On the Low Cycle Fatigue Life of Notched Specimens

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In this paper the fatigue life and the torque vs angle of twist relation ship of a shaft with one semi-circular longitudinal grove were investigated for establishing the simple rule which is valuable for estimating low-cycle fatigue life using the least amount of information on the strength of a material.

The form of a notch and the torsional loading were selected for making the mathematical treatment easier and for letting the experiment simpler. The torque-angle of twist relation ship of notched specimens under monotonic or cyclic loading were obtained mathematically and the result was compared with the experiment on aluminum alloy (A 3B 4-T 4 by Japanese Industrial Standard).

On the other hand, by comparing the fatigue life of notched and smooth specimens the adequancy of the assumption used was also investigated. The investigation shows that the assumption on strain distribution makes the simple estimation of fatigue life and the estimation of stress distribution possible in case of torsional low-cycle fatigue of a shaft with a longitudinal groove.

1. Introduction

The fatigue fracture of machines, airplanes and structures sometimes occures at the notch root under high strain repetition. For this reasion the fatigue problem under high strain repetitions has become much interested in recent years. In traditional work_ of notched members which are subjected to high strain repetitions, however, it has been investigated in such a way as experimental data are deduced as simple as possible without regarding the mechanics of strain cycling.

In this paper the prediction of the fatigue life of notched members using the least-amount of information on the strength of a material and using a simple assumption on strain distribution has been made. In this paper it is intended to clarify the followings:

(1) Calculating the nonlinear stress distribution of longitudinally notched bar under monotonic or cyclic twisting.

(2) Establishing the method that predicts the fatigue life of plastically stressed notched specimens using the fatigue data of plain specimens.

2. Torque-Angle of Twist Relationship of a Shaft with One Semi-Circular Longitudinal Groove under Static Loading

In this paper the torsional problem in the elastic-plastic state was analyzed by using a simple assumption: shearing strain on the cross section is proportional to an

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angle of twist per unit length

$$\gamma = K(r, \varphi) \cdot \theta \qquad (1)$$

where the proportional constant $K(r, \varphi)$ is assumed to be only a function of coordinates and not that of the material used. Namely it is assumed to be the same as elastic one even if the specimen deforms plastically. This means $K(r, \varphi)$ can be determined from the elastic solution. The stress at any point in the cross section can be calculated easily using this assumption and the stress-strain relationship of material. Followingly the torque applied to the notched member can be simply determined. The comparison of the experimentally obtained results with the torque-angle of twist relationship obtained by this method helps to make sure the adequancy of the assumption.

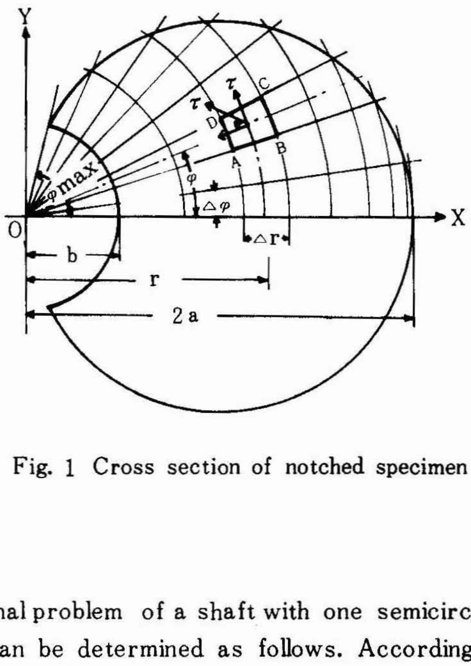

Fig. 1 Cross section of notched specimen

In case of the torsional problem of a shaft with one semicircular longitudinal groove the constant $K(r, \varphi)$ can be determined as follows. According to the theory of elasticity, the stress function ϕ of the torsional problem must satisfy the following equation

$$\frac{\partial^2 \phi}{\partial r^2} + \frac{1}{r}\frac{\partial \phi}{\partial r} + \frac{1}{r^2}\frac{\partial^2 \phi}{\partial \varphi^2} = -2G\theta \qquad (2)$$

at every point in the cross section and

$$\phi = 0 \qquad (3)$$

on the boundary of cross section. By reference to Weber's theory eqs. (2) and (3) are satisfied by taking the stress function of the form[1)]

