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On the Effect of Impurity Correlation on Superconductivity

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Abstract

The effect of correlated impurity spins on superconductivity is investigated by extending the Gor'kov and Rusinov theory and taking account of the effective internal field acting on each localized spins. The superconducting transition temperature T_c is calculated by using a simpler approximation than Bennemann's, and it is found that a co-existence of superconductivity and impurity ferromagnetism is easily obtainable even if the spin-orbit interaction is weak. The dependence of the critical concentration on the effective internal field and the spin-orbit interaction is also examined. It is concluded that T_c cannot be higher than in the Abrikosov and Gor'kov theory, except in the case when the spin-orbit interaction is sufficiently weak and the s-d exchange coupling constant is considerably large in magnitude.

§ 1. Introduction

Many authors have investigated the effect of magnetic impurities on superconductivity and it is observed experimentally that the superconducting transition temperature T_c decreases gradually with the addition of impurities and finally it vanishes at the critical concentration of impurities of the order of smaller than 1%.¹⁾ According to the theory of Abrikosov and Gor'kov (AG), this phenomenon is ascribed to the destruction of Cooper pairs between conduction electrons by localized spins of impurities through the s-d exchange interaction.²⁾ In the theory of AG, localized spins are treated to rotate independently with each other. This treatment is undoubtedly valid when the concentration of impurities c is sufficiently small. When c is relatively large, however, localized spins interact with each other via the RKKY interaction and then correlation between them becomes more important. In this case, the assumption made by AG is no more valid.

The first attempt to consider the effect of correlated impurity spins on superconductivity was done by Gor'kov and Rusinov (GR).³⁾ They assumed conduction electrons are polarized owing to the appearance of ferromagnetic ordering of impurity spins. When the ferromagnetic ordering occurs, impurity spins are not free to rotate

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since they are fixed by the effective internal field. Then the pair-breaking effect due to the s-d exchange interaction is weakened and T_C has a tendency to be higher than that given by AG. On the other hand, Fermi surfaces for different spin directions are shifted oppositely as a consequence of spin polarization of electrons. This leads to the destruction of superconductivity and to reduce T_C stronger than in the AG theory. These two mechanisms compete with each other to affect T_C in the presence of the ferromagnetic ordering. GR examined only the effect contributed from the splitting of Fermi surfaces, and showed the critical concentration is fairly small compared with the AG theory. They also discussed to find a possible region of co-existence of superconductivity and impurity ferromagnetism in the temperature-concentration phase diagram. The co-existence region obtained by them is very narrow restricted to low temperatures and small concentrations. GR and Tsuzuki⁴⁾ pointed out that one needs to include a strong spin-orbit interaction due to nonmagnetic impurities in order to extend this narrow region of co-existence. The effect of the spin-orbit scattering was studied in detail by Fulde and Maki,⁵⁾ who showed that the qualitative behavior of T_C against c is the same as in the theory of AG when the spin-orbit interaction is strong enough. This situation is easily understood if one notes that the spin-orbit scattering does not conserve the electron spin and thus weakens the effect by spin polarization of electrons, which is completely suppressed when the spin-orbit interaction becomes strong infinitely. Even in the presence of a strong spin-orbit interaction, T_C is seen to be always lower than in the case of unpolarized electrons.

Contrary to the expectation from the GR theory, some alloy systems⁶⁾ show that T_C is higher than that predicted by AG. This suggests the pair-breaking effect due to the s-d exchange scattering weakened by the effective internal field acting on impurity spins is more important than the effect arising from spin polarization of electrons. Bennemann⁷⁾ investigated how the pair-breaking effect due to the s-d exchange interaction is decreased by the effective internal field. He determined the dependence of the pair-breaking parameter on the effective internal field and also on the temperature which was ignored in the GR theory. Although he obtained the result consistent with experimental facts, an assumption that the spatial variation of internal fields is approximated by a Lorentzian distribution function does not have a clear physical meaning. Furthermore, his treatment is much complicated to handle. The reduction mechanism in the pair-breaking parameter was reexamined by Keller and Benda.⁸⁾ According to their conclusion, the effective internal field must be anomalously large in order that T_C is higher than expected from the AG theory. They suggested deviations of T_C from the theory of AG observed in some alloy systems⁶⁾ cannot be explained only by the depression in the pair-breaking parameter produced by the effective internal field.

In this paper we study the effect of correlated spins on the pair-breaking mechanism by employing a simpler approximation compared with Bennemann's one.

The presence of the RKKY interaction between localized spins is not taken account of explicitly in all the theoretical works explained above. From the beginning we introduce this interaction, by which the magnetic state of localized spins is considered to be determined independently of conduction electrons. Although we have not so much understood on the impurity ferromagnetism, the molecular field approximation will be applied to the RKKY interaction for simplicity. The spin-orbit interaction between conduction electrons and nonmagnetic impurities is also included.

One of the main purposes in this paper is to show that we are able to obtain a wide region of co-existence of superconductivity and impurity ferromagnetism in the temperature-concentration phase diagram even in the absence of a strong spin-orbit interaction. We also intend to point out that T_C cannot be higher than in the AG theory except in the case when the spin-orbit interaction is absent or weak and the strength of the s-d exchange interaction is considerably large in magnitude.

In §2, we introduce a model Hamiltonian which describes a system consisted of conduction electrons and impurities. Formulations are also presented in a similar way as AG and GR. We calculate the superconducting transition temperature T_C in §3. Using the result obtained above, we examine the concentration dependence of T_C in §4. Moreover, a possibility of co-existence of superconductivity and impurity ferromagnetism is discussed. Also we study how the critical concentration depends on the effective internal field and spin-orbit interaction. In §5, conclusion and further remarks are given.

§2. Model Hamiltonian and Formulations

Let us consider a system consisted of conduction electrons and localized spins of magnetic impurities. The model Hamiltonian which describes this system is

$$H = H_{BCS} + H_{sd} + H_{so} + H_{spin},$$

$$H_{BCS} = \sum_{\mathbf{k}\mu} \epsilon_{\mathbf{k}} C_{\mathbf{k}\mu}^{\dagger} C_{\mathbf{k}\mu} - \Delta \sum_{\mathbf{k}} (C_{\mathbf{k}1}^{\dagger} C_{-\mathbf{k}1} + C_{-\mathbf{k}1} C_{\mathbf{k}1}), \quad (2.1)$$

$$H_{sd} = -\frac{1}{N} \sum_j \sum_{\mathbf{k}, \mathbf{k}'} \sum_{\mu, \mu'} J_{sd}(\mathbf{k} - \mathbf{k}') e^{-i(\mathbf{k} - \mathbf{k}') \cdot \mathbf{R}_j} S_j \cdot \boldsymbol{\sigma}_{\mu\mu'} C_{\mathbf{k}\mu}^{\dagger} C_{\mathbf{k}'\mu'}, \quad (2.2)$$

$$H_{so} = \frac{1}{N} \sum_{\ell} \sum_{\mathbf{k}, \mathbf{k}'} \sum_{\mu, \mu'} U_{so}(\mathbf{k} - \mathbf{k}') e^{-i(\mathbf{k} - \mathbf{k}') \cdot \mathbf{R}_{\ell}} i(\mathbf{n} \times \mathbf{n}') \cdot \boldsymbol{\sigma}_{\mu\mu'} C_{\mathbf{k}\mu}^{\dagger} C_{\mathbf{k}'\mu'}, \quad \mathbf{n} = \mathbf{k}/|\mathbf{k}|, \quad (2.3)$$

$$H_{spin} = - \sum_{j > j'} J_{RKKY}(\mathbf{R}_j - \mathbf{R}_{j'}) S_j \cdot S_{j'}. \quad (2.4)$$

Here $C_{\mathbf{k}\mu}^{\dagger} (C_{\mathbf{k}\mu})$ is the creation (annihilation) operator of a conduction electron with

the wave number \mathbf{k} and spin μ (\uparrow or \downarrow). S_j is the spin operator of an impurity atom located at the lattice site \mathbf{R}_j , and σ is the 2×2 Pauli matrix with respect to the spin indices of the electronic operator.

