Periodic Wake behind a Circular Cylinder

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Abstract

The features of periodic wakes behind a circular cylinder in the range of $40 < R < 150$, R being the Reynolds number based on the speed and diameter of the cylinder, are investigated in a water tank using flow-visualization techniques.

The relationship between the Reynolds number (R) and wavelength in the waving-line wake or longitudinal spacing of vortices in the vortex-street (λ^*) is obtained as follows:

$$\lambda^* = 3.25 + 220/R, \qquad 40 < R < 150.$$

It is found, by examining the wake parameters which are derived from the presented empirical formula for λ^* (R) and the Roshko's formula for S (R), that the critical Reynolds number at which the vortex streets begin to form is about 64 and that the vortex streets do not develop in the range of the Reynolds number lower than this critical value.

§ 1. Introduction

The characteristics of the periodic wake behind a two-dimentional body have been investigated by many workers[1)~10)], and various features of the wake have been released besides a summary on the known phenomena found in the treatise by Goldstein[11)]. It seems, however, that the study of the wake structure is not sufficient yet, and that some misunderstandings about the growth and development of the periodic wake still remain.

On the other hand, it has been recently revealed[13), 14)] that the satellite imagery of mesoscale cloud wake in the lee of large oceanic islands bears a close resemblance to the wake behind a cylindrical body observed in laboratory experiments and that the values of various parameters which characterize the atmospheric wake are consistent with those obtained in laboratory investigations. In many cases of atmoshperic wake, the corresponding Reynolds number is not determined directly but is estimated indirectly by the aid of the Roshko's graph[3)] which shows the relationship between the Reynolds number R and Strouhal number S.

It seems that for determination of Reynolds number of atmospheric wakes, it

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is also convenient to use the relationship between the Reynolds number R and dimensionless wavelength or longitudinal spacing of vortices λ^*. On account of the difficulties in measuring $\lambda^*(R)$, however, only a few experimental data are available at present[4),5)].

One of the purposes of this paper is to present an empirical formula representing the relationship between R and λ^* for which no report has been found yet. Another purpose of this paper is to make clear the difference between the periodic non-vortex wakes and vortex-streets by estimating the values of parameters that characterize the periodic wakes.

The experimental equipments and methods were the same as those used in a previous work[12)]. Flow-visualization techniques were used and wake patterns behind a circular cylinder were recorded with a still camera. Experimental results were obtained through photographs of the streak-lines developed by color dyes in a water tank. The experimental conditions were controlled carefully before each experiment.

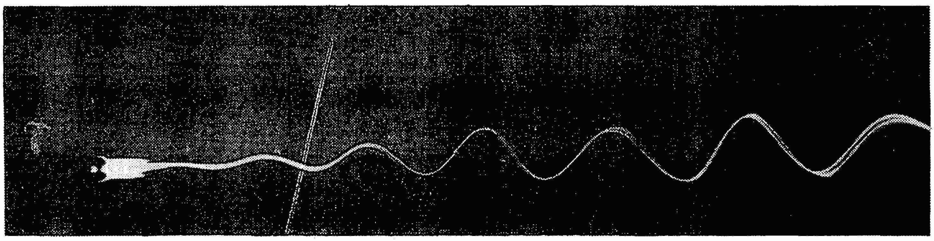
List of symbols

symbols	dimensionless expressions
d=diameter of cylinder	
h=lateral spacing of vortices	$h^*=h/d$
K=rate at which vorticity passes downstream in one row of vortex-streets	$K^*=K/U^2$
N=frequency of waves or vortices shedding in one row of vortex-streets	$N^*=Nd/U=S$ (Strouhal number)
U=velocity of cylinder	
V=velocity of waves or vortices relative to still water	$V^*=V/U$
v=velocity of waves or vortices relative to cylinder $v=U-V$	$v^*=v/U$ $v^*=1-V^*$
λ=wavelength or longitudinal spacing	$\lambda^*=\lambda/d$
ν=kinematic viscosity	$1/\nu^*=Ud/\nu=R$ (Reynolds number)
Γ=strength of vortex	$\Gamma^*=\Gamma/(Ud)$