$$\phi = \frac{G\theta}{2} (r^2 - b^2)(1 - 2a\cos\varphi / r) \tag{4}$$

Therefore, the shearing stress $\tau\varphi$ in the radial direction and the shearing stress τt in the tangential direction became respectively as follows:

$$\tau\varphi = \frac{1}{r}\frac{\partial\phi}{\partial\varphi} = G\theta\,(a - ab^2/r^2)\sin\varphi \tag{5}$$

$$\tau t = -\frac{\partial\phi}{\partial r} = G\theta\,\{a(1 + b^2/r^2) - r\}$$

From Eqs. (5) and (6) the total stress at any point in the cross section is represented in the form

$$\tau = \sqrt{\tau\phi^2 + \tau t^2}$$

$$= G\theta\,[(a - ab^2/r^2)^2\sin^2\varphi + \{a(1 + b^2/r^2)\cos\varphi - r\}^2]^{1/2} \tag{7}$$

$$= G\gamma$$

$$\therefore \gamma = [(a - ab^2/r^2)^2\sin^2\varphi + \{a(1 + b^2/r^2)\cos\varphi - r\}^2]^{1/2}\cdot\theta \tag{8}$$

Therefore, the constant $K(r, \varphi)$ becomes

$$K(r, \varphi) = [(a - ab^2/r^2)^2\sin^2\varphi + \{a(1 + b^2/r^2)\cos\varphi - r\}^2]^{1/2} \tag{9}$$

The magnitude ot the torque corresponding to any unit angle of twist is represented as follows:

where $$Mt = 2\int_0^{\varphi max}\int_b^{2a\cos\varphi} \tau\, tr^2 dr d\varphi \tag{10}$$

$$\tau t = \tau \sin\psi$$

$$\Psi = \tan^{-1}(\gamma t / |\gamma\varphi|) = \tan^{-1}\left\{\frac{a(1 + b^2/r^2)\cos\varphi - r}{|(a - ab^2/r^2)\sin\varphi|}\right\} \tag{11}$$

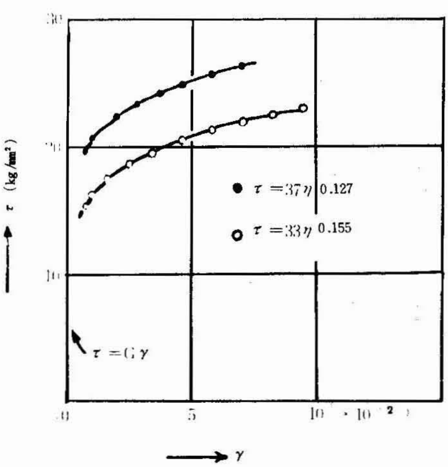

Fig. 2 Monotonic and cyclic stress strain curves for aluminum alloy (A 3B 4-T 4)

The stress strain diagram obtained from monotonic torsion tests of solid shaft specimens can be represented in the form

$$\tau = G\gamma \tag{12}$$

in the elastic region and

$$(\tau = F\gamma^{n}) \tag{13}$$

in the plastic region.

For aluminum alloy (A3B4-T4) used in our experiments the values of $F = 33.0 kg/mm^2$, $n = 0.155$, $G = 2725.0 kg/mm^2$ were obtained and the stress-strain diagram is shown in Fig. 2. The strain γ for any unit angle of twist can be obtained from eq. (8) and the stress τ in the elastic and plastic regions corresponding to this γ can be obtained from eq. (12) and (13). since the component τt in the tangential direction of τ can be obtained from eq. (11), the torque Mt applied to the notched member can be obtained by integrating eq. (10) over the whole cross section. To obtained numerical value of the torque Mt by digital computer, the cross section was divided into smaller parts as shown in Fig. 1. Since the torque about the point O by the shearing force acting on the small element ABCD is equal to $\triangle M = \tau t\, r^2 \triangle r \triangle \varphi$, eq. (10) can be represented approximately as

$$Mt \simeq \sum_{S} \triangle Mt \tag{14}$$

Where S is the area of the cross section of the notched member.