H_{BCS} is the usual BCS Hamiltonian, in which $\epsilon_{\mathbf{k}}$ is the kinetic energy of the conduction electron measured from the Fermi surface and Δ is the superconducting order parameter. The quantity Δ is given by solving the self-consistent equation,

$$\Delta = \frac{g}{N} \sum_{\mathbf{k}} \langle C_{\mathbf{k}\uparrow} C_{-\mathbf{k}\downarrow}^+ \rangle, \quad (2.5)$$

where g (>0) is the coupling constant of Cooper pairs ($\mathbf{k}\uparrow, -\mathbf{k}\downarrow$) and $\langle \dots \rangle$ means the thermal average. Equation (2.5) gives us a nonvanishing solution of Δ only at the temperature T lower than the certain (transition) temperature T_c . In other words, conduction electrons are in the superconducting state when $T < T_c$, while in the normal state when $T > T_c$.

Conduction electrons interact with localized spins through the s-d exchange interaction H_{sd} , the strength of which is J_{sd} . We assume impurity atoms which possess a localized spin S are distributed randomly with the fixed number $N_i = cN$ (N ; the total number of atoms in the system). Since eq. (2.2) involves the spin exchange term between conduction electrons and localized spins, and also the Fermi surface sliding term, the s-d exchange interaction is expected to destroy Cooper pairs and to suppress various superconducting properties remarkably. Following the method of AG, in this paper, we will treat with H_{sd} perturbationally up to the second order in J_{sd} . The Kondo effect is not taken account of here.

H_{so} represents the spin-orbit interaction acted on conduction electrons by nonmagnetic impurities whose concentration c_{so} is usually large. As mentioned in the Introduction, the spin-orbit scattering tends to reduce ferromagnetism of electrons and then to prevent Fermi surfaces from splitting for different spin directions. It will be shown later, when the strength U_{so} is infinitely large, the contribution arising from the splitting of Fermi surfaces is completely suppressed. H_{so} will be treated up to the second order in U_{so} and positions \mathbf{T}_i of nonmagnetic impurities are supposed to be distributed at random, in the same way as H_{sd} .

H_{spin} is the RKKY interaction between localized spins which is arising through the presence of conduction electrons. According to the RKKY theory,⁹⁾ the strength J_{RKKY} is proportional to $J_{sd}^2 \cdot \rho_F$, where ρ_F is the density of states at the Fermi surface. Since the concentration of magnetic impurities $c = N_i/N$ is practically small, the average distance \bar{R} between localized spins is expected to be large, so that J_{RKKY} may be approximated roughly by $J_{sd}^2 \cdot \rho_F / \bar{R}^3$. Note \bar{R} is related to c as $4\pi \bar{R}^3/3 = V/N_i = V/(cN)$, if the volume of the system is V . Then we may estimate J_{RKKY} as $J_{RKKY} = \lambda \cdot c J_{sd}^2 \cdot \rho_F$, where λ is a dimensionless quantity of the order of unity.¹⁰⁾ If one adopts the molecular field approximation, the RKKY interaction is replaced by

$$H_{spin} = -h \sum_j S_{jz}, \quad (2.6)$$

where $h = J_{KKY} \langle S_z \rangle$, representing the effective internal field acting on each localized spins.

Other interactions independent of spin such as the potential scattering are neglected since they exert little influences on superconductivity.¹¹⁾

For the aim to investigate various properties of a superconductor containing magnetic impurities, it is more convenient to make use of a Nambu formalism,¹²⁾ in which we use four-component operators for the conduction electron,

$$\Psi_k = \begin{pmatrix} C_{k1} \\ C_{-k1} \\ C_{k1}^+ \\ -C_{k1}^+ \end{pmatrix}, \quad \Psi_k^+ = \begin{pmatrix} C_{k1}^+, & C_{-k1}^+, & C_{-k1}, & -C_{k1} \end{pmatrix}, \quad (2.7)$$

and, in addition to the Pauli matrices σ_i , the other kind of the Pauli matrices ρ_i represented in the particle-hole space of the electronic state.* The notation of direct matrix product $\rho_i \times \sigma_j$, or $\rho_i \sigma_j$ in an abbreviated form will be often used. For example, $\rho_1 \sigma_2 \cdot \Psi_k$ is read as

$$\rho_1 \sigma_2 \cdot \Psi_k = i \begin{pmatrix} C_{k1}^+ \\ C_{-k1}^+ \\ -C_{-k1} \\ C_{k1} \end{pmatrix}$$

In terms of the above notation, eqs.(2.1), (2.2) and (2.3) are expressed as

$$H_{BCS} = \sum_k \Psi_k^+ (\varepsilon_k \rho_3 - \Delta \rho_1) \Psi_k \quad (2.8)$$

$$H_{sd} = -\frac{1}{N} \sum_j \sum_{k,k'} J_{sd}(k-k') e^{-i(k-k') \cdot R_j} \Psi_k^+ S_j \cdot \sigma \Psi_{k'}, \quad (2.9)$$

$$H_{so} = \frac{1}{N} \sum_\ell \sum_{k,k'} U_{so}(k-k') e^{-i(k-k') \cdot T_\ell} \Psi_k^+ \rho_3 i(n \times n') \cdot \sigma \Psi_{k'}, \quad (2.10)$$

respectively.

We now introduce the single-particle Green function,¹³⁾

$$\hat{G}_k(\omega_n) = \ll \Psi_k | \Psi_k^+ \gg_{\omega_n}, \quad (2.11)$$

$$\omega_n = (2n+1) \pi T, \quad n=0, \pm 1, \pm 2, \dots,$$

which is in general useful to derive many informations from the model Hamiltonian. The symbol "roof" above the letter G indicates that \hat{G} is a matrix of the dimension 4×4 . Hereafter, we will use this symbol which denotes 4×4 matrices to avoid the misunderstanding.

*A somewhat different notation from ours is used frequently in other references. Our notation, however, seems much easier to handle and any differences are not found in the final result.

