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26. $d_5(a_2x) = -g_{1,1}a_1b_{21}$ *if* $p=3$

27. $d_{2p-1}(h_2b_{02}^2) = 2\ b_{01}b_{11}h_2(1,2) + 2\ b_{11}^2h_1(2,1)$ *if* $p \geqq 5$

$d_5(h_2b_{02}^2) = -b_{11}^2h_1(2,1) + b_{01}^2b_{21}$ *if* $p=3$

28. $d_5(h_0b_{02}x) = -a_0b_{01}^2b_{21}$ *if* $p=3$

29. $d_5(h_2b_{02}a_2) = b_{01}a_1b_{21}$ *if* $p=3$

30. $d_3(x_2) = ub_{21}$ *if* $p=3$

31. $d_{2h-1}(h_2a_2^2) = 2\ a_0^{p+1}x_2$ *if* $p \geqq 5$

$d_5(h_2a_2^2) = a_1^2b_{21} - a_0^4x_2$ *if* $p=3$

32. $d_{p^2-3p+3}(a_1^{p-4}wb_{02} - b_{01}a_1^{p-5}wa_2) = -\gamma a_0^{p^2-3p-2}h_2b_{11}^2$ *if* $p \geqq 5$

33. $d_{2p-1}(a_1^lub_{02}a_2) = 2\ b_{01}a_1^{l+1}G - 2\ a_0^{p+1}a_1^{l-1}w(a_1b_{02} - a_2b_{01}),\ 1 \leqq l \leqq p-1,$ $l \neq p-4$ *if* $p \geqq 5$

$d_{2p-1}(a_1^{p-4}ub_{02}a_2) = 2\ b_{01}a_1^{p-3}G$ *if* $p \geqq 5$

$d_5(a_1ub_{02}a_2) = -b^0{}_1a_1^2G + a_0^4a_1wb_{02}$ *if* $p=3$

$d_5(a_1^2ub_{02}a_2) = -b_{01}a_1^3G$ *if* $p=3$

34. $d_{2p-1}(a_1^lub_{02}^2) = 2\ b_{01}^2a_1^lG,\ 1 \leqq l \leqq p-1$ *if* $p \geqq 5$

$d_5(a_1ub_{02}^2) = -b_{01}^2a_1G$ *if* $p=3$

$d_5(a_1^2ub_{02}^2) = -b_{01}^2a_1^2G - a_0^8b_{02}^3$ *if* $p=3$

35. $d_{p^2-2p+2}(a_1^{p-4}ub_{02}^2 - b_{01}a_1^{p-5}ua_2b_{02}) = \alpha a_0^{p^2-3p-1}b_{11}^3$ *if* $p \geqq 5$

36. $d_{2p-1}(a_1^lua_2^2) = 2\ a_1^{l+2}G - 2(1+3)\ a_0^{p+1}a_1^lwa_2,\ 1 \leqq l \leqq p-1$ *if* $p \geqq 5$

$d_5(a_1ua_2^2) = -a_1^3G + a_0^4a_1a_2w + a_0^5b_{11}a_2^2$ *if* $p=3$

$d_5(a_1^2ua_2^2) = -a_1^4G - a_0^4a_1^2a_2w$ *if* $p=3$

37. $d_{2p-1}(a_1h_2b_{02}a_2) = -a_0^{p+1}Gb_{02} - 2\ a_0^pa_2h_1(2,1)b_{11}$ *if* $p \geqq 5$

$d_5(a_1h_2b_{02}a_2) = b_{01}a_1^2b_{21} - a_0^4Gb_{02} + a_0^3b_{11}a_2h_1(2,1)$ *if* $p=3$

$d_5(a_1^2h_2b_{02}a_2) = b_{01}a_1^3b_{21} + a_0^4a_1Gb_{02}$ *if* $p=3$

38. $d_3(b_{12}) = -b_{11}h_3 + b_{21}h_2$ *if* $p=3$

$d_3(b_{03}) = h_1b_{12} - b_{02}h_3$ *if* $p=3$

$d_3(a_3) = -h_3a_2$ *if* $p=3$

39. $d_5(a_1h_2a_2^2) = a_1^3b_{21}$ *if* $p=3$

$d_5(a_1^2h_2a_2^2) = a_1^4b_{21} + a_0^4a_1^2x_2 - a_0^6h_1(2,1)a_2^2$ *if* $p=3$

40. $d_9(b_{03}^2) = -b_{01}^3h_3$ *if* $p=3$

$d_9(a_2^3) = -a_1^3h_3$ *if* $p=3$

41. $d_5(h_1x_2) = a_0^2b_{11}b_{21}$ *if* $p=3$

42. $d_5(h_2b_{02}w) = -a_0b_{11}^2h_2(1,2)$ *if* $p=3$

43. $d_5(h_1b_{02}^2a_2) = b_{11}^3a_2 + a_0b_{01}(Gb_{02} - b_{01}x_2) - a_0^3h_1(2,1)b_{02}^3$ *if* $p=3$

$d_5(h_1b_{02}a_2^2) = a_0a_1(b_{01}x_2 - Gb_{02})$ *if* $p=3$

PROOF. 1, ..., 10, 38 and 40 are determined by J.P.May (see Theorem 4.2.). But I cannot determine the coefficients α (in 2 and 8), β (in 3) and γ (in 9), and my results are different from Theorem 4.2 in 6, 7, 8($p=3$), 9($p=3$) and 40(first one).

We first verify 6. I know no proof of this except by the method 1 (imbedding method). Since h_1a_2 has bidegree $(p+1, (p^2+2p)q-1)$, $d_r(h_1a_2)$ has bidegree $(p+2, (p^2+2p)q-2)$. For dimensional considerations we see that there is only one element $a_0^p h_1(2,1)$ with bidegree $(p+2, (p^2+2p)q-2)$. Then we consider a differential $d^{2p-1}(a_0^p h_1(2,1))^*$ in dual spectral sequence. It is a routine matter to verify that $(a_0^p h_1(2,1))^*$ and $(h_1a_2)^*$ are appear in the bar construction $\bar{B}(A)$ as $\{P_2^1\}*\{P_1^1\}*\{Q_0\}^p$ and $\{P_1^1\}*\{Q_2\}^p$, respectively. In order to verify 6, we must show that in $\bar{B}(A)$ $d(\{P_2^1\}*\{P_1^1\}*\{Q_0\}^p)$ is homologous to $-2\{P_1^1\}*\{Q_2\}^p$ modulo terms of weight greater than $2p$. By the routine calculations, we have

$$
\begin{aligned}
&d(\{P_2^1\}*\{P_1^1\}*\{Q_0\}^p \\
&+\sum_{i=1}^{p-1}(-1)^i(i-1)!\,\{P_2^1\}*\{[Q_1]^i\,|\,(P_1^0)^{p-i}\}*\{Q_0\}^{p-i} \\
&+\sum_{i=0}^{p-2}(-1)^i i!\,\{[P_3^0]*[Q_1]^i\,|\,(P_1^0)^{p-i-1}\}*\{Q_0\}^{p-i} \\
&+\{P_1^2\}*\{P_1^1\}*\{Q_1\}^p-\{P_1^2\}*\{P_2^0\}*\{Q_0\}*\{Q_1\}^{p-1} \\
&+\sum_{i=1}^{p-1}(-1)^i(i-1)!\,\{[Q_2]^i\,|\,(P_2^0)^{p-i}\}*\{P_1^1\}*\{Q_0\}^{p-i} \\
&+\sum_{i=1}^{p-1}(-1)^i(i-1)!\,\{P_2^0\}*\{[Q_2]^i\,|\,(P_1^1)^{p-i}\}*\{Q_0\}*\{Q_1\}^{p-i-1} \\
&+\sum_{i=1}^{p-1}(-1)^i(i-1)!\,\{[Q_2]^i\,|\,(P_1^1)^{p-i}\}*\{P_1^1\}*\{Q_1\}^{p-i}) \\
&=-2\{P_1^1\}*\{Q_2\}^p \text{ modulo terms of weight greater than } 2p.
\end{aligned}
$$

Then we have 6.

More examples of proofs by the imbedding method are given in ([3]. Lemma II.6.2. and 6.3.) (which give the proofs of 1, 4, 5 and 38) and ([9]. chapter 5. in the case $p=2$). Since the proofs by the imbedding method are tedious but routine,we omit these.

Next we consider 2($p\geqq 5$). This is proved by the method 3. The element $a_0^n b_{11}$ cannot survive to $H^*(A)$ for large n, by the A. Liulevicious' vanishing theorem [2]. Since it is a cocycle for every d_r, it must be the image of some differential. But the elements in E_2^{s,p^2q+s-1} for large s are $a_0^{s-p^2+p}a_1^{p-2}u$ and $a_0^{s-1}h_2$. Since $a_0^{s-1}h_2$ is a permanent cocycle, we have $d_r(a_0^{s-p^2+p}a_1^{p-2}u)=\alpha a_0^{s-1}b_{11}$, $\alpha\not\equiv 0 (\text{mod } p)$, for large s. For dimensional and filtrational reasons we have the differential 2($p\geqq 5$). 3, 9($p\geqq 5$), 16 and 17 are proved by the similar method.

8 is proved by the differential 2 and the method 2; that is, by 2 and a relation $b_{11}a_1^{p-2}u = -a_0^p a_1^{p-3}ub_{02}$ obtained from Theorem 3.3. 15.d), we have $d_{p^2-2p+2}(a_0^p a_1^{p-3}ub_{02}) = -\alpha a_0^{p^2-p-1}b_{11}^2$, *here* $\alpha=1$ if $p=3$. Then for dimensional and filtrational reasons

we have 8. The explicit determinations of the coefficient α, β, γ, ε (in 16) and η (in17) (which may be possible by the method 5) are not requested for the computations of the structure of $E_\infty = E^0H^*(A)$.

From now on, for the proof, we assume that all differentials of lower order than the one considered are already obtained.