Fig. (1) shows experimental results and culculated values obtained by using

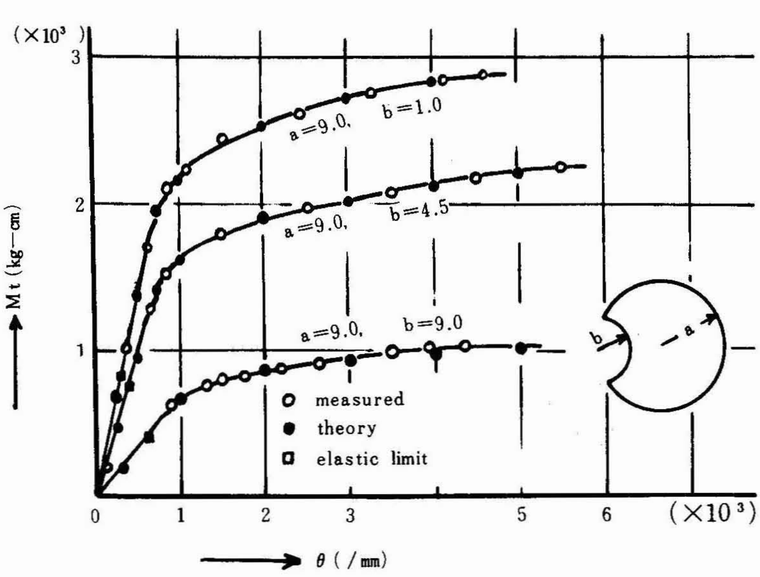

Fig. 3 Torque-angle of twist relationship under static loading (notched specion)

eq. (14). This figure shows that the values calculated for three aifferent notch radii give a very good agreement with experimentally obtained results.

3. Torque-Angle of Twist Relationship of a Shaft with One Semi-Circular Longitudinal Groove under Cyclic Loading

A group of stable hysteresis loops obtained by fatigue test of solid shaft specimens under constant strain amplitude is shown in Fig. (4). The cyclic stress-strain

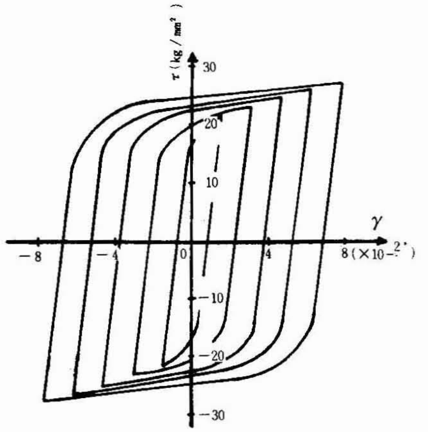

Fig. 4 A group of hysteresis loops obtained from torsional test of solid shafts

diagram shown in Fig. 2 was obtained by combining together the tips of the companion hysteresis loops in the figure.

•When the notched specimen made of aluminum alloy (A3B4-T4) is subjected to the repeated torsional loading under constant strain amplitude, it hardens initially and is stabilized after some number of cycles when the stable hysteresis loop can be obtained. In Fig. 5 it is shown that the calculated cyclic Mt-θ relationship determined

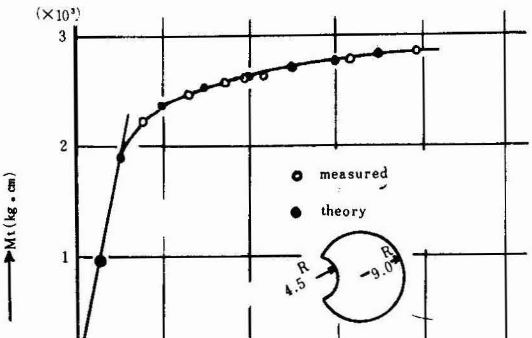

Fig. 5 Torque-angle of twist relationship un er cyclic loading (notched specimen)

as a locus of the tips on the stable hysteresis loops and the experimental results agreed quite well. For obtaining the calculated Mt-θ relationship of notched specimens the cyclic stress-strain diagram of the plain specimens was used.

Now, let us descrive the method for calculating the torque Mt under intermidiate stage of cyclic twisting using the simple assumption as was used above. The torque Mt_1 of any tip θ' on the stable hysteresis loop shown in Fig. 6 can be obtained

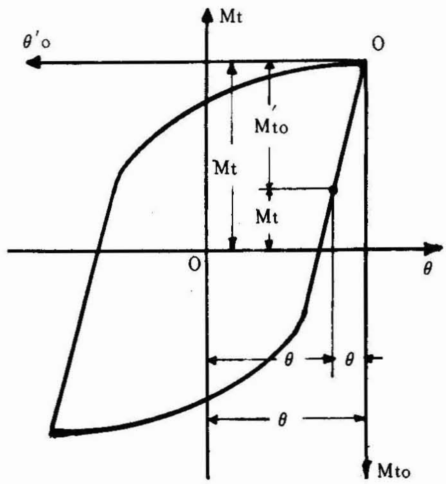

Fig. 6 Schematic figure of a hystersis loop

from the above-mentioned calculation for the cyclic twisting. If the notched member is twisted only θ' in the inverse-direction from the point O′ to the point P, the following relation is obtained.