The self-energy $\hat{\Sigma}$ is related to \hat{G}_n as

$$\hat{G}_n^{-1}(\omega_n) = \hat{g}_n^{-1}(\omega_n) - \hat{\Sigma}(\omega_n), \quad (2.12)$$

where \hat{g}_n is the unperturbed Green function without any interactions;

$$\hat{g}_n^{-1}(\omega_n) = i\omega_n - \varepsilon_k \rho_3 + \Delta \rho_1. \quad (2.13)$$

In the Born approximation where H_{sd} and H_{so} are treated perturbationally up to the second order in J_{sd} and U_{so} , respectively, the self-energy $\hat{\Sigma}$ is given by

$$\begin{aligned} & \hat{\Sigma}_{Born}(\omega_n) \\ &= -cJ_{sd}(0) \langle S_z \rangle \sigma_3 \\ &+ c \frac{1}{N} \sum_{\mathbf{k}'} |J_{sd}(\mathbf{k} - \mathbf{k}')|^2 \cdot \{ \langle S_z^2 \rangle \sigma_3 \hat{g}_{k'}(\omega_n) \sigma_3 \\ &+ \langle S_x^2 + S_y^2 \rangle [\sigma_- \hat{g}_{k'}(\omega_n + i\hbar) \sigma_+ + \sigma_+ \hat{g}_{k'}(\omega_n - i\hbar) \sigma_-] \} \\ &+ c_{so} \frac{1}{N} \sum_{\mathbf{k}'} \frac{1}{3} |U_{so}(\mathbf{k} - \mathbf{k}')|^2 \\ &\cdot \rho_3 [\sigma_1 \hat{g}_{k'}(\omega_n) + \sigma_2 \hat{g}_{k'}(\omega_n) \sigma_2 + \sigma_3 \hat{g}_{k'}(\omega_n) \sigma_3] \rho_3, \end{aligned} \quad (2.14)$$

where $\sigma_{\pm} = (\sigma_x \pm i\sigma_y) / 2$.

Following the treatment of AG, we replace the unperturbed Green function $\hat{g}_{k'}$ involved in the above equation by the true Green function $\hat{G}_{k'}$. Then, the self-energy is renormalized as

$$\begin{aligned} & \hat{\Sigma}(\omega_n) \\ &= -I \sigma_3 \\ &+ \frac{1}{\tau_{ex}} [\langle S_z^2 \rangle \sigma_3 \hat{F}(\omega_n) \sigma_3 + \langle S_x^2 + S_y^2 \rangle \hat{F}_{\pm}(\omega_n)] \\ &+ \frac{1}{3} \cdot \frac{1}{\tau_{so}} \rho_3 [\sigma_1 \hat{F}(\omega_n) \sigma_1 + \sigma_2 \hat{F}(\omega_n) \sigma_2 + \sigma_3 \hat{F}(\omega_n) \sigma_3] \rho_3, \end{aligned} \quad (2.15)$$

where

$$I = cJ_{sd}(0) \langle S_z \rangle, \quad (2.16a)$$

$$\frac{1}{\tau_{ex}} = \frac{1}{4} c \rho_F \int d\Omega |J_{sd}(\mathbf{k} - \mathbf{k}')|^2, \quad (2.16b)$$

$$\frac{1}{\tau_{so}} = \frac{1}{4} c_{so} \rho_F \int d\Omega \sin^2(\theta_{k-k'}) |U_{so}(\mathbf{k} - \mathbf{k}')|^2, \quad (2.16c)$$

$$\hat{F}(\omega_n) = \frac{1}{\pi} \int d\varepsilon_k \hat{G}_k(\omega_n), \quad (2.17a)$$

and

$$\hat{F}_{\pm}(\omega_n) = \sigma_- \hat{F}(\omega_n + ih) \sigma_+ + \sigma_+ \hat{F}(\omega_n - ih) \sigma_- . \quad (2.17b)$$

Here τ_{ex} and τ_{so} are the relaxation times corresponding to the s-d exchange and spin-orbit interactions, respectively, while I is the parameter causing the shift of Fermi surfaces for each spin directions. The parameter h comes out in the spin-flip term with the factor $\langle S_x^2 + S_y^2 \rangle$. Once the scattering process accompanied by the spin-flip is occurred, the localized spin feels the effective field of a more or less strength by h than just before the scattering. We should take account of this situation and thus have included the energy difference h of the localized spin between the initial state and the intermediate one in deriving the above equation for the self-energy $\hat{\Sigma}$. This effect is not considered in the GR theory. Naturally, if one puts $h=0$ in eq.(2.15), the same expression as GR is obtained for $\hat{\Sigma}$.

We must solve eqs.(2.12), (2.15), (2.17a) and (2.17b) self-consistently. We suppose that the true Green function \hat{G}_k takes the form,

$$\begin{aligned} \hat{G}_k^{-1}(\omega_n) = & (i\tilde{\omega}_{n+} - \tilde{\epsilon}_{k+}\rho_3 + \tilde{\Delta}_{n+}\rho_1) \cdot \frac{1}{2} (1 + \sigma_3) \\ & + (i\tilde{\omega}_{n-} - \tilde{\epsilon}_{k-}\rho_3 + \tilde{\Delta}_{n-}\rho_1) \cdot \frac{1}{2} (1 - \sigma_3), \end{aligned} \quad (2.18)$$

in terms of which eq.(2.17a) is calculated as

$$\begin{aligned} \hat{F}(\omega_n) = & -\frac{1}{\pi} \int d\epsilon_k \frac{i\tilde{\omega}_{n+} + \tilde{\epsilon}_{k+}\rho_3 - \tilde{\Delta}_{n+}\rho_1}{\tilde{\omega}_{n+}^2 + \tilde{\epsilon}_{k+}^2 + \tilde{\Delta}_{n+}^2} \cdot \frac{1}{2} (1 + \sigma_3) \\ & -\frac{1}{\pi} \int d\epsilon_k \frac{i\tilde{\omega}_{n-} + \tilde{\epsilon}_{k-}\rho_3 - \tilde{\Delta}_{n-}\rho_1}{\tilde{\omega}_{n-}^2 + \tilde{\epsilon}_{k-}^2 + \tilde{\Delta}_{n-}^2} \cdot \frac{1}{2} (1 - \sigma_3) \\ = & -\frac{i\tilde{\omega}_{n+} + \delta_+\rho_3 - \tilde{\Delta}_{n+}\rho_1}{s\sqrt{\tilde{\omega}_{n+}^2 + \tilde{\Delta}_{n+}^2}} \cdot \frac{1}{2} (1 + \sigma_3) \\ & -\frac{i\tilde{\omega}_{n-} + \delta_-\rho_3 - \tilde{\Delta}_{n-}\rho_1}{s\sqrt{\tilde{\omega}_{n-}^2 + \tilde{\Delta}_{n-}^2}} \cdot \frac{1}{2} (1 - \sigma_3), \end{aligned} \quad (2.19)$$

where

$$s\sqrt{\tilde{\omega}_{n\pm}^2 + \tilde{\Delta}_{n\pm}^2} = \sqrt{\tilde{\omega}_{n\pm}^2 + \tilde{\Delta}_{n\pm}^2} \cdot \text{sgn Re} \sqrt{\tilde{\omega}_{n\pm}^2 + \tilde{\Delta}_{n\pm}^2},$$

and $\delta_{\pm} = \tilde{\epsilon}_{k\pm} - \epsilon_k$, being assumed negligibly small.