Next we verify 7. We make use of the method 6. We first consider in the case $p \geqq 5$. a). $l=0$: Since $d_p(a_1) = -a_0^p h_1$, Massey product $< h_1a_2, h_1, -a_0^p >$ is defined in E_{p+1} and is equal to $h_1a_1a_2$. Since $d_{2p-1}(h_1a_2) = -2\,a_0^p h_1(2,1)$, it is easy to see that this Massey product satisfies all conditions of Theorem 4.1. Then we have $d_{2p-1}(< h_1a_2, h_1, -a_0^p >) = -< -2\,a_0^p h_1(2,1),\ h_1, -a_0^p >$. Since $-< -2\,a_0^p h_1(2,1), h_1, -a_0^p >$ is equal to $-2\,a_0^p h_1(2,1)a_1 = -2\,a_0^{p+2}w$, we have $d_{2p-1}(h_1a_1a_2) = -2\,a_0^{p+2}w$. For dimensional and filtrational reasons and by a relation $h_1a_1 = a_0u$ listed in Theorem 3.3. 1,f), we have $d_{2p-1}(a_2u) = -2\,a_0^{p+1}w$. b). $1 \leqq l \leqq p-4$ or $l=p-2$: Since $d_p(a_1^{l+1}) = -(l+1)\,a_0^{p+1}ua_1^{l-1}$, Massey product $(l+1) < h_1a_2, ua_1^{l-1}, -a_0^{p+1} >$ is defined in E_{p+1} and is equal to $h_1a_1^{l+1}a_2$. Furthermore it is easy to see that this Massey product satisfies all conditions of Theorem 4.1. Then we have $d_{2p-1}((l+1) < h_1a_2, ua_1^{l-1}, -a_0^{p+1} >) = -(l+1) < -2\,a_0^p h_1(2,1), ua_1^{l-1}, -a_0^{p+1} >$. Since $-(l+1) < -2a_0^p h_1(2,1), ua_1^{l-1}, -a_0^{p+1} > = -2a_0^p a_1^{l+1}h_1(2,1) = -2a_0^{p+2}a_1^l w$, we have $d_{2p-1}(h_1a_1^{l+1}a_2) = -2\,a_0^{p+2}a_1^l w$. Then, by the manipulative method, we have $d_{2p-1}(a_1^l a_2u) = -2\,a_0^{p+1}a_1^l w$. c). $l=p-1$: Since $d_{2p-1}(a_0a_1^{p-1}a_2u) = d_{2p-1}(a_1^p h_1a_2) = -2\,a_0^p h_1(2,1)a_1^p$ by a), we have $d_{2p-1}(a_1^{p-1}a_2u) = -2\,a_0^{p-1}h_1(2,1)a_1^p = -2\,a_0^{p+1}a_1^{p-1}w$. Next we consider in the case $p=3$. I know no proof of the first one except by the imbedding method. The second one is proved by the method 2. Since $d_5(a_0a_1^2a_2u) = d_5(a_1^3h_1a_2) = a_0^3a_1^3h_1(2,1) = a_0^5a_1^2w$, we have $d_5(a_1^2a_2u) = a_0^4a_1^2w$ for dimensional and filtrational reasons.

14, 29 and 43 are proved by the similar method.

14. Since $d_p(b_{02}) = -b_{01}h_2 + h_1b_{11}$, matric Massey product $< (b_{01}\ b_{11}), \begin{pmatrix} -h_2 \\ h_1 \end{pmatrix}, h_1a_2 >$ is defined in E_{p+1} and is equal to $h_1b_{02}a_2$. Since $d_{2p-1}(h_1a_2) = -2\,a_0^p h_1(2,1)$, it is easy to see that this matric Massey product satisfies all conditions of Theorem 4.1. Then we have $d_{2p-1}(< (b_{01}\ b_{11}), \begin{pmatrix} -h_2 \\ h_1 \end{pmatrix}, h_1a_2 >) = -< (-b_{01}\ -b_{11}), \begin{pmatrix} -h_2 \\ h_1 \end{pmatrix}, -2\,a_0^p h_1(2,1) >$. The last matric Massey product is equal to $-a_0b_{01}G + a_0^2b_{11}w + a_0^3h_1(2,1)b_{02} = -a_0b_{01}G$ if $p=3$ and to $2\,a_0b_{01}G - 2\,a_0^p h_1(2,1)b_{02}$ if $p \geqq 5$. Then we have 14.

29. Matric Massey product $< (b_{01}\ b_{11}), \begin{pmatrix} -h_2 \\ h_1 \end{pmatrix}, h_2a_2 >$ is defined in E_{p+1} and is equal to $h_2b_{02}a_2$. Furthermore it is easy to see that this matric Massey product satis-

fies all conditions of Theorem 4.1. Since $d_5(h_2a_2) = a_0^3h_2(1,2)$, we have $d_5(<(b_{01}\ b_{11}), \begin{pmatrix} -h_2 \\ h_1 \end{pmatrix}, h_2a_2>) = -<(-b_{01}\ -b_{11}), \begin{pmatrix} -h_2 \\ h_1 \end{pmatrix}, a_0^3h_2(1,2)>$. The last matric Massey product is equal to $a_1b_{01}b_{21}$. Then we have 29.

43. Matric Massey product $< h_1b_{02}^2, h_2, h_1, a_0^3 >$ is defined in E_4 and is equal to $h_1b_{02}^2a_2$. Then by Theorem 4.1, we have $d_5(< h_1b_{02}^2, h_2, h_1, a_0^3 >) = -< h_0b_{01}v_0 + b_{11}^3, h_2, h_1, a_0^3 >$. Since the last matric Massey product is equal to $b_{11}^3a_2 + a_0b_{01}(Gb_{02} - b_{01}x_2) - a_0^3h_1(2,1)b_{02}^3$, we have the first one. The second is obtained by the method 2; $d_5(a_0^3h_1b_{02}a_2^2) = -d_5(a_0ub_{11}a_2^2) = a_0b_{11}a_1^2G = -a_0^4a_1Gb_{02}$. Since $a_1(b_{01}x_2 - Gb_{02})$ is permanent cocycle for dimensional and filtrarional reasons, we have $d_5(h_1b_{02}a_2^2) = a_0a_1(b_{01}x_2 - Gb_{02})$.

Next we consider $13(p=3)$. By dimensional and filtrational considerations we have $d_5(h_1b_{02}^2) = \alpha h_0b_{01}v_0 + \beta b_{11}^3$, where $\alpha, \beta \in Z_3$(possibly zero).

By the imbedding method, we have $d_5(h_0b_{01}v_0)^* = (h_1b_{02}^2)^*$. Then $\alpha = 1$.

On the other hand, $\beta a_0^3b_{11}^3 = d_5(a_0^3h_1b_{02}^2) = -d_5(b_{11}a_1h_1b_{02}) = -d_5(a_0b_{11}ub_{02}) = a_0^3b_{11}^3$. Then we have $\beta = 1$. The result is follow.

$9(p=3)$, 11, 12, 18, 19, 20, 21, 22, 23, 27, 30, 32, 33, 34, 35, 36, 37, 39, 41 and 42 are proved by the similar method.

$9(p=3)$. For dimensional and filtrational reasons we have $d_3(w) = \alpha a_0h_1h_1(2,1)$, where $\alpha \in Z_3$(possibly zero). $\alpha a_0^3h_1h_1(2,1) = d_3(a_0^2w) = d_3(h_1(2,1)a_1) = -a_0^3h_1(2,1)h_1$. Then we have $\alpha = -1$.

11. For dimensional and filtrational reasons we have $d_p(G) = \alpha a_0^{p-1}h_1h_2(1,2)$, where $\alpha \in Z_p$(possibly zero). $\alpha a_0^ph_1h_2(1,2) = d_p(a_0G) = d_p(h_2(1,2)a_1) = -a_0^ph_1h_2(1,2)$. Then we have $\alpha = -1$.

12. Since $d_p(b_{02}a_2) = h_1b_{11}a_2 - h_2b_{01}a_2 - a_1h_2b_{02}$, $d_{2p-1}(h_1b_{11}a_2) = -2a_0^ph_1(2,1)b_{11} \neq 0$ in E_{2p-1} and $d_{2p-1}(b_{01}h_2a_2) = a_0^pb_{01}h_2(1,2) = 0$, we have $d_{2p-1}(a_1h_2b_{02}) = d_{2p-1}(h_1b_{11}a_2) = -2a_0^ph_1(2,1)b_{11}$.

18. We make use of relations 3.3. 4.b). and 3.3. 4.a). For dimensional and filtrational reasons we have $d_{2p-1}(g_{2,l}a_2^2) = \alpha_l g_{1,l+1}a_1G$, $\alpha_l \in Z_p$ (possibly zero), for $0 \leqq l \leqq p-3$. Since $d_{2p-1}(g_{1,p-l-3}g_{2,l}a_2^2) = 1/(l+2)\ d_{2p-1}(a_0^{p-3}b_{01}h_1a_2^2) = 2/(l+2)a_0^{p-2}b_{01}a_1G = 2g_{1,p-l-3}g_{1,l+1}a_1G$ for $0 \leqq l \leqq p-3$, we have $d_{2p-1}(g_{2,l}a_2^2) = -2g_{1,l+1}a_1G$ for $0 \leqq l \leqq p-3$.

19. For dimensional and filtrational reasons we have $d_{2p-1}(g_{2,1}b_{02}^2) = \alpha h_0k_{1,1}b_{11}^2 + \beta b_{01}^2v_1$, where $\alpha, \beta \in Z_p$, if $p \geqq 5$ and $d_{2p-1}(g_{2,l}b_{02}^2) = \gamma_l b_{01}^2v_l$, where $\gamma_l \in Z_p$, for $2 \leqq l \leqq p-3$ if $p \geqq 5$. By the imbedding method, we have $d_{2p-1}(h_0k_{1,1}b_{11}^2)^* = 2$

$(g_{2,1}b_{02}^2)^*$. Then we have $\alpha = 2$.
Since $d_{2p-1}(g_{1,p-l-3}g_{2,l}b_{02}^2) = 1/(l+2)\ a_0^{p-3}b_{01}d_{2p-1}(h_1b_{02}^2) = 2/(l+2)\ a_0^{p-3}b_{01}^3h_2(1,2) = 2\,g_{1,p-l-3}b_{01}^2v_1$ for $0 \leqq l \leqq p-3$, we have $\beta = \gamma_l = -2$ for $2 \leqq l \leqq p-3$.
(Remark. $d_p(g_{2,0}b_{02}^2) = 2\ h_0k_{1,0}b_{11}b_{02}$.)