$$Mt = Mt_1 - M\acute{t}o \tag{15}$$

Therefore, the experimental values of the torque Mt are the same quantities as those subtracted Mt́o from Mt_1. After all, we have only to calculate the torque Mt́o for a given unit angle of twist θ' in the inverse-direction. When the notched member is twisted only θ' in the inverse-direction, every point in its cross section follows each hysteresis loop which is determined at the peak strain value. In order to get the numerical result on twisting moment it is important to express the companion loops by a simple mathematical expression. In case of aluminum alloy (A3B4-T4) it was found that the hysteresis loops were simply generated by parallel-shifting of a curve. That is, if we move parallel the axis of abscissa of all hysteresis loops as shown in Fig. 4 and superpose those curves respectively, we notice that all of the these hysteresis loops show a good agreement and can be approximated by one curve (Say"master curve") as shown in Fig. 8. The torque Mt́o can be calculated by using the master curve as follows. When the state of stress-strain of a point in the cross section of the notched specimen changes from the point O′ to the point P corresponding to the change of angle of twist θ' of the specimen as shown in Fig. 7, the following is obtained.

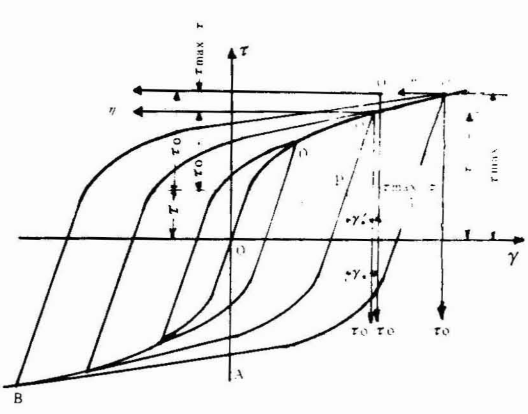

Fig. 7 Stress - strain relationship on hysteresis loop

$$\tau = \tau_1 - \tau'_0 \tag{16}$$

from which

$$\sum_S \tau \, r^2 \triangle r \triangle \varphi = \sum_S \tau_1 \, r^2 \triangle r \triangle \varphi - \sum_S \tau'_0 \, r^2 \triangle r \triangle \varphi \tag{17}$$

This equation is equivalent to eq. (15). Therefore, using the stress τ'_0 obtained by using the master curve, the torque M_{t0} can be determined. Now, if the curve O_0AB shown in Fig. 7 is selected as the master curve, we get

$$\tau'_0 = \tau_0 - (\tau_{max} - \tau_1) \tag{18}$$

Let us approximate the representative curve in the form

$$\tau_0 = G\gamma_0$$

in the elastic region and (19)

$$\tau_0 = F_0 \gamma_0^{n}$$

in the plastic region, where

$$\gamma_0 = \frac{\tau_{max} - \tau_1}{G}$$

$$= K(r, \varphi)\theta' + \frac{\tau_{max} - \tau_1}{G} \tag{20}$$

Using eqs. (19), (20) and τ_{max}, τ_1 which can be obtained from Fig. 2, τ'_0 in eq. (19) for any given θ' can be obtained. Therefore, the torque M'_{t0} for this θ' may be obtained by the numerical integration. Fig. 9 shows experimental results and calculated values obtained by using the representative approximate curve shown in Fig. 8. They give a good agreement.