Substituting eq.(2.19) into eqs.(2.15) and (2.17b), we find

$$\begin{aligned} \hat{\Sigma}(\omega_n) = & -\left[I + \frac{\langle S_z^2 \rangle}{\tau_{ex}} \cdot \frac{i\tilde{\omega}_{n+} + \delta_+\rho_3 - \tilde{\Delta}_{n+}\rho_1}{s\sqrt{\tilde{\omega}_{n+}^2 + \tilde{\Delta}_{n+}^2}} \right. \\ & + \frac{\langle S_x^2 + S_y^2 \rangle}{\tau_{ex}} \cdot \frac{i\tilde{\omega}_{n-} + h + \delta_-\rho_3 - \tilde{\Delta}_{n-}\rho_1}{s\sqrt{(\tilde{\omega}_{n-} - ih)^2 + \tilde{\Delta}_{n-}^2}} \\ & \left. + \frac{1}{3} \cdot \frac{1}{\tau_{so}} \cdot \frac{i\tilde{\omega}_{n+} + \delta_+\rho_3 + \tilde{\Delta}_{n+}\rho_1}{s\sqrt{\tilde{\omega}_{n+}^2 + \tilde{\Delta}_{n+}^2}} \right] \end{aligned}$$

$$\begin{aligned}
& + \frac{2}{3} \cdot \frac{1}{\tau_{so}} \cdot \frac{i\tilde{\omega}_{n-} + \delta_- \rho_3 + \tilde{\Delta}_{n-} \rho_1}{s \sqrt{\tilde{\omega}_{n-}^2 + \tilde{\Delta}_{n-}^2}} \cdot \frac{1}{2} (1 + \sigma_3) \\
& - [\dots] \cdot \frac{1}{2} (1 - \sigma_3). \quad (2.20)
\end{aligned}$$

We have here omitted to write down explicitly $[\dots]$, which is obtained by interchanging suffices \pm and changing signs of I and h in the corresponding expression in front of $(1 + \sigma_3)/2$. Therefore, one can express the renormalized quantities $\tilde{\omega}_{n\pm}$ and $\tilde{\Delta}_{n\pm}$ as

$$\begin{aligned}
\tilde{\omega}_{n\pm} = & \omega_n \mp iI + \left(\frac{\langle S_z^2 \rangle}{\tau_{ex}} + \frac{1}{3} \cdot \frac{1}{\tau_{so}} \right) \frac{\tilde{\omega}_{n\pm}}{s \sqrt{\tilde{\omega}_{n\pm}^2 + \tilde{\Delta}_{n\pm}^2}} \\
& + \frac{\langle S_x^2 + S_y^2 \rangle}{\tau_{ex}} \cdot \frac{\tilde{\omega}_{n\mp} \mp ih}{s \sqrt{(\tilde{\omega}_{n\mp} \mp ih)^2 + \tilde{\Delta}_{n\mp}^2}} \\
& + \frac{2}{3} \cdot \frac{1}{\tau_{so}} \cdot \frac{\tilde{\omega}_{n\mp}}{s \sqrt{\tilde{\omega}_{n\mp}^2 + \tilde{\Delta}_{n\mp}^2}}, \quad (2.21)
\end{aligned}$$

and

$$\begin{aligned}
\tilde{\Delta}_{n\pm} = & \Delta - \left(\frac{\langle S_z^2 \rangle}{\tau_{ex}} - \frac{1}{3} \cdot \frac{1}{\tau_{so}} \right) \frac{\tilde{\Delta}_{n\pm}}{s \sqrt{\tilde{\omega}_{n\pm}^2 + \tilde{\Delta}_{n\pm}^2}} \\
& - \frac{\langle S_x^2 + S_y^2 \rangle}{\tau_{ex}} \cdot \frac{\tilde{\Delta}_{n\mp}}{s \sqrt{(\tilde{\omega}_{n\mp} \mp ih)^2 + \tilde{\Delta}_{n\mp}^2}} \\
& + \frac{2}{3} \cdot \frac{1}{\tau_{so}} \cdot \frac{\tilde{\Delta}_{n\mp}}{s \sqrt{\tilde{\omega}_{n\mp}^2 + \tilde{\Delta}_{n\mp}^2}}, \quad (2.22)
\end{aligned}$$

respectively. These equations for $\tilde{\omega}_{n\pm}$ and $\tilde{\Delta}_{n\pm}$ seem too complicated to be solved simultaneously. For the calculation of the superconducting transition temperature T_c , however, it is sufficient to seek for the solutions with respect to the lowest order in Δ , which are easily obtained as seen in the next section.

§ 3. Superconducting Transition Temperature T_c

In this section we calculate the superconducting transition temperature T_c by making use of eqs. (2.5), (2.21) and (2.22). By the definition of the Green function, eq. (2.5) is rewritten as

$$\begin{aligned}
\Delta = & \frac{g}{N} \sum_{\mathbf{k}} T \sum_n \text{Tr} [\rho_1 \cdot \hat{G}_{\mathbf{k}}(\omega_n)] \\
= & g \rho_F \cdot \pi T \sum_n \text{Tr} [\rho_1 \cdot \hat{F}(\omega_n)] \\
= & g \rho_F \cdot \pi T \sum_n \frac{1}{2} \left\{ \frac{\tilde{\Delta}_{n+}}{s \sqrt{\tilde{\omega}_{n+}^2 + \tilde{\Delta}_{n+}^2}} + \frac{\tilde{\Delta}_{n-}}{s \sqrt{\tilde{\omega}_{n-}^2 + \tilde{\Delta}_{n-}^2}} \right\}. \quad (3.1)
\end{aligned}$$

Since the superconducting order parameter Δ vanishes at $T = T_C$, one obtains

$$\frac{1}{g\rho_F} = \pi T_C \sum_n \frac{1}{2} \lim_{\Delta \rightarrow 0} \frac{1}{\Delta} \left\{ \frac{\tilde{\Delta}_{n+}}{\tilde{\omega}_{n+}} \operatorname{sgn} \operatorname{Re} \tilde{\omega}_{n+} + \frac{\tilde{\Delta}_{n-}}{\tilde{\omega}_{n-}} \operatorname{sgn} \operatorname{Re} \tilde{\omega}_{n-} \right\}. \quad (3.2)$$

This equation determines T_C and implies that we need to consider the ratio $\tilde{\Delta}_{n\pm}/\tilde{\omega}_{n\pm}$ only up to the first order in Δ . When $\Delta \rightarrow 0$, eqs. (2.21) and (2.22) are approximated by

$$\begin{aligned} \tilde{\omega}_{n\pm} = \omega_n \mp iI + \left(\frac{\langle S_z^2 \rangle}{\tau_{ex}} + \frac{1}{3} \cdot \frac{1}{\tau_{so}} \right) \operatorname{sgn} \operatorname{Re} \tilde{\omega}_{n\pm} \\ + \left(\frac{\langle S_x^2 + S_y^2 \rangle}{\tau_{ex}} + \frac{2}{3} \cdot \frac{1}{\tau_{so}} \right) \operatorname{sgn} \operatorname{Re} \tilde{\omega}_{n\mp}, \end{aligned} \quad (3.3)$$

and

$$\begin{aligned} \tilde{\Delta}_{n\pm} = \Delta - \left(\frac{\langle S_z^2 \rangle}{\tau_{ex}} - \frac{1}{3} \cdot \frac{1}{\tau_{so}} \right) \frac{\tilde{\Delta}_{n\pm}}{\tilde{\omega}_{n\pm}} \operatorname{sgn} \operatorname{Re} \tilde{\omega}_{n\pm} \\ - \left\{ \frac{\langle S_x^2 + S_y^2 \rangle}{\tau_{ex}} \cdot \frac{\tilde{\Delta}_{n\mp}}{\tilde{\omega}_{n\mp} \mp ih} - \frac{2}{3} \cdot \frac{1}{\tau_{so}} \cdot \frac{\tilde{\Delta}_{n\mp}}{\tilde{\omega}_{n\mp}} \right\} \operatorname{sgn} \operatorname{Re} \tilde{\omega}_{n\mp}, \end{aligned} \quad (3.4)$$

respectively. From eq. (3.3), one can get immediately

$$\operatorname{sgn} \operatorname{Re} \tilde{\omega}_{n\pm} = \operatorname{sgn} \omega_n. \quad (3.5)$$