20. For dimensional and filtrational reasons we have $d_{2p-1}(g_{2,l}b_{02}a_2) = \alpha_l\ g_{1,l+1}b_{01}G$, where $\alpha_l \in Z_p$ (possibly zero), for $1 \leqq l \leqq p-3$. Since $d_{2p-1}(g_{1,p-l-3}g_{2,l}b_{02}a_2) = 1/(l+2)\ d_{2p-1}(a_0^{p-3}b_{01}h_1b_{02}a_2) = 2/(l+2)a_0^{p-2}b_{01}^2G = 2\,g_{1,p-l-3}g_{1,l+1}b_{01}G$ for $0 \leqq l \leqq p-3$, we have $\alpha_l = -2$ for $1 \leqq l \leqq p-3$.

21. For dimensional and filtrational reasons we have $d_{2p-1}(ub_{02}^2) = \alpha\,b_{01}^2G$, where $\alpha \in Z_p$ (possibly zero). If $p \geqq 5$, then we have $d_{2p-1}(g_{1,2}b_{01}ub_{02}^2) = d_{2p-1}(b_{01}a_1g_{2,1}b_{02}^2) = -2\ b_{01}^3a_1v_1 = -2\ g_{1,2}b_{01}^3G$ and then $d_{2p-1}(ub_{02}^2) = 2\ b_{01}^2G$. If $p=3$, then we have $d_5(a_0b_{01}ub_{02}^2) = d_5(b_{01}a_1h_1b_{02}^2) = h_0b_{01}^2a_1v_0 = -\ a_0b_{01}^3G$ and then $d_5(ub_{02}^2) = -\ b_{01}^2G$.

22. For dimensional and filtrational reasons we have $d_{2p-1}(ub_{02}a_2) = \alpha\,b_{01}a_1G + \beta\,a_0^{p+1}wb_{02}$, where $\alpha,\ \beta \in Z_p$, if $p \geqq 5$ and $d_5(ub_{02}a_2) = \gamma\,b_{01}a_1G + \varepsilon\,a_0^2(b_{11}^2a_2 + a_0^2wb_{02})$, where $\gamma,\ \varepsilon \in Z_3$, if $p=3$. Since $d_{2p-1}(a_0^pub_{02}a_2) = -\ d_{2p-1}(a_1ua_2b_{11})$ is equal to $-\ a_0^4(a_1w - a_0b_{11}a_2)b_{11} = a_0^5b_{11}^2a_2 + a_0^7wb_{02}$ if $p=3$ and to $2\ a_0^{p+1}a_1wb_{11} = -2\ a_0^{2p+1}wb_{02}$ if $p \geqq 5$, we have $\beta = -2$ and $\varepsilon = 1$. Furthermore, we have $d_{2p-1}(a_0b_{01}ub_{02}a_2) = d_{2p-1}(b_{01}a_1h_1b_{02}a_2) = 2\,a_0b_{01}^2a_1G$ and then we have $\alpha = 2$ and $\gamma = -1$.

23. For dimensional and filtrational reasons we have $d_{2p-1}(ua_2^2) = \alpha\ (a_1^2G - 3\ a_0^{p+1}wa_2)$, where $\alpha \in Z_p$, if $p \geqq 5$ and $\beta\,a_1^2G$, where $\beta \in Z_3$, if $p=3$. Since $d_{2p-1}(a_0b_{01}ua_2^2) = d_{2p-1}(b_{01}a_1h_1a_2^2) = 2\ a_0b_{01}a_1^2G$, we have $\alpha = 2$ and $\beta = -1$.

27. For dimensional and filtrational reasons we have $d_{2p-1}(h_2b_{02}^2) = \alpha\ b_{01}b_{11}h_2(1,2) + \beta\,b_{11}^2h_1(2,1)$, where $\alpha,\ \beta \in Z_p$, if $p \geqq 5$ and $\gamma\,b_{11}^2h_1(2,1) + \varepsilon\,b_{01}^2b_{21}$, where $\gamma, \varepsilon \in Z_3$, if $p=3$. If $p \geqq 5$, then we have $d_p(b_{02}^3) = 3\ h_1b_{11}b_{02}^2 - 3\ h_2b_{01}b_{02}^2$ and $d_{2p-1}(h_1b_{11}b_{02}^2) = 2\ b_{01}^2b_{11}h_2(1,2) + 2\ b_{01}b_{11}^2h_1(2,1)$. Then we have $d_{2p-1}(h_2b_{01}b_{02}^2) = 2\ b_{01}^2b_{11}h_2(1,2) + 2\ b_{01}b_{11}^2h_1(2,1)$ and therefore $\alpha = \beta = 2$. If $p=3$, then, by the imbedding method, we have $d_5(b_{01}^2b_{21})^* = (h_2b_{02}^2)^*$. Then we have $\varepsilon = 1$. Since $d_5(a_0^3h_2b_{02}^2) = -\ d_5(b_{11}h_2a_1b_{02}) = -\ a_0^3h_1(2,1)b_{11}^2$, we have $\gamma = -1$.

30. For dimensional reasons we have $d_3(x_2) = \alpha\,ub_{21}$, whree $\alpha \in Z_3$. Since $d_3(a_0x_2) = d_3(h_2(1,2)a_2) = -\ h_2(1,2)h_2a_1 = b_{21}h_1a_1 = a_0b_{21}u$, we have $\alpha = 1$.

32. For dimensional and filtrational reasons there is only one element $a_0^{p^2-3p-2}h_2b_{11}^2$ which is possibly killed by $a_1^{p-4}wb_{02} - b_{01}a_1^{p-5}wa_2$. Since $d_{p^2-3p+3}(a_0^pa_1^{p-4}wb_{02}) = -\ d_{p^2-3p+3}(a_1^{p-3}wb_{11}) = -\ \gamma\,a_0^{p^2-2p-2}h_2b_{11}^2$ by 9, we have the result.

33. We first consider in the case $p \geqq 5$. For dimensional and filtrational reasons we

have $d_{2p-1}(a_1^l u b_{02} a_2) = \alpha_l\ b_{01} a_1^{l+1} G + \beta_l\ a_0^{p+1} a_1^l w b_{02}$ for $1 \leqq l \leqq p-1$ and $l \neq p-4$, and $\gamma b_{01} a_1^{p-3} G$ for $l=p-4$. Since $d_{2p-1}(a_0^p a_1^l u b_{02} a_2) = - d_{2p-1}(a_1^{l+1} b_{11} u a_2) = 2 a_0^{2+1} a_1^{l+1} w b_{11} = -2 a_0^{2p+1} a_1^l w b_{02}$ for $1 \leqq l \leqq p-1$ and $l \neq p-4$, we have $\beta_l = -2$. Since $d_{2p-1}(h_0 a_1^l u b_{02} a_2) = -2\ h_0 a_1^l b_{01} a_1 G$ for $1 \leqq l \leqq p-1$, we have $\alpha_l = \gamma = 2$.
Next we consider in the case $p=3$.
For dimensional and filtrational reasons we have $d_5(a_1 u b_{02} a_2)$
$= \alpha b_{01} a_1^3 G + \beta a_0^4 a_1 w b_{02}$ and $d_5(a_1^2 u b_{02} a_2) = \gamma b_{01} a_1^3 G$. Since $d_3(a_1^3 b_{02} a_2)$
$= - a_0^4 a_1 u b_{02} a_2 - a_1^3 h_2 b_{02} a_2 - a_1^4 b_{02} h_2$, $d_5(a_1^3 h_2 b_{02} a_2) = 0$ and $d_5(a_1^4 b_{02} h_2)$
$= - a_0^8 a_1 b_{02} w$, we have $d_5(a_0^4 a_1 u b_{02} a_2) = a_0^8 a_1 b_{02} w$. Then we have $\beta = 1$.
Since $d_5(h_0 a_1^l u b_{02} a_2) = h_0 a_1^l b_{01} a_1 G$ for $1 \leqq l \leqq 2$, we have $\alpha = \gamma = -1$.

34. We first consider in the case $p \leqq 5$. For dimensional and filtrational reasons we have $d_{2p-1}(a_1^l u b_{02}^2) = \alpha_l b_{01}^2 a_1^l G$, where $\alpha_l \in Z_p$, for $1 \leqq l \leqq p-1$.
Since $d_{2p-1}(h_0 a_1^l u b_{02}^2) = -2\ h_0 a_1^l b_{01}^2 G$ for $1 \leqq l \leqq p-1$, we have $\alpha_l = 2$.
Next we consider in the case $p=3$. For dimensional and filtrational reasons we have $d_5(a_1 u b_{02}^2) = \alpha b_{01}^2 a_1 G$ and $d_5(a_1^2 u b_{02}^2) = \beta b_{01}^2 a_1^2 G + \gamma a_0^8 b_{02}^3$.
Since $d_5(a_0^3 a_1^2 u b_{02}^2) = - d_5(a_1^3 b_{11} u b_{02}) = a_0^2 b_{11}^3 a_1^3 = - a_0^{11} b_{02}^3$, we have $\gamma = -1$.
Since $d_5(h_0 a_1^l u b_{02}^2) = h_0 a_1^l b_{01}^2 G$ for $1 \leqq l \leqq 2$, we have $\alpha = \beta = -1$.

35. For dimensional and filtrational reasons there is only one possible element $a_0^{p^2-3p-1} b_{11}^3$. Since $d_{p^2-2p+2}(a_0^p a_1^{p-4} u b_{02}^2) = - d_{p^2-2p+2}(a_1^{p-3} u b_{02} b_{11})$
$= \alpha a_0^{p^2-2p-1} b_{11}^3$ by 8, we have the result.

36. We first consider in the case $p \geqq 5$. For dimensional considerations we have $d_{2p-1}(a_1^l u a_2^2) = \alpha_l (a_1^{l+2} G - (l+3) a_0^{p+1} a_1^l w a_2)$ for $1 \leqq l \leqq p-1$, where $\alpha_l \in Z_p$ (possibly zero). Since $d_{2p-1}(h_0 a_1^l u a_2^2) = -2\ h_0 a_1^l a_1^2 G$ for $1 \leqq l \leqq p-1$, we have $\alpha_l = 2$.
Next we consider in the case $p=3$. For dimensional and filtrational reasons we have $d_5(a_1 u a_2^2) = \alpha\ (a_1^3 G - a_0^4 a_1 a_2 w - a_0^5 b_{11} a_2^2)$ and $d_5(a_1^2 u a_2^2) = \beta\ (a_1^4 G + a_0^4 a_1^2 a_2 w)$, where α, $\beta \in Z_3$. Since $d_5(h_0 a_1^l u a_2^2) = h_0 a_1^l a_1^2 G$ for $1 \leqq l \leqq 2$, we have $\alpha = \beta = -1$.

