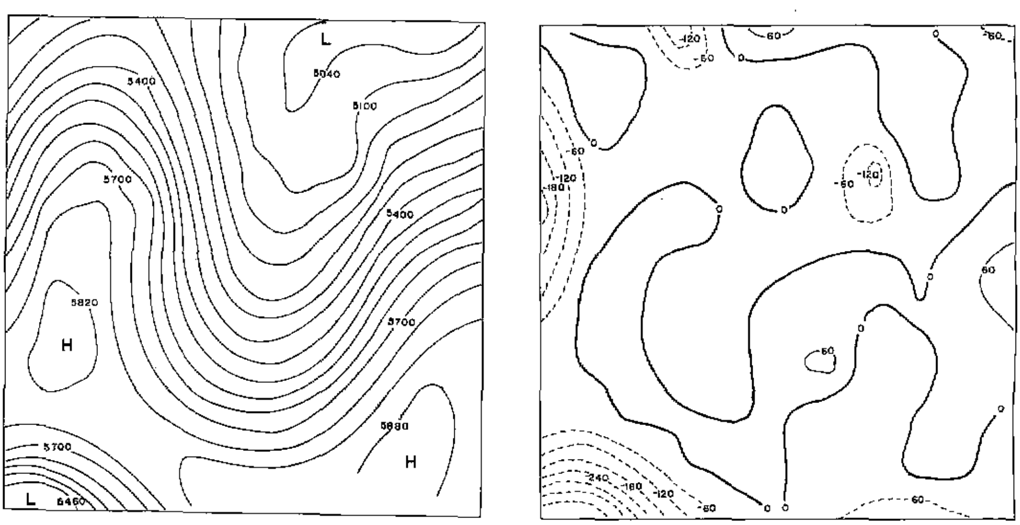
(a) (b)

Fig. 6 Objective 500-mb height analysis by Type B (a), and its error distribution (b). Contour interval is 60 m. RMS = 122.3 m.



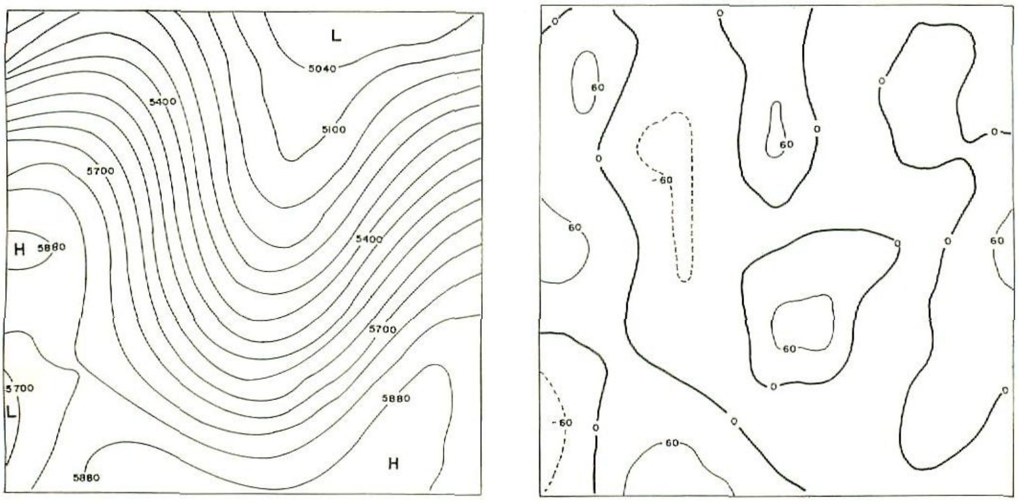
(a) (b)

Fig. 7 Objective 500-mb height analysis by Type C (a), and its error distribution (b). Contour interval is 60 m. RMS = 70.8 m.