Then, if we introduce a new variable $u_{n\pm} \equiv \tilde{\omega}_{n\pm} / \tilde{\Delta}_{n\pm}$, eqs. (3.3) and (3.4) are reduced to

$$\begin{aligned} u_{n\pm} \cdot \Delta = \omega_n \mp iI + \left\{ \frac{S(S+1) + \langle S_z^2 \rangle}{\tau_{ex}} + \frac{2}{3} \cdot \frac{1}{\tau_{so}} \right\} \operatorname{sgn} \omega_n \\ + \frac{u_{n\pm}}{u_{n\mp}} \left\{ \frac{\langle S_x^2 + S_y^2 \rangle}{\tau_{ex}} \cdot \frac{1}{1 \mp \frac{ih}{\tilde{\omega}_{n\mp}}} - \frac{2}{3} \cdot \frac{1}{\tau_{so}} \right\} \operatorname{sgn} \omega_n \\ = \frac{(|\omega_n| + a)^2 + I^2 - |b_{n\pm}|^2}{\omega_n \pm iI + (a - b_{n\mp}) \operatorname{sgn} \omega_n}, \end{aligned} \quad (3.6)$$

where

$$a = \frac{1}{\tau_{ex}} \left\{ S(S+1) + \langle S_z^2 \rangle \right\} + \frac{2}{3} \cdot \frac{1}{\tau_{so}}, \quad (3.7a)$$

$$b_{n\pm} = \frac{\langle S_x^2 + S_y^2 \rangle}{\tau_{ex}} \cdot \frac{1}{1 \pm \frac{ih}{\tilde{\omega}_{n\pm}}} - \frac{2}{3} \cdot \frac{1}{\tau_{so}}. \quad (3.7b)$$

Thus, substituting eq. (3.6) into eq. (3.2), we find

$$\ln \frac{T_C}{T_{C0}} = 2\pi T_C \sum_{n \geq 0} \left(\frac{1}{x_n} - \frac{1}{\omega_n} \right), \quad (3.8)$$

where

$$\frac{1}{x_n} = \frac{\omega_n + \left\{ \frac{2\langle S_z^2 \rangle}{\tau_{ex}} + \frac{4}{3} \cdot \frac{1}{\tau_{so}} - f_n \right\} \operatorname{sgn} \omega_n}{\left\{ |\omega_n| + \frac{2S(S+1)}{\tau_{ex}} \right\} \left\{ |\omega_n| + \frac{2\langle S_z^2 \rangle}{\tau_{ex}} + \frac{4}{3} \cdot \frac{1}{\tau_{so}} \right\} + I^2 - g_n}, \quad (3.9)$$

$$f_n = \frac{\langle S_x^2 + S_y^2 \rangle}{\tau_{ex}} \cdot \frac{(I-h)h}{\tilde{\omega}_n^2 + (I-h)^2}, \quad (3.10a)$$

$$g_n = \left[\frac{\langle S_x^2 + S_y^2 \rangle}{\tau_{ex}} \right]^2 \cdot \frac{(2I-h)h}{\tilde{\omega}_n^2 + (I-h)^2} - \frac{4}{3} \cdot \frac{1}{\tau_{so}} f_n, \quad (3.10b)$$

$$\tilde{\omega}_n = \omega_n + \left\{ \frac{S(S+1)}{\tau_{ex}} + \frac{1}{\tau_{so}} \right\} \text{sgn } \omega_n. \quad (3.10c)$$

Here T_{c0} is the transition temperature of a pure superconductor, and use of the relations,

$$\frac{1}{g\rho_F} = \ln \frac{2\gamma\omega_D}{\pi T_{c0}} \quad \text{and} \quad 2\pi T_c \cdot \sum_{\omega_n \geq 0} \frac{\omega_D}{\omega_n} = \ln \frac{2\gamma\omega_D}{\pi T_c}, \quad (3.11)$$

has been made in deriving eq.(3.8). $\gamma = 1.781$ is the Euler constant and ω_D the Debye frequency.

If we consider the case of $h=I=0$, eq.(3.8) reduces to

$$\ln \frac{T_c}{T_{c0}} = \sum_{n \geq 0} \left\{ \frac{1}{n + \frac{1}{2} + \frac{S(S+1)}{\pi T_c \tau_{ex}}} - \frac{1}{n + \frac{1}{2}} \right\}, \quad (3.12)$$

which is the same expression as obtained by AG.²⁾ It is to be noted that the relaxation time τ_{so} does not appear in eq.(3.12). According to this AG equation, T_c decreases monotonously with the concentration of magnetic impurities and it vanishes at the critical concentration given by

$$c_0^{AG} = c \frac{\pi T_{c0} \cdot \tau_{ex}}{4\gamma S(S+1)}. \quad (3.13)$$

In the case when $h=0$ but $I \neq 0$, the same equation as GR³⁾ is derived from eqs. (3.8) and (3.9). It is therefore expected that our result includes both ones of AG and GR.

When the spin-orbit interaction becomes infinitely strong, the relaxation time τ_{so} is very small and then we find eq.(3.8) is identical to eq.(3.12). This is consistent with the conclusion obtained by Fulde and Maki,⁵⁾ and means the effect on the pair-breaking mechanism produced by the splitting of Fermi surfaces is completely suppressed in the limit of $\tau_{so} \rightarrow 0$. The critical concentration, in this limit, coincides with eq.(3.13) and consequently we can hardly expect that T_c is higher than in the AG theory.

§ 4. Discussions

Before examining in detail the result obtained in the preceding section, we give a brief comment on the impurity ferromagnetism.

When conduction electrons exist in the normal state, it is easily understood that conduction electrons exert no influence on ferromagnetism of localized spins. On the other hand, in the case when conduction electrons are in the superconducting

state, the magnetic state of localized spins is slightly affected by them, since the energy needed to destroy ferromagnetism is of the order of J_{RKKY} per an atom which is much larger than the one needed to suppress superconductivity being of the order of $T_C^2 \cdot \rho_F$. This statement is not valid in the case $c \rightarrow 0$, but we are not interested in this case. Thus we cannot consider that the impurity ferromagnetism is affected by superconductivity in practice. Then, ferromagnetism of localized spins is formed irrespectively to whether the state of conduction electrons is superconducting or not, and it is determined only by the RKKY interaction.

If the molecular field approximation is applied for the RKKY interaction, the ferromagnetic ordering temperature T_M , at which the ferromagnetic ordering of localized spins $\langle S_z \rangle$ vanishes, is given by

$$T_M = \frac{1}{3} J_{RKKY} S(S+1) . \quad (4.1)$$

As noted in §2., J_{RKKY} is proportional to the concentration c of magnetic impurities so that the ferromagnetic ordering temperature T_M decreases linearly with c . In the theory of GR, however, T_M is not always a linearly decreasing function of c . Especially, T_M varies as c^2 for sufficiently small c in the absence of the spin-orbit interaction. Such a behavior of T_M against c makes it difficult for us to find a possibility of co-existence of superconductivity and impurity ferromagnetism. Only when the spin-orbit interaction is very strong, T_M is seen to be a linear function of c and a co-existence of two phases becomes possible extensively. On the contrary, in our treatments, a linear-dependence of T_M is always maintained, as seen in eq.(4.1), so that superconductivity and impurity ferromagnetism are expected to co-exist with relative ease.