37. We first consider in the case $p \geqq 5$. Since $d_p(b_{02} a_2^2) = h_1 b_{11} a_2^2 - b_{01} h_2 a_2^2 -2 h_2 a_1 b_{02} a_2$, $d_{2p-1}(b_{01} h_2 a_2^2) = 0$ and $d_{2p-1}(h_1 b_{11} a_2^2) = -2\ a_0^{p+1} G b_{02} -4\ a_0^p a_2 h_1(2,1) b_{11}$, we have the result. Next we consider in the case p=3. Since $d_3(b_{02} a_2) = h_1 b_{11} a_2^2 - b_{01} h_2 a_2^2 + h_2 a_1 b_{02} a_2$, $d_5(h_1 b_{11} a_2^2) = a_0^4 G b_{02} - a_0^3 b_{11} a_2 h_1(2,1)$ and $d_5(b_{01} h_2 a_2^2) = b_{01} a_1^2 b_{21}$, we have $d_5(a_1 h_2 b_{02} a_2) = - a_0^4 G b_{02} + a_0^3 b_{11} a_2 h_1(2,1) + b_{01} a_1^2 b_{21}$. Since $d_3(a_1 b_{02} a_2^2) = a_0^3 h_1 b_{02} a_2^2 - b_{01} a_1 h_2 a_2^2 + a_1^2 h_2 b_{02} a_2$, $d_5(a_0^3 h_1 b_{02} a_2^2) = - d_5(a_0 u b_{11} a_2^2)$
$= a_0 b_{11} a_1^2 G = - a_0^4 a_1 G b_{02}$ and $d_5(b_{01} a_1 h_2 a_2^2) = b_{01} a_1^3 b_{21}$, we have $d_5(h_1 b_{02} a_2^2)$
$= a_0 a_1 (b_{01} x_2 - G b_{02})$ *and* $d_5(a_1^2 h_2 b_{02} a_2) = b_{01} a_1^3 b_{21} + a_0^4 a_1 G b_{02}$.

39. For dimensional and filtrational reasons we have $d_5(a_1 h_2 a_2^2) = \alpha\ a_1^3 b_{21}$ and

$d_5(a_1^2 h_2 a_2^2) = \beta\,(a_1^4 b_{21} + a_0^4 a_1^2 x_2 - a_0^6 h_1(2,1) a_2^2)$, where $\alpha, \beta \in Z_3$.
Since $d_5(h_0 a_1^l h_2 a_2^2) = -h_0 a_1^l a_1^2 b_{21}$ for $1 \leqq l \leqq 2$, we have $\alpha = \beta = 1$.

41. For dimensional reasons we have $d_5(h_1 x_2) = \alpha a_0^2 b_{11} b_{21}$, where $\alpha \in Z_3$.
Since $d_5(a_0 h_1 x_2) = d_5(h_1 a_2 h_2(1,2)) = a_0^3 h_1(2,1) h_2(1,2) = a_0^3 b_{11} b_{21}$, we have $\alpha = 1$.

42. For dimensional reasons we have $d_5(h_2 b_{02} w) = \alpha a_0 b_{11}^2 h_2(1,2)$, where $\alpha \in Z_3$. Since $d_5(a_0^3 h_2 b_{02} w) = -d_5(h_2 a_1 b_{11} w) = -a_0^4 b_{11}^2 h_2(1,2)$, we have $\alpha = -1$.

I know no proof of the rest of the proposition except by the imbedding method. We omit these.

The differentials in Proposition 4.3 give all the essential imformation about the May spectral sequence in the range $t - s \leqq 2(p-1)(3p^2+3p+4)-2$.
Countless other differentials must be cranked out, but they all follow from those given above by more or less elementary arguments or routine calculations. In the actual work, I make use of the ideas of [9]. This is the very long but routine work, and probably not desirable in any case; so in this paper we omit it.

We collect the results into the following theorem. The algebra structure of $E_\infty = E^0H^*(A)$ is easily derived from Theorem 3.3 and Proposition 4.3.

THEOREM 4.4. *The following elements of $H^*(E^0A)$ survive to E_∞. These elements are linearly independent over Z_p and iuclude all elements of a basis for $E_\infty^{s,t}$ in the range $0 \leqq t-s \leqq 2(p-1)(3p^2+3p+4)-2$, ($j \geqq 0$, and $k \geqq 0$ unless otherwise specified).*

1. $a_0^i \in (i,0)$, $0 \leqq i$

2. a). $g_{1,l} a_1^j b_{01}^k \in (jp+2k+l+1, (jp+kp+l+1)q-2k-1)$, $0 \leqq l \leqq p-2$
 b). $a_0^i a_1^j b_{01}^k \in (jp+2k+i, (jp+kp)q-2k)$, $0 \leqq i \leqq p-2$, $k \geqq 1$
 c). $a_0^i h_1 \in (i+1, pq-1)$, $0 \leqq i \leqq p-1$
 $a_0^i h_1 b_{01}^k \in (2k+i+1, (kp+p)q-2k-2)$, $0 \leqq i \leqq p-3$, $k \geqq 1$
 d). $b_{01}^k g_{2,l} \in (2k+l+2, (kp+p+l+2)q-2k-2)$, $0 \leqq l \leqq p-3$
 e). $a_0^i a_1^j u \in (pj+p+i, (jp+2p)q-1)$, $0 \leqq i \leqq p$, $j \not\equiv p-2 \bmod p$
 $a_0^i b_{01}^k a_1^j u \in (jp+p+2k+i, (jp+kp+2p)q-2k-1)$, $0 \leqq i \leqq p-2$, $k \geqq 1$
 f). $g_{1,l} b_{01}^k a_1^j u \in (jp+p+2k+l+1, (kp+jp+2p+l+1)q-2k-2)$, $0 \leqq l \leqq p-2$

3. a). $b_{01}^k k_{1,l} \in (2k+l+2, (kp+lp+2p+l+1)q-2k-2)$, $0 \leqq l \leqq p-3$
 b). $h_0 b_{01}^k k_{1,l} \in (2k+l+3, (kp+lp+2p+l+2)q-2k-3)$, $0 \leqq l \leqq p-3$

4. a). $a_0^i b_{11} \in (i+2, p^2q-2)$, $0 \leqq i \leqq p^2-p-2$
 b). $a_0^i h_2 \in (i+1, p^2q-1)$, $0 \leqq i \leqq p^2-1$

5. a). $a_0^i b_{01}^k b_{11} \in (2k+i+2, (kp+p^2)q-2k-2)$, $0 \leqq i \leqq p-3$, $k \geqq 1$
 b). $g_{1,l} b_{01}^k b_{11} \in (2k+l+3, (p^2+kp+l+1)q-2k-3)$, $0 \leqq l \leqq p-3$
 c). $b_{01}^k g_{2,l} b_{11} \in (2k+l+4, (p^2+kp+p+l+2)q-2k-4)$, $0 \leqq l \leqq p-3$

d). $a_0^i b_{01} h_2 \in (i+3, (p^2+p)q-3)$, $0 \leqq i \leqq p-2$

$a_0^i b_{01}^k h_2 \in (i+2k+1, (p^2+kp)q-2k-1)$, $0 \leqq i \leqq p-3$, $k \geqq 2$

e). $g_{1,l} h_2 \in (l+2, (p^2+l+1)q-2)$, $0 \leqq l \leqq p-2$

6. a). $b_{01}^k k_{1,l} b_{11} \in (2k+l+4, (p^2+kp+lp+2p+l+1)q-2k-4)$, $0 \leqq l \leqq p-3$

b). $h_0 b_{01}^k k_{1,l} b_{11} \in (2k+l+5, (p^2+kp+lp+2p+l+2)q-2k-5)$, $1 \leqq l \leqq p-3$

7. a). $a_0^i h_1 b_{02} \in (i+3, (p^2+2p)q-3)$, $0 \leqq i \leqq p-1$

b). $a_0^i b_{01}^k h_1 b_{02} \in (2k+i+3, (p^2+kp+2p)q-2k-3)$, $0 \leqq i \leqq p-3$, $k \geqq 1$

c). $a_0^i b_{01}^k h_1 a_2 \in (2k+p+i+1, (p^2+kp+2p)q-2k-1)$, $0 \leqq i \leqq p-3$, $k \geqq 1$

d). $a_0^i c \in (i+2, (p^2+2p)q-2)$, $0 \leqq i \leqq p-1$, *where* $c = h_1(2,1)$

e). $a_0^i b_{01}^k c \in (2k+i+2, (p^2+kp+2p)q-2k-2)$, $0 \leqq i \leqq p-4$, $k \geqq 1$

8. a). $b_{01}^k a_1^j e_l \in (jp+p+2k+l+2, (p^2+kp+jp+2p+l)q-2k-3)$, $1 \leqq l \leqq p-1$,

where $e_l = g_{1,l-1}(a_1 b_{02} - a_2 b_{01})$

b). $a_0^i b_{01}^k a_1^j f \in (jp+p+2k+i+4, (p^2+kp+jp+3p)q-2k-4)$, $0 \leqq i \leqq p-2$,

where $f = b_{01}(a_1 b_{02} - a_2 b_{01})$

9. a). $b_{01}^k g_{2,l} b_{02} \in (2k+l+4, (p^2+kp+2p+l+2)q-2k-4)$, $1 \leqq l \leqq p-3$

b). $g_{1,l} b_{01}^k a_1^j u b_{02} \in (jp+p+2k+l+3, (p^2+kp+jp+3p+l+1)q-2k-4)$, $0 \leqq l \leqq p-2$

c). $b_{01}^k g_{2,l} a_2 \in (p+2k+l+2, (p^2+kp+2p+l+2)q-2k-2)$, $0 \leqq l \leqq p-3$

d). $g_{1,l} b_{01}^k a_1^j u a_2 \in (jp+2p+2k+l+1, (p^2+kp+jp+3p+l+1)q-2k-2)$, $0 \leqq l \leqq p-2$

e). $a_0^i b_{01}^k a_1^j u a_2 \in (jp+2p+2k+i, (p^2+kp+jp+3p)q-2k-1)$, $0 \leqq i \leqq p-2$, $k \geqq 1$

f). $a_0^i a_1^j u b_{02} \in (jp+p+i+2, (p^2+jp+3p)q-3)$, $0 \leqq i \leqq p$, $j \not\equiv p-3 \bmod p$

g). $a_0^i b_{01}^k a_1^j u b_{02} \in (jp+p+2k+i+2, (p^2+jp+kp+3p)q-2k-3)$, $0 \leqq i \leqq p-2$, $k \geqq 1$