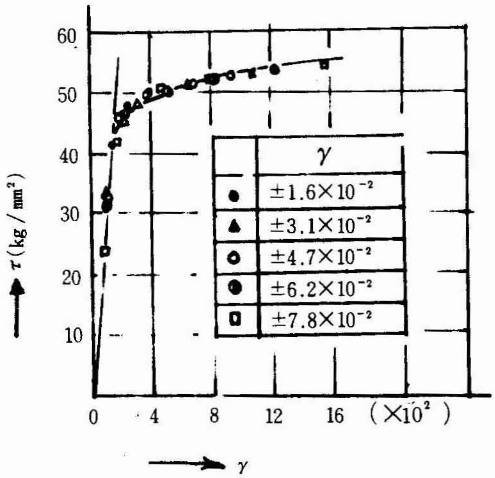

Fig. 8 Characteristic of hysteresis loops

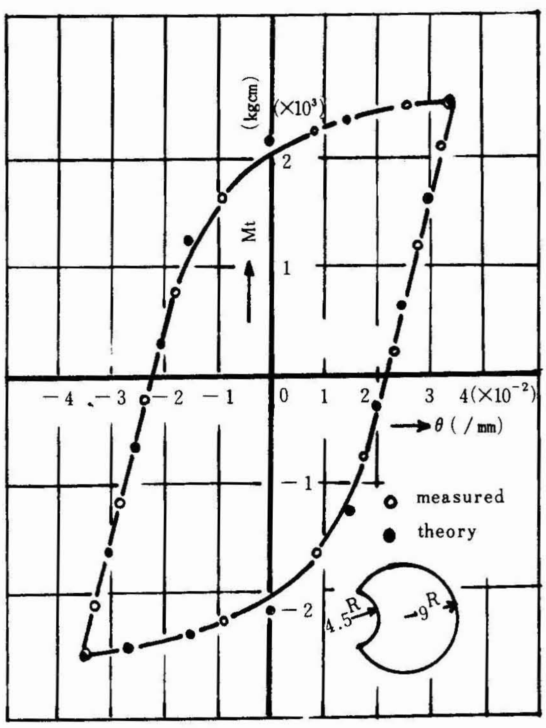

Fig. 9 Torque-angle of twist relationship on hysteresis loop

4· Torsion Fatigue Test of Notched Members under Constant Strain Amplitude

Fig. 10 shows the strain amplitude vs the number of cycles relationship under constant twisting amplitude zero mean cycling. The number of cycles was determined when the crack length reached up to 3～5 mm. Strain of notched specimens in this figure represent those at the notch root calculated by using the assumption. The results of notched specimens in torsional fatigue test are slightly lower than

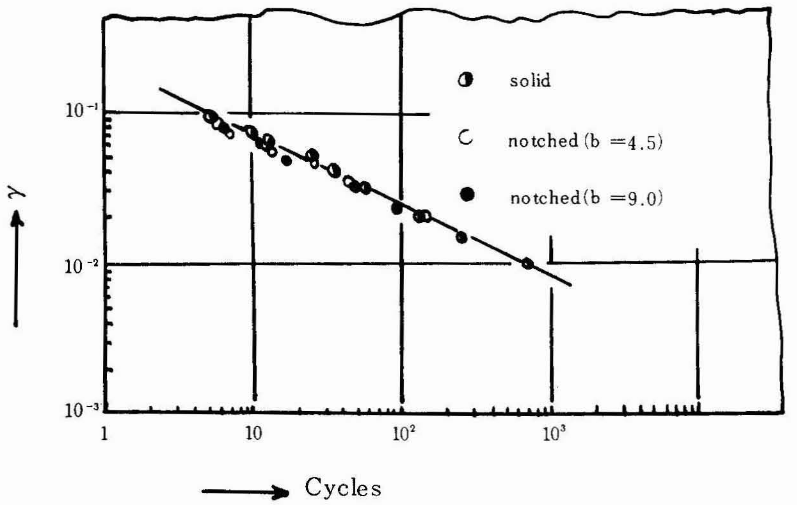

Fig. 10 Smooth and notched specimen fatigue data

those of plain specimens. It is perhaps the main reasons that the one is that the strain quantity at the notch root is more difficult than that of smooth specimen. we conclusively say that our assumption is apparently the first order good approximation for predicting the fatigue life of notched menbers from plain specimen fatigue data and good enough for usual engineering purpose.

5. Conclusion

To estimate the low cycle fatigue life of notched members in consideration of the strain hardening property of a material, the notch effect was investigated on the shaft with a semi-circular longitudinal groove by using the simple assumption on the strain distribution. The following conclusions were obtained.

(1) Experimental results and calculated values of the monotonic torque-angle ot twist relationship gave a good agreement, especially in case of small notch.

(2) Experimental results and calculated values of the cyclic torque-angle of twist relationship gave a good agreement.

(3) Experimental results and calculated values of the torque at the intermidiate state of loading also give a good agrement .

(4) The fatigue life of notched members can be predicted from smooth specimen fatigue data using the simple assumption on strain distribution.

6. References References

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