When c is relatively small, the ferromagnetic ordering temperature T_M is always smaller than the superconducting transition temperature T_C which is determined by the AG equation (3.12), since the ferromagnetic ordering of localized spins $\langle S_z \rangle$ is absent, namely, $h = I = 0$, at $T = T_C > T_M$. As c is increased gradually, T_M increases linearly, while T_C decreases correspondingly to eq.(3.12). Therefore T_M and T_C curves against c should intersect at a certain point in the temperature-concentration phase diagram. The intersection point (T^*, c^*) is given by

$$\ln \frac{T^*}{T_{C0}} = \sum_{n \geq 0} \left\{ \frac{1}{n + \frac{1}{2} + \frac{3}{2\pi\xi}} - \frac{1}{n + \frac{1}{2}} \right\}, \quad (4.2a)$$

$$\frac{c^*}{c_0^{AG}} = \frac{12\gamma}{2\pi\xi} \cdot \frac{T^*}{T_{C0}} = \frac{3.402}{\xi} \cdot \frac{T^*}{T_{C0}}, \quad (4.2b)$$

$$\xi = \frac{1}{2} J_{RKKY} \cdot \tau_{ex} \quad (4.2c)$$

which is easily derived from eqs.(3.12), (3.13) and (4.1). The dimensionless parameter ξ is independent of the characteristic of the system, since both J_{RKKY} and

τ_{ex}^{-1} are proportional to $cJ_{sd}^2 \cdot \rho_F$. The dependences of T^* and c^* on ζ are shown in Fig. 1.

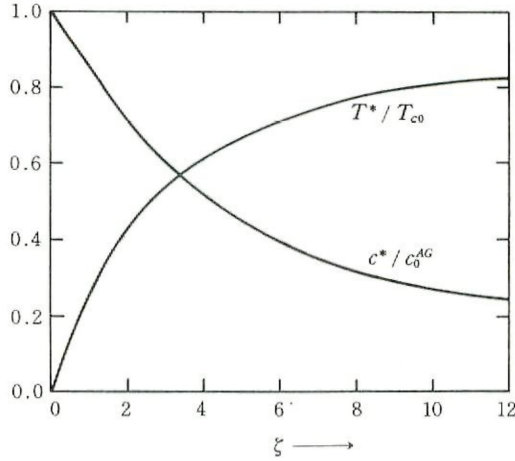


Fig. 1 The dependence of the intersection point of T_C and T_M on the parameter $\zeta = J_{KKY} \cdot \tau_{ex} / 2$.

In order to study the c -dependence of T_C , we have performed numerical calculations by using eqs. (3.8), (3.9) and (3.10). In doing so, it is necessary for us to calculate the thermal averages $\langle S_z \rangle$ and $\langle S_z^2 \rangle$. According to the molecular field approximation, $\langle S_z \rangle$ is given by solving the self-consistent equation,

$$\langle S_z \rangle = SB_S (SJ_{KKY} \langle S_z \rangle / T), \quad (4.3a)$$

where $B_S(x)$ is the Brillouin function defined by

$$B_S(x) = \left(1 + \frac{1}{2S}\right) \coth \left(1 + \frac{1}{2S}\right) x - \frac{1}{2S} \coth \frac{1}{2S} x.$$

In terms of $\langle S_z \rangle$, the average $\langle S_z^2 \rangle$ is expressed as

$$\langle S_z^2 \rangle = S(S+1) - \langle S_z \rangle \coth (J_{KKY} \langle S_z \rangle / 2T). \quad (4.3b)$$

In Figs. 2 and 3, we show the behavior of T_C against c for the values of $J_{sd} \cdot \rho_F = -0.05$ and -0.10 , respectively.* For comparison, the result of the GR theory is also depicted. A similar behavior of T_C against c is found for positive values of $J_{sd} \cdot \rho_F$. As seen in these Figures, a co-existence of superconductivity and impurity ferromagnetism is easily obtained even if the spin-orbit interaction is weak. For the

*The strength of the s-d exchange interaction $J_{sd}(\mathbf{k} \cdot \mathbf{k}')$ is assumed independent of $\mathbf{k} \cdot \mathbf{k}'$ for simplicity.

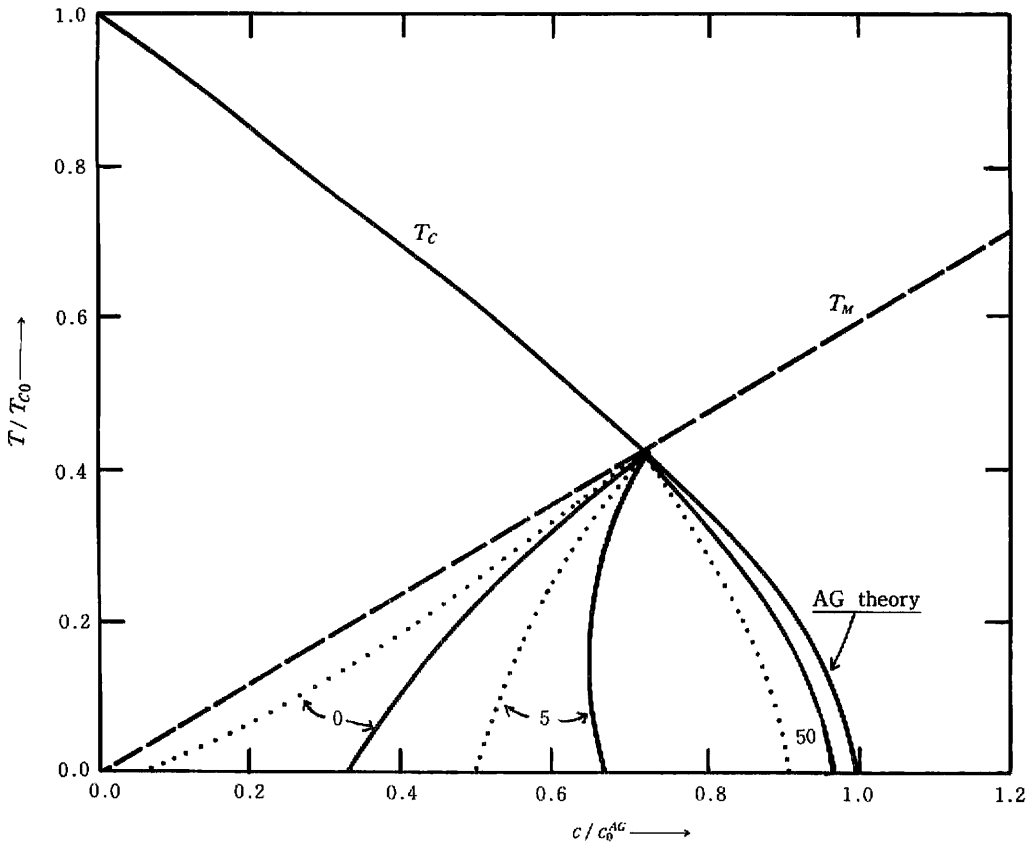


Fig. 2 The behaviors of the superconducting transition temperature T_c and the ferromagnetic ordering temperature T_M against the concentration c of magnetic impurities for $S=1/2$, $J_{sd} \cdot \rho_F = -0.05$, $\zeta = 2$ and various values of $\eta = \tau_{ex}/\tau_{so}$ which are denoted by the numbers attached to each curves. T_{c0} is the transition temperature of a pure superconductor and c_0^{AG} the critical concentration given by the AG theory. The solid curves represent T_c , while the dashed line T_M . For comparison, T_c in the GR theory is also depicted by the dotted curves. A similar behavior of T_c is obtained for the positive value of $J_{sd} \cdot \rho_F$.