10. a). $b_{01}^k g_{3,l} \in (2k+l+3, (p^2+kp+2p+l+3)q-2k-3)$, $0 \leqq l \leqq p-4$

b). $b_{01}^k h_1 g_{3,0} \in (2k+4, (p^2+3p+kp+3)q-2k-4)$, *if* $p \geqq 5$

c). $b_{01}^k j_l \in (2k+l+4, (p^2+lp+kp+3p+l+3)q-2k-4)$, $1 \leqq l \leqq p-4$

11. a). $a_0^i a_1^j w \in (jp+p+i, (p^2+jp+3p)q-2)$, $0 \leqq i \leqq p$, $j \not\equiv p-3 \bmod p$ *if* $p \geqq 5$

$a_0^i a_1^{3l} m \in (9l+i+6, 36l+82)$, $0 \leqq i \leqq 3$, $l \geqq 0$, *where* $m = a_1 w - a_0 b_{11} a_2$ *if* $p=3$

$a_0^i a_1^{3l+2} w \in (91+i+12, 36l+106)$, $0 \leqq i \leqq 3$, $l \geqq 0$ *if* $p=3$

b). $a_0^i b_{01}^k a_1^j w \in (jp+p+2k+i, (p^2+kp+jp+3p)q-2k-2)$, $0 \leqq i \leqq p-2$, $k \geqq 1$

c). $g_{1,l} b_{01}^k a_1^j w \in (jp+p+2k+l+1, (p^2+kp+jp+3p+l+1)q-2k-3)$, $0 \leqq l \leqq p-2$

12. a). $b_{01}^k k_{1,l} b_{02} \in (2k+l+4, (p^2+kp+lp+3p+l+1)q-2k-4)$, $0 \leqq l \leqq p-4$

b). $b_{01}^k k_{1,l} a_2 \in (2k+l+p+2, (p^2+kp+lp+3p+l+1)q-2k-2)$, $0 \leqq l \leqq p-3$

c). $h_0 b_{01}^k k_{1,l} b_{02} \in (2k+l+5, (p^2+kp+lp+3p+l+2)q-2k-5)$, $0 \leqq l \leqq p-3$

d). $h_0 b_{01}^k k_{1,l} a_2 \in (2k+l+p+3, (p^2+kp+lp+3p+l+2)q-2k-3)$, $0 \leqq l \leqq p-3$

e). $b_{01}^k k_{2,l}$ $(2k+l+3, (p^2+kp+lp+3p+l+1)q-2k-3)$, $0 \leqq l \leqq p-4$

13. a). $a_0^i b_{11}^2 \in (i+4, 2p^2q-4)$, $0 \leqq i \leqq p^2-2p-2$

$a_0^i b_{01}^k b_{11}^2 \in (2k+i+4, (2p^2+kp)q-2k-4)$, $0 \leqq i \leqq p-3$, $k \geqq 1$

b). $g_{1,l} b_{01}^k b_{11}^2 \in (2k+l+5, (2k+l+5, (2p^2+kp+l+1)q-2k-5)$, $0 \leqq l \leqq p-4$

14. a). $a_0^i b_{11} h_2 \in (i+3, 2p^2q-3)$, $0 \leqq i \leqq p^2-2p-3$

b). $a_0^i b_{01}^k b_{11} h_2 \in (2k+i+3, (2p^2+kp)q-2k-3)$, $0 \leqq i \leqq p-4$, $k \geqq 1$

c). $g_{1,l} b_{11} h_2 \in (l+4, (2p^2+l+1)q-4)$, $0 \leqq l \leqq p-4$

15. a). $a_0^i a_1^{pt} l \in (tp^2+p^2-p+i+2, (tp^2+2p^2)q-2)$, $0 \leqq i \leqq p^2-2$, $0 \leqq t \leqq p-3$,

where $l = a_1^{p-2}(a_1 b_{02} - a_2 b_{01})$

$a_0^i a_1^3 l \in (i+17, 106)$, $0 \leqq i \leqq 7$ if $p=3$

$a_0^i a_1^6 l \in (i+26, 142)$, $0 \leqq i \leqq 7$ if $p=3$

b). $a_0^i a_1^{(t+1)p-3} a_2 u \in (tp^2+p^2-p+i, (t+2)p^2q-1)$, $0 \leqq i \leqq p^2+p$, $0 \leqq t \leqq p-3$

$a_0^i a_1^6 a_2 u \in (i+34, 143)$, $0 \leqq i \leqq 12$ if $p=3$

16. a). $b_{01}^k x \in (2k+p, (2p^2+kp+p-1)q-2k-2)$,

b). $h_0 b_{01}^k x \in (2k+p+1, (2p^2+kp+p)q-2k-3)$

c). $b_{01}^k h_1 x \in (2k+p+1, (2p^2+kp+2p-1)q-2k-3)$, if $p \geqq 5$

$b_{01}^k h_1 x \in (2k+4, 10k+89)$, if $p=3$

d). $b_{01}^k g_{2,0} x \in (2k+p+2, (2p^2+kp+2p+1)q-2k-4)$

17. a). $a_0^i h_2 b_{02} \in (i+3, (2p^2+p)q-3)$, $0 \leqq i \leqq p-1$

$a_0^i b_{01} h_2 b_{02} \in (i+5, (2p^2+2p)q-5)$, $0 \leqq i \leqq p-2$

$a_0^i b_{01}^k h_2 b_{02} \in (2k+i+3, (2p^2+kp+p)q-2k-3)$, $0 \leqq i \leqq p-3$, $k \geqq 2$

b). $g_{1,l} h_2 b_{02} \in (l+4, (2p^2+p+l+1)q-4)$, $0 \leqq l \leqq p-2$

c). $b_{01}^k g_{2,l} b_{11} b_{02} \in (2k+l+6, (2p^2+kp+2p+l+2)q-2k-6)$, $1 \leqq l \leqq p-3$

d). $a_0^i d \in (i+2, (2p^2+p)q-2)$, $0 \leqq i \leqq p-1$, where $d = h_2(1,2)$

$a_0^i b_{01} d \in (i+4, (2p^2+2p)q-4)$, $0 \leqq i \leqq p-3$

$b_{01}^k d \in (2k+2, 10k+82)$, $k \geqq 2$ if $p=3$

e). $a_0^i b_{11} c \in (i+4, (2p^2+2p)q-4)$, $0 \leqq i \leqq p-1$

$a_0^i b_{01}^k b_{11} c \in (2k+i+4, (2p^2+kp+2p)q-2k-4)$, $0 \leqq i \leqq p-4$, $k \geqq 1$

f). $b_{01}^k v_l \in (2k+l+3, (2p^2+kp+p+l+2)q-2k-3)$, $0 \leqq l \leqq p-3$, $k \leqq 1$

$b_{01}^k v_0 \in (2k+3, (2p^2+kp+p+2)q-2k-3)$, $k \geqq 2$

g). $b_{01}^k k_{1,l} b_{11}^2 \in (2k+l+6, (2p^2+kp+lp+2p+l+1)q-2k-6)$, $0 \leqq l \leqq p-3$

h). $h_0 b_{01}^k k_{1,l} b_{11}^2 \in (2k+l+7, (2p^2+kp+lp+2p+l+2)q-2k-7)$, $1 \leqq l \leqq p-4$

i). $b_{01}^k b_{11} g_{3,l} \in (2k+l+5, (2p^2+kp+2p+l+3)q-2k-5)$, $0 \leqq l \leqq p-4$

18. a). $g_{1,l} b_{01}^k b_{11} a_2 \in (p+2k+l+3, (2p^2+kp+p+l+1)q-2k-3)$, $0 \leqq l \leqq p-3$

b). $a_0^i b_{01}^k b_{11} a_2 \in (p+2k+i+2, (2p^2+kp+p)q-2k-2),\ 0 \leqq i \lneqq p-3,\ k \geqq 1$

c). $g_{1,l} b_{01}^k h_2 a_2 \in (p+2k+l+2, (2p^2+kp+2p+l+2)q-2k-2),\ 0 \lneqq l \leqq p-2,\ k \lneqq 1$

d). $a_0^i b_{01} h_2 a_2 \in (p+i+3, (2p^2+2p)q-3),\ 0 \lneqq i \leqq p-2$

$a_0^i b_{01}^k h_2 a_2 \in (p+2k+i+1, (2p^2+kp+p)q-2k-1),\ k \geqq 2$

e). $b_{01}^k g_{2,l} b_{11} a_2 \in (p+2k+l+4, (2p^2+kp+2p+l+2)q-2k-4),\ 0 \lneqq l \leqq p-3$

19. a). $g_{1,p-3} b_{01}^k b_{11} b_{02} \in (p+2k+2, (2p^2+kp+2p-2)q-2k-5),$

b). $a_0^i h_1 d \in (i+3, (2p^2+2p)q-3),\ 0 \lneqq i \leqq p-2$

c). $b_{01}^k g_{2,l} b_{11}^2 \in (2k+l+6, (2p^2+kp+p+l+2)q-2k-6),\ 0 \leqq l \leqq p-3$