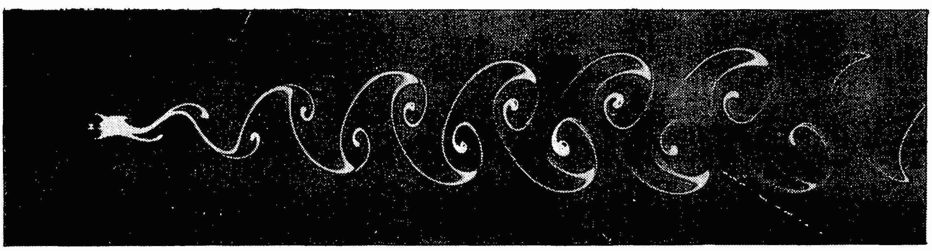
§ 2. Periodic non-vortex wakes and vortex streets

In a previous paper[7)], we emphasized the experimental results that the wake behind a circular cylinder for the range of the Reynolds number $10 < R < 150$

could be classified into quite different patterns as follows : (1) straight-line wake with standing vortex-pair ($R < 30$), (2) waving-line wake with oscillating vortex-pair ($30 < R < 63$), (3) vortex-streets with degenerated vortex-pair ($63 < R < 150$)



(a)



(b)

Plate I. (a) Periodic non-vortex wake (R=62.0)
(b) Periodic vortex-street (R=74.5)
(Cylinders move from right to left.)

The characteristics of the growth and development of these three patterns of wakes were discussed in detail and the differences of the structure in these patterns were made clear[7), 8)].

Out of these three patterns, the last two, (2) and (3), are periodic. Both of these periodic wakes are originally initiated by the vorticity shedding periodically from the vortices, called oscillating vortex pair or degenerated vortex-pair, formed at the rear side of the cylinder, but the wake development downstream are quite different in these two cases.

It should be emphasized that while the wake behind a circular cylinder becomes periodic at the Reynolds number of about 30, the so-called vortex-street patterns first occur at the Reynolds number of about 63[7)]: that is, for the Reynolds number lower than this critical value, only waving-line wakes take place and vortex streets do not develop owing to the weakness of vorticity in the vortex-layer springing out from the vortex-pair formed at the rear of the cylinder[8)].

In Plate I, these two patterns of wake are shown. All photographs of streak-lines have been taken at the horizontal layer of 2.5 cm below the surface of water which was illuminated horizontally through narrow slits from both sides of the tank.

The wake pattern shown in Plate I (a) is an example of periodic non-vortex wakes, the Reynolds number of which is comparatively high but slightly lower than the critical Reynolds number. In the waving-line wake, the amplitude of the waves increases and the wavelength decreases with increasing Reynolds number. The Plate I (b) shows a typical vortex street in a real fluid. In the vortex streets, both the longitudinal and lateral spacings of vortices decrease with increasing Reynolds number.

§ 3. Wavelengths and longitudinal spacings

For a fixed Reynolds number, the value of wavelength of waving-line wake or the longitudinal spacing of vortex streets increases linearly with distance downstream from the cylinder until a stable region is reached where a constant value is maintained, and the value for the further downstream increses again. The distance downstream from the cylinder up to the stable region decreases with increasing