reason why one can obtain a possibility of co-existence of two phases more easily compared with the GR theory, we can consider as follows. One is that we have explicitly taken account of the presence of the RKKY interaction between localized spins. In the molecular field approximation, this leads to the c -linear dependence of T_M as noted above, which is favorable to extend a co-existence region in the temperature-concentration phase diagram. The other is the inclusion of the effective internal field h in calculating the spin-flip scattering process for the self-energy $\hat{\Sigma}$ of the Green function. To see how the effective internal field weakens the effect due to

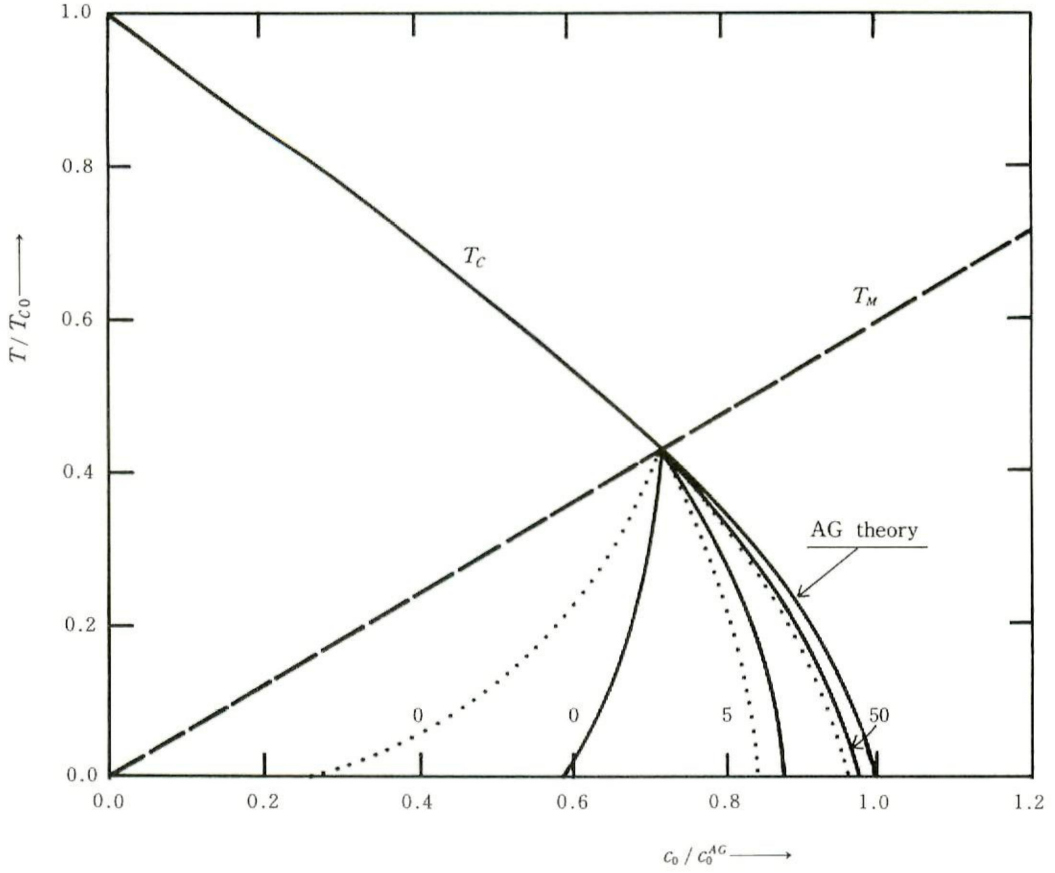


Fig. 3 The behavior of the superconducting transition temperature T_C and the ferromagnetic ordering temperature T_M against the concentration c of magnetic impurities for $S = 1/2$, $J_{sd} \cdot \rho_F = -0.10$, $\xi = 2$ and various values of $\eta = \tau_{ex} / \tau_{so}$. The notations are the same as in Fig. 2.

the splitting of Fermi surfaces, we estimate the slope of T_C just below the intersection point of the T_M and T_C curves. In the absence of the spin-orbit interaction, it is given by, after lengthy calculations,

$$Z_b \equiv \frac{c^*}{T^*} \left[\frac{dT_C}{dc} \right]_{T_C = T_-}, \quad c = c^* = 1 + \frac{1}{Y_b - 1}, \quad (4.4)$$

$$Y_b = \frac{3}{2\pi\xi} \phi'_6 - \frac{10}{3} \cdot \frac{1}{S^2 + (S+1)^2} \cdot \frac{1}{(2\pi J_{sd} \cdot \rho_F)^2} X, \quad (4.5a)$$

$$X = \alpha_0 + \alpha_1 (2\pi\xi J_{sd} \cdot \rho_F) + \alpha_2 (2\pi\xi J_{sd} \cdot \rho_F)^2, \quad (4.5b)$$

$$\alpha_0 = \frac{9}{4} \left(\phi_2 - \phi_6 + \frac{2}{2\pi\xi} \phi'_6 \right), \quad (4.6a)$$

$$\alpha_1 = \frac{4}{3} \left(4\phi_6 - 4\phi_3 - \frac{3}{2\pi\zeta} \phi'_6 - \frac{3}{2\pi\zeta} \phi'_3 \right), \quad (4.6b)$$

$$\alpha_2 = \frac{1}{3} \left(32\phi_3 - 27\phi_2 - 5\phi_6 + \frac{6}{2\pi\zeta} \phi'_6 - \frac{12}{2\pi\zeta} \phi'_3 \right), \quad (4.6c)$$

$$\phi_i = \phi \left(\frac{1}{2} + \frac{1}{2} \cdot \frac{\ell}{2\pi\zeta} \right), \quad (4.7a)$$

$$\phi'_i = \phi' \left(\frac{1}{2} + \frac{1}{2} \cdot \frac{\ell}{2\pi\zeta} \right). \quad (4.7b)$$

Here $\psi(x)$ is the digamma function defined by

$$\psi(x) = -\ln \gamma - \sum_{n=0}^{\infty} \left\{ \frac{1}{n+x} - \frac{1}{n+1} \right\},$$

and $\psi'(x)$ its derivative. One can easily verify that α_0 and α_1 are negative, while α_2 positive. In the GR theory, a similar result is obtained except that $\alpha_1 = \alpha_2 = 0$ and then X is replaced by $X^{GR} = \alpha_0$. The parameters α_1 and α_2 appear in X owing to the inclusion of the effective internal field which is not taken account of in the GR theory. When the condition

$$J_{sd} \cdot \rho_F > \frac{1}{2\pi\zeta} \cdot \frac{|\alpha_1|}{\alpha_2} \quad \text{or} \quad J_{sd} \cdot \rho_F < 0, \quad (4.8)$$

is satisfied, X is larger than X^{GR} . Since $|\alpha_1| / (2\pi\zeta \cdot \alpha_2)$ is seen to be a sufficiently small quantity for $\zeta \gtrsim 0.5$, almost values of $J_{sd} \cdot \rho_F$ satisfy the above condition. For $\zeta < 0.5$, however, a more careful treatment is needed. As shown in Fig. 1, we have $T^*/T_{c0} < 0.15$ for $\zeta < 0.5$ and we will not consider this case still more. Therefore one finds that Y_b and then Z_b^{-1} are smaller than the ones given by the GR theory. This means a higher concentration c of magnetic impurities corresponds to the given value of T_c in comparison with the GR theory. In other words, the effective internal field has a tendency to reduce the effect due to the splitting of Fermi surfaces and then to broaden a co-existence region of superconductivity and impurity ferromagnetism. Even if the spin-orbit interaction is taken account of, this tendency is not altered.