20. a). $a_0^i h_1 b_{02}^2 \in (i+5, (2p^2+3p)q-5),\ p-2 \leqq i \lneqq 2p-2\ if\ p \geqq 5$

b). $a_0^i c b_{02} \in (i+4, (2p^2+3p)q-4),\ p-3 \leqq i \lneqq 2p-3\ if\ p \geqq 5$

c). $g_{1,l} G \in (p+l+2, (2p^2+2p+l+1)q-3),\ 0 \lneqq l \lneqq p-2$

d). $b_{01} G \in (p+3, (2p^2+3p)q-4)$

e). $g_{1,l} b_{01} G \in (p+l+4, (2p^2+3p+l+1)q-5),\ 0 \leqq l \lneqq 1$

21. a). $a_0^i h_2 w \in (p+i+1, (2p^2+3p)q-3),\ 0 \leqq i \lneqq 1$

b). $g_{1,l} h_2 w \in (p+l+2, (2p^2+3p+l+1)q-4),\ 0 \lneqq l \lneqq 1$

22. a). $a_0^i (b_{01} h_1 b_{02} a_2 - a_0 u b_{02}^2) \in (p+i+5, (2p^2+4p)q-5),\ 0 \leqq i \lneqq p-1$

$a_0^i b_{01}^k (b_{01} h_1 b_{02} a_2 - a_0 u b_{02}^2) \in (p+2k+i+5, (2p^2+kp+4p)q-2k-5),\ 0 \lneqq i \leqq p-3,$
$k \geqq 1$

b). $a_0^i (b_{01} a_2 c - a_0^2 w b_{02}) \in (p+i+4, (2p^2+4p)q-4),\ 0 \lneqq i \leqq p-1\ if\ p \geqq 5$

$a_0^i a_{01}^k (b_{01} a_2 c - a_0^2 w b_{02}) \in (p+2k+i+4, (2p^2+kp+4p)q-2k-4),\ 0 \lneqq i \leqq p-4,$
$k \geqq 1\ if\ p \geqq 5$

$a_0^i (b_{11}^2 a_2 + a_0^2 w b_{02}) \in (i+7, 116),\ 0 \leqq i \lneqq 2\ if\ p=3$

$b_{01}^k b_{11}^2 a_2 \in (2k+7, 10k+116),\ k \geqq 1\ if\ p=3$

c). $b_{01}^k (g_{1,l+2} w b_{02} - b_{01} g_{3,l} a_2) \in (p+2k+l+5, (2p^2+kp+4p+l+3)q-2k-5),\ 0 \lneqq l$
$\lneqq p-4$

d). $b_{01}^k (g_{1,l+2} u b_{02}^2 - b_{01} g_{2,l+1} b_{02} a_2) \in (p+2k+l+7, (2p^2+kp+4p+l+3)q-2k-6),$
$0 \lneqq l \leqq p-4$

23. a). $a_0^i b_{01}^k (b_{01} h_1 a_2^2 - a_0 u b_{02} a_2) \in (2p+2k+i+3, (2p^2+kp+4p)q-2k-3),\ 0 \lneqq i \leqq p-3,$
$k \geqq 1$

b). $b_{01}^k (b_{01} g_{2,l} a_2^2 - g_{1,l+1} u b_{02} a_2) \in (2p+2k+l+4, (2p^2+kp+4p+l+2)q-2k-4),$
$0 \lneqq l \leqq p-3$

24. a). $a_0^i b_{01}^k a_1^j (Z) \in (jp+2p+2k+i+4, (2p^2+kp+jp+4p)q-2k-4),\ 0 \leqq i \lneqq p-2,$
$k \geqq 1,\ where\ Z = a_1^2 b_{02}^2 - 2 b_{01} a_1 b_{02} a_2 + b_{01}^2 a_2^2$

b). $g_{1,l} b_{01}^k a_1^j (Z) \in (jp+2p+2k+l+5, (2p^2+kp+jp+4p+l+1)q-2k-5),\ 0 \leqq l \lneqq$
$p-2$

c). $a_0^i a_1^{p-4}$(乙) $\in (p^2-2p+i+4, 3p^2q-4)$, $0 \leqq i \leqq p^2-2$ *if* $p \geqq 5$

$a_0^i a_1^2$(乙) $\in (i+16, 140)$, $0 \leqq i \leqq 7$ *if* $p=3$

25. a). $a_0^i b_{01}^k a_1^j u b_{02}$(甲) $\in (jp+2p+i+4, (2p^2+kp+jp+5p)q-2k-5)$, $0 \leqq i \leqq p-2$, $k \geqq 1$, *where* 甲 $= a_1 b_{02} - a_2 b_{01}$

b). $a_0^i a_1^j u b_{02}$(甲) $\in (jp+2p+i+4, (2p^2+jp+5p)q-5)$, $0 \leqq i \leqq p$, $j \not\equiv p-5$ *mod* p *if* $p \geqq 5$

$a_0^i a_1^j u b_{02}$(甲) $\in (3j+i+10, 12j+127)$, $0 \leqq i \leqq 3$, $j \not\equiv 1$ *mod* 3 *if* $p=3$

c). $g_{1,l} b_{01}^k a_1^j u b_{02}$(甲) $\in (jp+2p+2k+l+5,\ 2(p^2+kp+jp+5p+l+1)q-2k-6)$, $0 \leqq l \leqq p-2$

26. a). $a_0^i b_{01}^k a_1^j u a_2$(甲) $\in (jp+3p+2k+i+2, (2p^2+kp+jp+5p)q-2k-1)$, $0 \leqq i \leqq p-2$, $k \geqq 1$

b). $a_0^i a_1^{p-5} u a_2$(甲) $\in (p^2-2p+i+2, 3p^2q-3)$, $0 \leqq i \leqq p^2-2$ *if* $p \geqq 5$

$a_0^i a_1 u a_2$(甲) $\in (i+14, 141)$, $0 \leqq i \leqq 7$ *if* $p=3$

c). $g_{1,l} b_{01}^k a_1^j u a_2$(甲) $\in (jp+3p+2k+l+3, (2p^2+kp+jp+5p+l+1)q-2k-2)$, $0 \leqq l \leqq p-2$

27. a). $a_0^i b_{01}^k a_1^j w$(甲) $\in (jp+2p+2k+i+2, (2p^2+kp+jp+5p)q-2k-4)$, $0 \leqq i \leqq p-2$, $k \geqq 1$

b). $a_0^i a_1^j w$(甲) $\in (jp+2p+i+2, (2p^2+jp+5p)q-4)$, $0 \leqq i \leqq p$, $j \not\equiv p-5$ *mod* p *if* $p \geqq 5$

$a_0^i a_1^j w$(甲) $\in (3j+i+8,\ 12j+128)$, $0 \leqq i \leqq 3$, $j \not\equiv 1$ *mod* 3 *if* $p=3$

c). $g_{1,l} b_{01}^k a_1^j w$(甲) $\in (jp+2p+2k+l+3, (2p^2+kp+jp+5p+l+1)q-2k-5)$, $0 \leqq l \leqq p-2$

28. a). $a_0^i a_1 b_{02}^2 \in (i+7, 104)$, $2 \leqq i \leqq 7$ *if* $p=3$

b). $a_0^i h_1 b_{02} a_2 \in (i+6, 105)$, $1 \leqq i \leqq 6$ *if* $p=3$

26. a). $a_0^i b_{11}^3 \in (i+6, 3p^2q-6)$, $0 \leqq i \leqq p^2-3p-2$

$a_0^i b_{01}^k b_{11}^3 \in (2k+i+6, (3p^2+kp)q-2k-6)$, $0 \leqq i \leqq p-3$, $k \geqq 1$ *if* $p \geqq 5$

b). $g_{1,l} b_{01}^k b_{11}^3 \in (2k+l+7, (3p^2+kp+l+1)q-2k-7)$, $0 \leqq l \leqq p-4$

30. a). $a_0^i b_{11}^2 h_2 \in (i+5, 3p^2q-5)$, $0 \leqq i \leqq p^2-3p-3$

b). $a_0^i b_{01}^k b_{11}^2 h_2 \in (i+2k+5, (3p^2+kp)q-2k-5)$, $0 \leqq i \leqq p-4$, $k \geqq 1$

c). $g_{1,l} b_{11}^2 h_2 \in (l+6, (3p^2+l+1)q-6)$, $0 \leqq l \leqq p-5$

31. a). $a_0^i h_2 b_{11} b_{02} \in (i+5, (2p^2+2p)q-5)$, $0 \leqq i \leqq p-1$

$a_0^i b_{01}^k b_{11} h_2 b_{02} \in (2k+i+5, (2p^2+kp+2p)q-2k-5)$, $0 \leqq i \leqq p-4$, $k \geqq 1$

b). $g_{1,l} h_2 b_{11} b_{02} \in (l+6, (2p^2+2p+l+1)q-6)$, $0 \leqq l \leqq p-4$

32. a). $a_0^i b_{11} d \in (i+4, (3p^2+p)q-4)$, $0 \leqq i \leqq p-1$

$a_0^i b_{01}^k b_{11} d \in (2k+i+4, (3p^2+kp+p)q-2k-4)$, $0 \leqq i \leqq p-4$, $k \geqq 1$

b). $b_{01}^k g_{3,l} b_{11}^2 \in (2k+l+7, (3p^2+kp+2p+l+3)q-2k-7),\ 0 \leqq l \leqq p-4$

33. a). $a_0^i b_{01}^k b_{11}^2 a_2 \in (2k+i+p+4, (3p^2+kp+p)q-2k-4),\ 0 \leqq i \leqq p-3,\ k \geqq 1$ *if* $p \geqq 5$

b). $g_{1,l} b_{01}^k b_{11}^2 a_2 \in (2k+p+l+5, (3p^2+kp+p+l+1)q-2k-5),\ 0 \leqq l \leqq p-4$

34. a). $a_0^i b_{01}^k b_{11} h_2 a_2 \in (p+2k+i+3, (3p^2+kp+p)q-2k-2),\ 0 \leqq i \leqq p-4,\ k \geqq 1$

b). $g_{1,l} b_{11} h_2 a_2 \in (p+l+4, (3p^2+p+l+1)q-2k-3),\ 0 \leqq l \leqq p-4$

35. a). $a_0^i h_2 b_{02} a_2 \in (p+i+3, (3p^2+2p)q-3),\ 0 \leqq i \leqq p-1$ *if* $p \geqq 5$

$a_0^i b_{01} h_2 b_{02} a_2 \in (p+i+5, (3p^2+3p)q-5),\ 0 \leqq i \leqq p-2$ *if* $p \geqq 5$