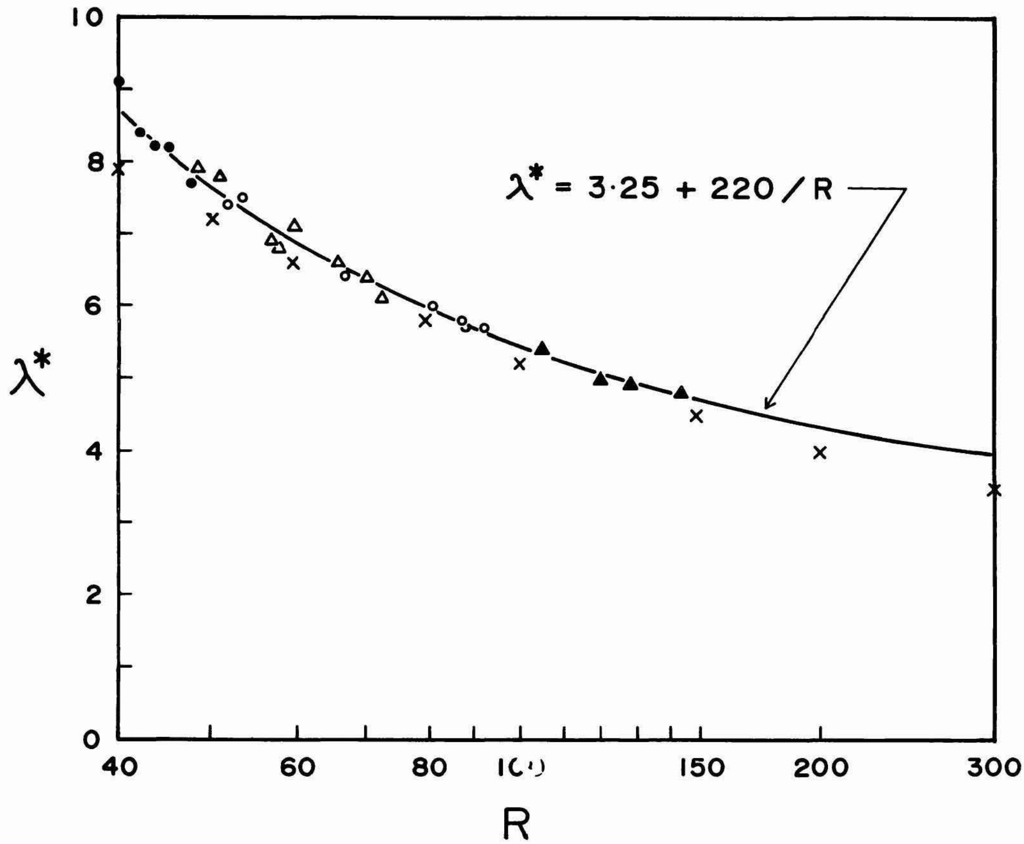


Fig. 1. Wavelengths or longitudinal spacings of vorticies against Reynolds number. ▲, d=9.82 *mm*; △, d=6.65 *mm*; ○, d=5.33 *mm*; ●, d=4.21 *mm*; ×, Taneda's experiment.

Reynolds number. For instance, it is about 30 diameters for the Reynolds number 41, about 20 diameters for the Reynolds number 56, and about 9 diameters for the Reynolds number 135.

The variation with the Reynolds number of dimensionless wavelength or longitudinal spacing λ^*, which was measured at the stable region, is shown in Fig. 1. The relationship between R and λ^* is well represented by the following formula:

$$\lambda^* = 3.25 + 220/R, \qquad 40 < R < 150. \tag{1}$$

§ 4. Velocities of waves and vortex streets

In the case of periodic wakes behind a circular cylinder, not only the vortices in the vortex streets but also the waves in the waving-line wake move towards the cylinder, and there exist the following relations:

$$v = U - V, \tag{2}$$

$$N = v/\lambda = (U-V)/\lambda. \tag{3}$$

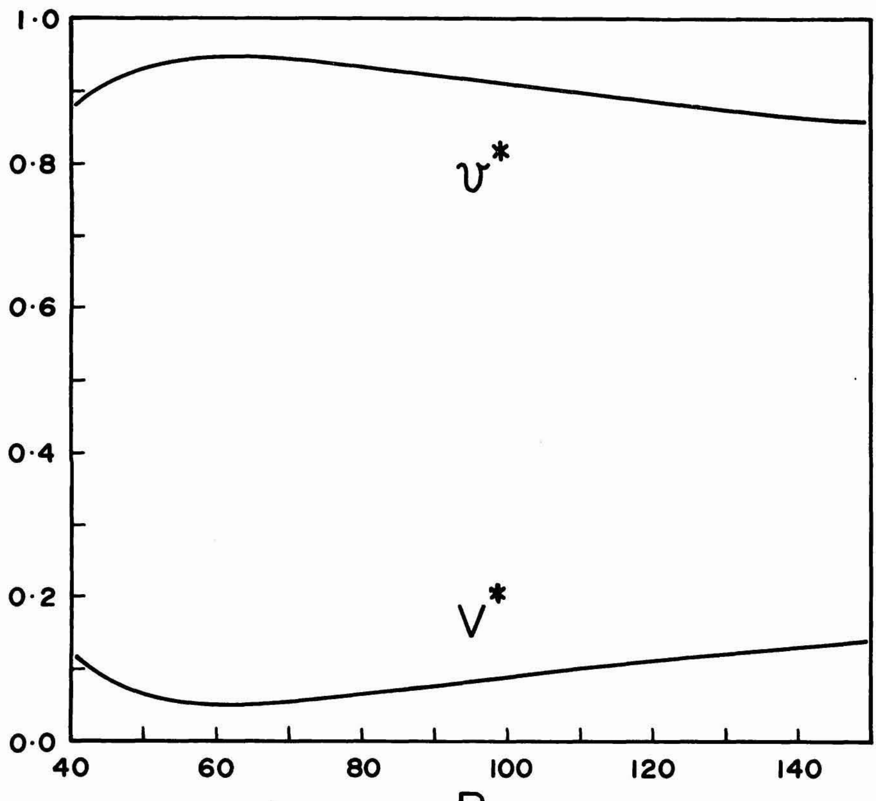


Fig. 2. Velocity of waves and vortex-streets against Reynolds number.

By means of the reference quantities U and d, the following dimensionless relations are obtained:

$$v^* = 1 - V^* \tag{4}$$

$$S = Nd/U = v^*/\lambda^*,$$

$$v^* = S\lambda^* \tag{5}$$

On the other hand, for the Reynolds number in the range of $50 < R < 150$, Roshko[3] (1954) gave the relationship between the Reynolds number R and Strouhal number S as

$$S = 0.212 - 4.5/R, \quad 50 < R < 150. \tag{6}$$

Fig. 2 shows the graph of calculated values of V^* and v^* based on the Eqs. (4) and (5) using the values of S and λ^* which in turn are calculated using the Eqs. (1) and (6). It is seen from this Figure that the minimum value of V^* or the maximum value of v^* appears at the Reynolds number of 64. This suggests that in the two ranges of Reynolds number, lower than or higher than the critical Reynolds number, there exist different patterns of wake: that is, one is the waving-line wake and the other is the vortex street. The value of this critical Reynolds number is very close to that obtained in the previous experiment[7].

§ 5. Vortex strength and passing rate of vorticity

In the Kármán vortex streets, there exist the following relations among the wake parameters[11]:

$$V = (\Gamma/2\lambda)\tanh(\pi h/\lambda), \tag{7}$$

$$\tanh(\pi h/\lambda) = 1/\sqrt{2}. \tag{8}$$

With the reference quantities U and d, the following dimensionless relations are obtained:

$$\Gamma^* = 2\sqrt{2}\,\lambda^* V^*, \tag{9}$$

$$K^* = \Gamma^* S = 2\sqrt{2}\,V^*(1 - V^*). \tag{10}$$

In the real vortex streets, the stable region is assumed to be a good approximation of the idealized Kármán vortex streets. Thus, the vortex strength Γ^* and the passing rate of vorticity K^* can be estimated from the Eqs. (9) and (10) using the values of S and λ^*. Fig. 3 shows the graph of the calculated values of Γ^* and K^*.