In Fig. 4, we show the dependence of the critical concentration c_0 at which T_c is vanishing on the effective internal field and the spin-orbit interaction. The parameter $\eta = \tau_{ex} / \tau_{so}$ denotes how the spin-orbit interaction is strong compared with the s-d exchange interaction. According to Fig. 4, the critical concentration c_0 does not depend on the strength ζ of the effective internal field so much, except for large values of $J_{sd} \cdot \rho_F$ in magnitude and small η . We find that c_0 is smaller than c_0^{AG} when η is sufficiently large, and it approaches to c_0^{AG} as η is increased to infinity. Then, in the case when the spin-orbit interaction is strong enough, the superconducting transition temperature T_c cannot be higher than in the theory of AG, as conjectured at the end of the preceding section.

Only in the case when $\tau_{so}^{-1} \rightarrow 0$ and $J_{sd} \cdot \rho_F$ is relatively large in magnitude, T_c is expected to exceed the one given by the AG theory. The slope just above the intersection point (T^*, c^*) is calculated as

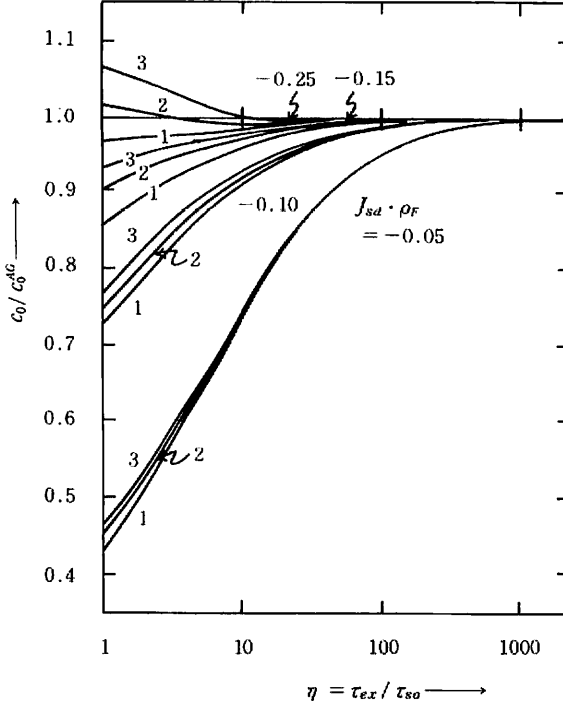


Fig. 4 The dependence of the critical concentration c_0 on the parameter $\eta = \tau_{ex} / \tau_{so}$ for $S = 1/2$, $J_{sd} \cdot \rho_F = -0.05, -0.10, -0.15$ and -0.25 . The numbers attached to each curve denote the values of $\zeta = J_{KKV} \cdot \tau_{ex} / 2$. A similar behavior is obtained for the positive value of $J_{sd} \cdot \rho_F$.

$$Z_a \equiv \frac{c^*}{T^*} \left[\frac{dT_C}{dc} \right]_{T_C = T^*, c = c^*} = 1 + \frac{1}{Y_a - 1}, \quad (4.9)$$

where

$$Y_a = \frac{3}{2\pi\zeta} \psi'_0.$$

In order that the slope of T_C just above the intersection point coincides with the one just below the intersection point, one needs $Z_a = Z_b$ from which the condition $X = 0$ is derived. Then, the T_C curve agrees with the one of the AG theory, at the intersection point, if $J_{sd} \cdot \rho_F$ takes the values equally to

$$\beta_{\pm} = \frac{1}{2\pi\zeta} \cdot \frac{1}{2\alpha_2} \left[-\alpha_1 \pm \sqrt{\alpha_1^2 - 4\alpha_0 \cdot \alpha_2} \right]. \quad (4.10)$$

The values of β_{\pm} are depicted in Fig. 5. For the value of $J_{sd} \cdot \rho_F$ larger than β_{\pm} in magnitude, T_C can be higher than in the AG theory. As Fig. 5 shows, however, the magnitude of β_{\pm} is too large to have a physical meaning. Thus, it is concluded that our treatment cannot give T_C higher than the AG theory if we take an appropriate value of $J_{sd} \cdot \rho_F$.

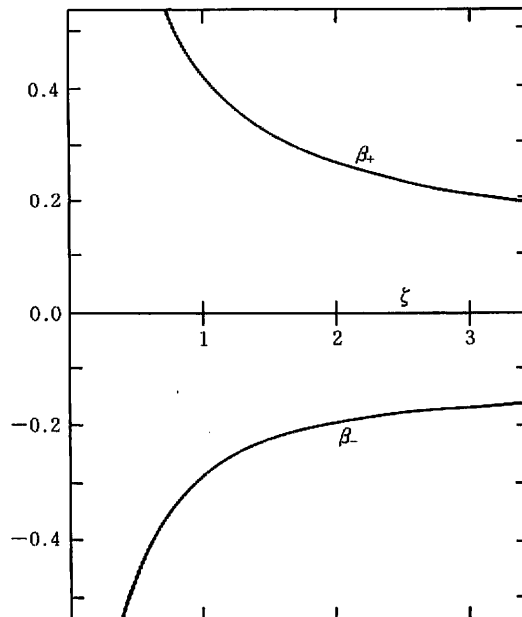


Fig. 5 The values of β_{\pm} as a function of the parameter $\xi = J_{RKKY} \cdot \tau_{ex} / 2$.

§ 5. Concluding Remarks

In the previous sections, we have discussed the effect of correlated impurity spins on superconductivity. There exist various mechanisms which have a considerable influence on the superconducting transition temperature T_c . They are summarized as follows:

- 1) The pair-breaking mechanism due to the s-d exchange interaction.
- 2) Spin polarization of electrons or the splitting of Fermi surfaces which leads to the depression of T_c .
- 3) The spin-orbit interaction which does not conserve the electron spin.
- 4) The effective internal field acting on localized spins of magnetic impurities.

The effect due to the mechanism 1) was first studied by AG, and GR examined how the mechanism 2) competes with 1) in affecting superconductivity, to obtain a possibility of co-existence of superconductivity and impurity ferromagnetism. GR and other authors showed that the effect due to 2) is remarkably suppressed by the presence of 3). In addition to these mechanisms, we have investigated the effect due to 4), and its relation with other mechanisms.

We have shown that the effective internal field reduces the effect due to the splitting of Fermi surfaces and then plays a role in enlarging a possibility of co-existence of superconductivity and impurity ferromagnetism, even in the absence of

a strong spin-orbit interaction.

When the spin-orbit interaction is sufficiently strong, the critical concentration c_0 is smaller than c_0^{AG} given by the theory of AG. This implies T_c is always lower than in the AG theory. Only in the case when the spin-orbit interaction is absent or fairly weak and the strength of the s-d exchange interaction is anomalously large, T_c can be higher than the AG theory predicts. Therefore, we can hardly expect that our treatment gives a result which is consistent with experimental facts.

In order to improve our treatment, it is necessary to calculate the self-energy of the Green function more carefully. We have assumed implicitly the effective internal field is spatially uniform. Since the impurity atoms are distributed randomly in space, the effective internal field should change for each positions of atoms. Then, we need to introduce a distribution function which takes account of the spatial variation of the effective internal field, as in the attempt of Bennemann's.⁷⁾ Elsewhere we will discuss how our treatment is improved by introducing a proper distribution function of the effective internal field.

In addition to the improvement of our treatment, there remains a task to investigate how the effective internal field exerts on other physical quantities such as the upper critical field, the specific heat jump and so on.

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