$a_0^i (a_0 h_2 b_{02} a_2 + h_2 a_2 x) \in (i+7, 129),\ 0 \leqq i \leqq 1$ *if* $p=3$

b). $a_0^i h_2 b_{02}$(甲) $\in (i+8, 139),\ 0 \leqq i \leqq 3$ *if* $p=3$

$b_{01} h_2 b_{02}$(甲) $\in (10, 149)$, *if* $p=3$

c). $g_{1,l} h_2 b_{01}^k b_{02} a_2 \in (p+2k+l+4, (3p^2+2p+kp+l+1)q-2k-4),\ 0 \leqq l \leqq p-2,$
$k \leqq 1$ *if* $p \geqq 5$

d). $b_{01}^k a_2 (b_{01} x - g_{1,1} h_2 b_{02}) \in (2k+8, 10k+136)$, *if* $p=3$

e). $h_0 b_{01}^k a_2 (b_{01} x - g_{1,1} h_2 b_{02}) \in (2k+9, 10k+139)$, *if* $p=3$

36. a). $a_0^i x_2 \in (p+i+1, (3p^2+2p)q-2),\ 0 \leqq i \leqq p$ *if* $p \geqq 5$

$a_0^i b_{01} x_2 \in (p+i+3, (3p^2+3p)q-4),\ 0 \leqq i \leqq p-2$ *if* $p \geqq 5$

$a_0^i (b_{11} a_2 c - a_0 G b_{02} + a_0 b_{01} x_2) \in (i+7, 140),\ 0 \leqq i \leqq 2$ *if* $p=3$

$a_0^i b_{01} (G b_{02} - b_{01} x_2) \in (i+8, 150),\ 0 \leqq i \leqq 1$ *if* $p=3$

b). $g_{1,l} b_{01}^k x_2 \in (p+2k+l+2, (3p^2+kp+2p+l+1)q-2k-3),\ 0 \leqq l \leqq p-2,\ k \leqq 1$
if $p \geqq 5$

$g_{1,l} b_{01}^k (G b_{02} - b_{01} x_2) \in (2k+l+7, 10k+4l+143),\ 0 \leqq l \leqq 1$ *if* $p=3$

37. a). $a_0^i b_{01}^k b_{11} a_2^2 \in (2p+2k+i+2, (3p^2+kp+2p)q-2k-2),\ 0 \leqq i \leqq p-3,\ k \geqq 1$

b). $g_{1,l} b_{01}^k b_{11} a_2^2 \in (2p+2k+l+3, (3p^2+kp+2p+l+1)q-2k-3),\ 0 \leqq l \leqq p-3$

38. a). $a_0^i b_{01} h_2 a_2^2 \in (2p+i+3, (3p^2+3p)q-3),\ 0 \leqq i \leqq p-2$ *if* $p \geqq 5$

b). $g_{1,l} b_{01}^k h_2 a_2^2 \in (2p+2k+l+2, (3p^2+kp+2p+l+1)q-2k-2),\ 0 \leqq l \leqq p-2,\ k \geqq 1$
if $p \geqq 5$

39. a). $a_0^i (b_{01} d b_{02} + c b_{11} b_{02}) \in (i+6, (3p^2+3p)q-6),\ 0 \leqq i \leqq p-1$ *if* $p \geqq 5$

b). $a_0^i h_1 d b_{02} \in (i+5, (3p^2+3p)q-5),\ 0 \leqq i \leqq p-2$

40. a). $a_0^i (c b_{11} a_2 + 1/2 a_0 G b_{02}) \in (p+i+4, (3p^2+3p)q-4),\ 0 \leqq i \leqq p-1$ *if* $p \geqq 5$

b). $a_0^i h_1 x_2 \in (p+i+2, (3p^2+3p)q-3),\ 0 \leqq i \leqq p-1$ *if* $p \geqq 5$

41. a). $b_{01}^k k_{1,l} a_2^2 \in (2p+2k+l+2, (2p^2+kp+lp+4p+l+2)q-2k-2),\ 0 \leqq l \leqq p-3$

b). $h_0 b_{01}^k k_{1,l} a_2^2 \in (2p+2k+l+3, (2p^2+kp+lp+4p+l+2)q-2k-3),\ 0 \leqq l \leqq p-3$

c). $b_{01}^k k_{1,l} b_{02} a_2 \in (p+2k+l+4, (2p^2+kp+lp+4p+l+1)q-2k-4),\ 0 \leqq l \leqq p-4$

d). $h_0 b_{01}^k k_{1,l} b_{02} a_2 \in (p+2k+l+5, (2p^2+kp+lp+4p+l+2)q-2k-5),\ 0 \leqq l \leqq p-3$

42. a). $b_{01}^{k}k_{1,l}b_{11}a_2 \in (p+2k+l+4,(2p^2+kp+lp+3p+l+1)q-2k-4),\ 0 \leqq l \leqq p-3$

b). $h_0b_{01}^{k}k_{1,l}b_{11}a_2 \in (p+2k+l+5,(2p^2+kp+lp+3p+l+2)q-2k-5),\ 1 \leqq l \leqq p-3$

43. a). $b_{01}^{k}h_2g_{3,l} \in (2k+l+4,(2p^2+kp+2p+l+3)q-2k-4),\ 0 \leqq l \leqq p-4,\ l=0\ if\ k \geqq 1.$

b). $a_1G-2a_0^{p-1}a_2c \in (2p+1,(2p^2+3p)q-2),$

c). $h_0a_1G \in (2p+2,(2p^2+3p+1)q-3),$

d). $b_{01}^{k}k_{1,0}x \in (p+2k+2,(2p^2+kp+3p)q-2k-4)$

e). $h_0b_{01}^{k}k_{1,0}x \in (p+2k+3,(2p^2+kp+3p+1)g-2k-5)$

44. a). $b_{01}^{k}k_{1,l}b_{11}b_{02} \in (2k+l+6,(2p^2+kp+lp+3p+l+1)q-2k-6),\ 0 \leqq l \leqq p-4$

b). $h_0b_{01}^{k}k_{1,l}b_{11}b_{02} \in (2k+l+7,(2p^2+kp+lp+3p+l+2)q-2k-7),\ 1 \leqq l \leqq p-3$

c). $b_{11}^{2}k_{2,l} \in (l+7,(3p^2+3p+lp+l+1)q-7),\ 0 \leqq l \leqq p-4$

d). $b_{01}^{k}k_{2,l}a_2 \in (p+2k+l+3,(2p^2+kp+lp+4p+l+1)q-2k-3),\ 0 \leqq l \leqq p-4$

e). $b_{01}^{k}k_{2,l}b_{02} \in (2k+l+5,(2p^2+kp+lp+4p+l+1)q-2k-5),\ 0 \leqq l \leqq p-5$

f). $h_0b_{01}^{k}k_{1,l}b_{02}^{2} \in (2k+l+7,(2p^2+kp+lp+4p+l+2)q-2k-7),\ 0 \leqq l \leqq p-4$

g). $b_{01}^{k}h_1g_{3,0}b_{02} \in (2k+6,(2p^2+kp+4p+3)q-2k-6),\ if\ p \geqq 5$

h). $b_{01}^{k}h_1g_{3,0}a_2 \in (p+2k+4,(2p^2+kp+4p+3)p-4),\ if\ p \geqq 5$

45. a). $b_{01}^{k}h_1^{l}(1,2,1) \in (2k+l+4,(2p^2+kp+lp+4p+l+2)q-2k-4),\ 0 \leqq l \leqq p-4$

b). $h_0b_{01}^{k}h_1^{l}(1,2,1) \in (2k+l+5,(2p^2+kp+lp+4p+l+3)q-2k-5),\ 0 \leqq l \leqq p-4$

c). $b_{01}^{k}b_{11}j_l \in (2k+l+6,(2p^2+kp+lp+3p+l+3)q-2k-6),\ 1 \leqq l \leqq p-4$

d). $b_{01}^{k}j_lb_{02} \in (2k+l+6,(2p^2+kp+lp+4p+l+3)q-2k-6),\ 1 \leqq l \leqq p-4$

e). $b_{01}^{k}j_la_2 \in (p+2k+l+4,(2p^2+kp+lp+4p+l+3)q-2k-4),\ 1 \leqq l \leqq p-4$

46. a). $b_{01}^{k}b_{11}x \in (p+2k+2,(3p^2+kp+p-1)q-2k-4)$

b). $h_0b_{01}^{k}b_{11}x \in (p+2k+3,(3p^2+kp+p)q-2k-6)$

47. a). $b_{01}^{k}g_{2,l}b_{11}^{3} \in (2k+l+8,(3p^2+kp+p+l+2)q-2k-8),\ 0 \leqq l \leqq p-4$

b). $b_{01}^{k}(g_{1,p-4}b_{11}^{2}b_{02}-b_{01}k_{2,p-4}b_{02}) \in (p+2k+3,(3p^2+kp+2p-3)q-2k-7),\ if\ p \geqq 5$

c). $b_{01}^{k}b_{11}v_l \in (2k+l+5,(3p^2+kp+p+l+2)q-2k-5),\ 0 \leqq l \leqq p-4$

48. a). $b_{01}^{k}xa_2 \in (2p+2k,(3p^2+kp+2p-1)q-2k-2),\ if\ p \geqq 5$

b). $h_0b_{01}^{k}xa_2 \in (2p+2k+1,(3p^2+kp+2p)q-2k-3),\ if\ p \geqq 5$

c). $h_0b_{01}^{k}xb_{02} \in (p+2k+3,(3p^2+kp+2p)q-2k-5)\ if\ p \geqq 5$

$b_{01}^{k}(h_0x+a_0h_2b_{02})b_{02} \in (2k+6,10k+127),\ k \geqq 1\ if\ p=3$

d). $b_{01}^{k}k_{1,l}b_{11}^{3} \in (2k+l+8,(3p^2+kp+lp+2p+l+1)q-2k-8),\ 0 \leqq l \leqq 1\ if\ p \geqq 5$