It should be noted again that the minimum values of Γ^* and K^* appear at the Reynolds number 64, respectively, suggesting that the patterns of wakes in the

Reynolds number ranges lower than and higher than the critical Reynolds number are not the same. Within the lower Reynolds number range, as has already been seen, *no* vortex street is formed. Therefore, the Eqs. (9) and (10), which are derived from the Kármán's theory on vortex streets, are not applicable for this region. The nonapplicability of these equations for the said region is well suggested by the dotted parts of the curves in Fig. 3 which are physically unrealistic.

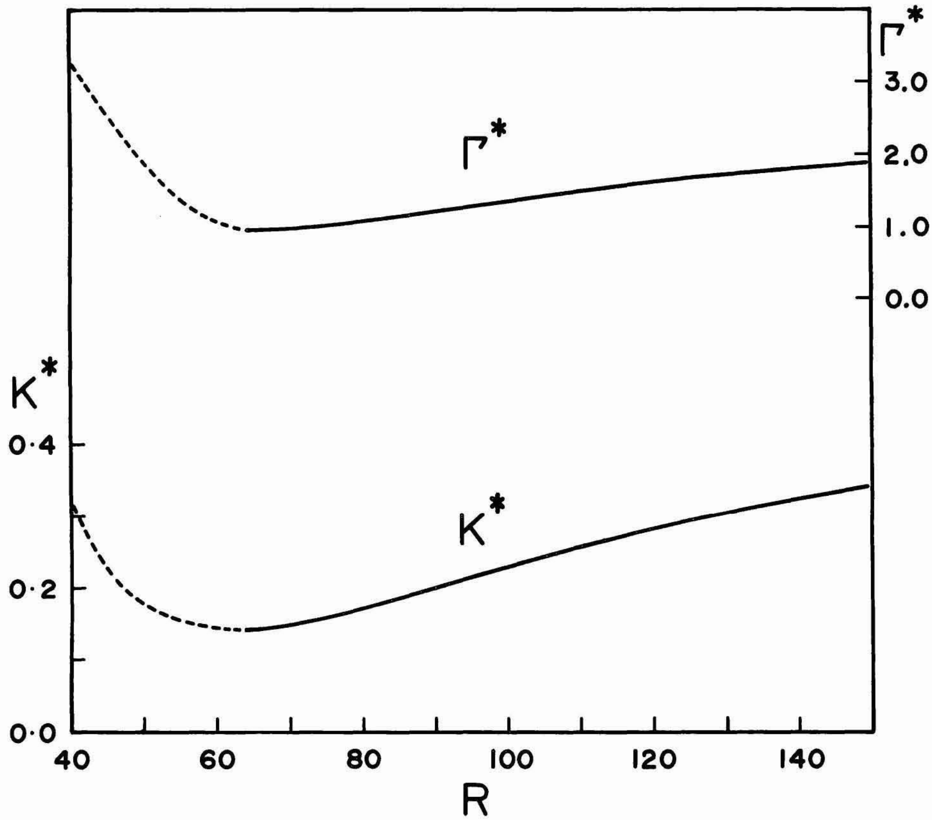


Fig. 3. Vortex strength Γ^* and passing rate of vorticity K^* against Reynolds number. (R > 64)

§ 6. Conclusions

An experimental investigation on the periodic wakes behind a circular cylinder at Reynolds number from 40 to 150 was carried out for the purpose of studying the behavior of wakes. The main results of this study are as follows:

(1) Wavelengths in the waving-line wake or longitudinal spacing in the vortex-street λ^* can be well represented by the empirical formula:

$$\lambda^* = 3.25 + 220/R, \quad 40 < R < 150.$$

(2) The wake parameters are derived from the empirical formula presented for λ^{*} (R) and the Roshko's formula for S (R). On examining these parameters, the critical Reynolds number of about 64 is obtained.

(3) Periodic wakes behind a circular cylinder could be classified into the following distinct Reynolds number ranges:

(a) waving-line wake for $R \lesssim 64$

(b) vortex-street wake for $R \gtrsim 64$

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