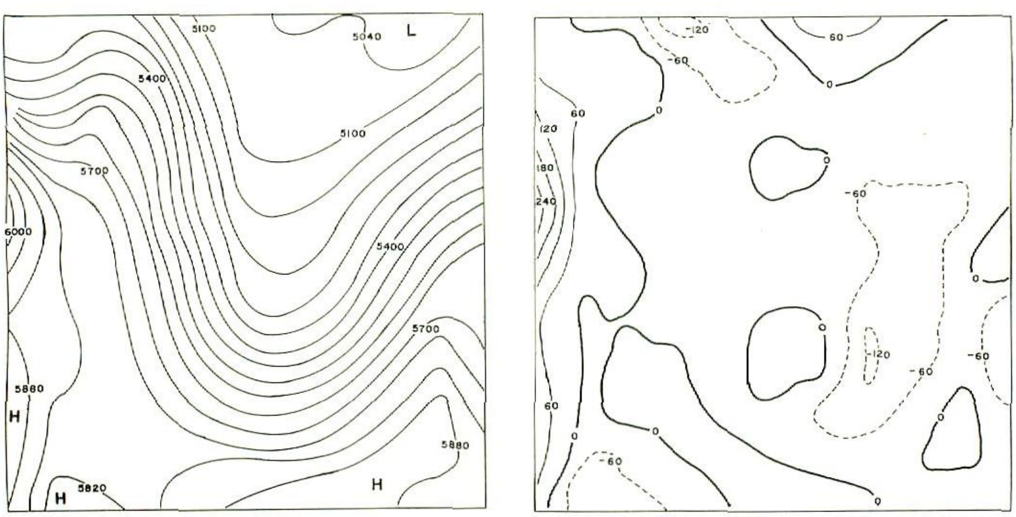
(a) (b)

Fig. 8 Objectiive 500-mb height analysis by Type D (a), and its error distribution (b). Contour interval is 60 m. RMS = 99.2 m.



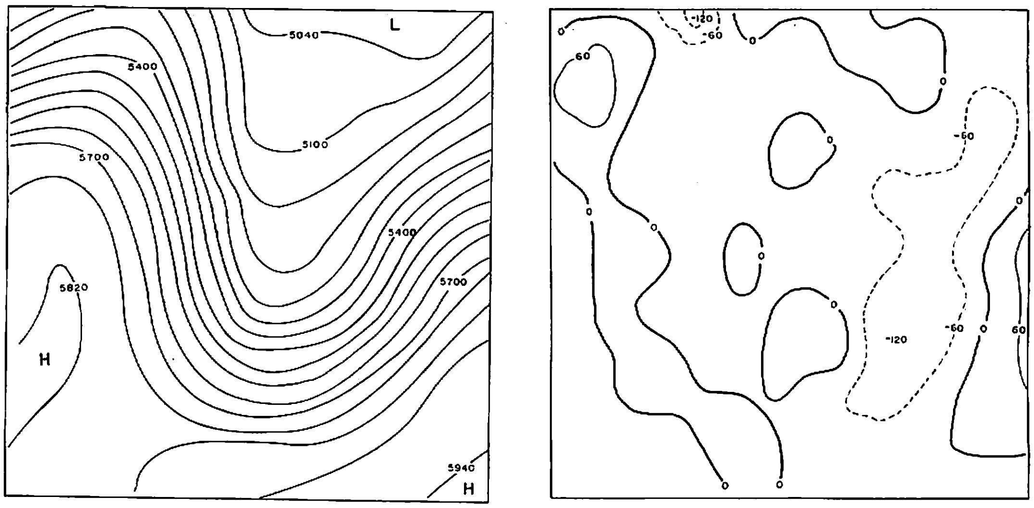
(a) (b)

Fig. 9 Objective 500-mb height analysis by Type E (a), and its error distribution (b). Contour interval is 60 m. *RMS* = 48.4 m.



(a) (b)

Fig. 10 Objective 500-mb height analysis by Type F (a), and its error distribution (b). Contour interval is 60 m. RMS = 68.9 m.



(a) (b)

Fig. 11 Objective 500-mb height analysis by Type G (a), and its error distribution (b). Contour interval is 60 m. RMS = 46.3 m.