49. a). $g_{1,l}h_2b_{02}^{2} \in (l+6,(3p^2+2p+l+1)q-6),\ 0 \leqq l \leqq p-2\ if\ p \geqq 5$

b). $b_{01}^{k}g_{2,l}b_{11}^{2}b_{02} \in (2k+l+8,(3p^2+kp+2p+l+2)q-2k-8),\ 1 \leqq l \leqq q-3$

c). $g_{1,p-3}b_{01}^{k}b_{11}b_{02}^{2} \in (p+2k+4,(3p^2+kp+3p-2)q-2k-5),\ if\ p \geqq 5$

d). $b_{01}^{k}h_1xb_{02} \in (p+2k+3,(3p^2+kp+3p-1)q-2k-5)$

e). $b_{01}^k h_1 x a_2 \in (2p+2k+1, (3p^2+kp+3p-1)q-2k-3)$

f). $b_{01}^k v_l b_{02} \in (2k+l+5, (3p^2+kp+2p+l+2)q-2k-5)$, $0 \leqq l \leqq p-3$ *if* $p \geqq 5$

g). $h_0 b_{01}^k v_{p-3} b_{02} \in (p+2k+3, (3p^2+kp+3p)q-2k-6)$, *if* $p \geqq 5$

h). $g_{1,p-3} b_{01}^k b_{11} b_{02} a_2 \in (2p+2k+2, (3p^2+kp+3p-2)q-2k-5)$

i). $b_{01}^k g_{2,l} b_{11}^2 a_2 \in (p+2k+l+6, (3p^2+kp+2p+l+2)q-2k-4)$, $0 \leqq l \leqq p-3$

50. a). $b_{01}^k b_{11} h_2 g_{3,0} \in (2k+6, (3p^2+kp+2p+3)q-2k-6)$, *if* $p \geqq 5$

$b_{11} h_2 g_{3,l} \in (l+6, (3p^2+2p+l+3)q-5)$, $1 \leqq l \leqq p-4$

b). $b_{01}^k g_{2,l} b_{11} a_2^2 \in (2p+2k+l+4, (3p^2+kp+3p+l+2)q-2k-4)$, $0 \leqq l \leqq p-3$, $k \leqq 1$

c). $g_{1,l} a_1 x_2 \in (2p+l+2, (3p^2+3p+l+1)q-3)$, $0 \leqq l \leqq p-2$ *if* $p \geqq 5$

d). $k_{1,0} b_{11}^2 b_{02} \in (8, (3p^2+3p+1)q-8)$, *if* $p \geqq 5$

e). $b_{01}^k k_{1,0} b_{11}^2 a_2 \in (p+2k+6, (3p^2+kp+3p+1)q-2k-6)$

f). $g_{1,l} G b_{02} \in (p+l+4, (3p^2+3p+l+1)q-5)$, $0 \leqq l \leqq p-2$ *if* $p \geqq 5$

g). $g_{2,p-3} b_{11} b_{02}^2 \in (p+5, (3p^2+4p-1)q-8)$, *if* $p \geqq 5$

h). $k_{2,0} b_{11}^2 \in (7, (3p^2+3p+1)q-7)$, *if* $p \geqq 5$

i). $b_{01}^k g_{2,l} x_2 \in (p+2k+l+3, (3p^2+kp+3p+l+2)q-2k-4)$, $0 \leqq l \leqq p-3$

j). $h_0 g_{2,0} x_2 \in (7, 151)$, *if* $p=3$

k). $g_{3,l} b_{11} a_2 \in (p+l+5, (3p^2+3p+l+3)q-5)$, $0 \leqq l \leqq p-4$

51. a). $b_{01}^k y \in (2k+3, (3p^2+kp+2p+1)q-2k-3)$, *if* $p \geqq 5$

b). $h_0 b_{01}^k y \in (2k+4, (3p^2+kp+2p+2)q-2k-4)$, *if* $p \geqq 5$

c). $h_1 y \in (4, (3p^2+3p+1)q-4)$, *if* $p \geqq 5$

d). $h_2 g_{3,l} b_{02} \in (l+6, (3p^2+3p+l+3)q-6)$, $0 \leqq l \leqq p-4$ *if* $p \geqq 5$

e). $g_{2,0} y \in (5, (3p^2+3p+3)q-5)$, *if* $p \geqq 5$

f). $g_{2,l} b_{11} b_{02} a_2 \in (p+l+6, (3p^2+3p+l+2)q-6)$, $1 \leqq l \leqq p-3$ *if* $p \geqq 5$

From now on, we assume $p=3$.

52. a). $a_0^i b_{21} \in (i+2, 106)$, $0 \leqq i \leqq 13$

$a_0^i b_{01} b_{21} \in (i+4, 116)$, $0 \leqq i \leqq 1$

$b_{01}^2 b_{21} \in (6, 126)$

b). $g_{1,l} b_{01}^k b_{21} \in (2k+l+3, 10k+4l+109)$, $0 \leqq l \leqq 1$, $k \leqq 1$

53. a). $a_0^i h_3 \in (i+1, 107)$, $0 \leqq i \leqq 26$

$a_0^i b_{01}^k h_3 \in (2k+i+1, 10k+107)$, $0 \leqq i \leqq 1$, $1 \leqq k \leqq 2$

$a_0^i b_{01}^k a_1 h_3 \in (2k+i+4, 10k+119)$, $0 \leqq i \leqq 1$, $1 \leqq k \leqq 3$

$a_0^i b_{01}^k a_1^2 h_3 \in (2k+i+7, 10k+131)$, $0 \leqq i \leqq 1$, $1 \leqq k \leqq 2$

b). $g_{1,l} b_{01}^k a_1^j h_3 \in (2k+3j+l+2, 10k+12j+4l+110)$, $0 \leqq l \leqq 1$, $k \leqq 2$, $j \leqq 2$

54. a). $a_0^i h_1 b_{21} \in (i+3, 117)$, $0 \leqq i \leqq 2$

b). $g_{2,0} b_{21} \in (4, 124)$

c). $h_0 g_{2,0} b_{21} \in (5,127)$

55. a). $a_0^i h_1 h_3 \in (i+2,118),\ 0 \leqq i \leqq 2$

$b_{01}^k h_1 h_3 \in (2k+2, 10k+118),\ 1 \leqq k \leqq 2$

b). $b_{01}^k g_{2,0} h_3 \in (2k+3,\ 10k+125),\ 0 \leqq k \leqq 2$

c). $h_0 a_1 b_{21} \in (6,121)$

56. a). $a_0^i u h_3 \in (i+4,130),\ 0 \leqq i \leqq 3$

$a_0^i b_{01}^k a_1^j u h_3 \in (3j+2k+i+4,\ 10k+12j+130),\ 0 \leqq i \leqq 1,\ k \geqq 1$

$a_1^2 u h_3 \in (10,154)$

b). $g_{1,l} b_{01}^k a_1^j u h_3 \in (3j+2k+l+5,\ 4l+10k+12j+133),\ 0 \leqq l \leqq 1$

57. a). $b_{01}^k k_{1,0} h_3 \in (2k+3, 10k+133)$

b). $h_0 b_{01}^k k_{1,0} h_3 \in (2k+4, 10k+136)$

c). $b_{01}^k k_{1,0} b_{11} x \in (2k+7, 10k+138)$

d). $b_{01}^k g_{2,0} b_{11} x \in (2k+7, 10k+130)$

58. a). $a_0^i b_{02}^3 \in (i+6,138),\ 2 \leqq i \leqq 7$

b). $a_0^i b_{11} b_{21} \in (i+4,140),\ 0 \leqq i \leqq 1$

c). $a_0^i b_{11} h_3 \in (i+3,141),\ 0 \leqq i \leqq 4$

$b_{01} b_{11} h_3 \in (5,151)$

d)· $h_0 b_{11} h_3 \in (4,145)$

59. a). $b_{01}^k g_{2,0} a_2 x \in (2k+8, 10k+144)$

b). $b_{01}^k b_{11}^2 x \in (2k+7, 10k+146)$

e). $a_0^i h_1 b_{02}^3 \in (i+7,149),\ 1 \leqq i \leqq 2$

d). $b_{11}^2 d \in (6,150)$

e). $\text{エ} \in (5,151)$

60. a). $k_{1,0} a_2 x \in (8,152)$

b). $h_0 k_{1,0} a_2 x \in (9,155)$

c). $a_1 (G b_{02} - b_{01} x_2) \in (9,152)$

d). $h_0 a_1 (G b_{02} - b_{01} x_2) \in (10,155)$

e). $k_{1,0} b_{11} a_2^2 \in (10,156)$

f). $a_0^i u x_2 \in (i+7,153),\ 0 \leqq i \leqq 1$

g). $h_0 u x_2 \in (8,156)$

h). $k_{1,0} x_2 \in (6,156)$

i). $h_0 k_{1,0} b_{02} x \in (8,153)$

j). $h_0 h_2 b_{02} w \in (7,154)$

k). $g_{1,1} h_2 b_{02} w \in (8,158)$

REMARK. In [3], J.P.May calculated the E_∞ term $E^0H^*(A)$ in the range $t-s \leqq (2p^2+p+2)q-4$. We point out the following differences between Theorem 4.4 and

Theorem II.6.10 [3] :

1. 6 listed in Theorem II.6.10 does not appear in Theorem 4.4.
2. 7.a), 11.a), 12.a), 13.a), 14.b), 14.c), 15.b) and 15.c) listed in Theorem II.6.10 are replaced in Theorem 4.4 by

7.a'). $k_1^l b_1^1 (b_1^0)^k$, $0 \leqq l \leqq p-3$

11.a'). $g_3^l (b_1^0)^k$, $0 \leqq l \leqq p-4$

12.a'). $a_0^i a_1^j w$, $0 \leqq i \leqq p$, $j \not\equiv p-3 \bmod p$ *if* $p > 3$.

$a_0^i m$, $0 \leqq i \leqq 3$, $m = a_1 w - a_0 a_2 b_1$ *if* $p=3$

$a_0^i a_1^j w (b_1^0)^k$, $0 \leqq i \leqq p-2$, $1 \leqq k$

13.a'). $k_1^l b_2^0 (b_1^0)^k$, $0 \leqq l \leqq p-4$

14.b'). $h_0 (b_1^1)^2 (b_1^0)^k$, $p > 3$

14.c'). $g_1^l (b_1^1)^2 (b_1^0)^k$, $1 \leqq l \leqq p-4$

15.b'). $h_0 h_2 b_1^1$, $p > 3$

15.c'). $g_1^l h_2 b_1^1$, $1 \leqq l \leqq p-4$, respectively.

3. The following elements do not appear in Theorem II.6.10.

21. $a_0^i (b_1^0)^k a_1^j u a_2$, $0 \leqq i \leqq p-2$, $k \geqq 1$.

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