The Third Sum Rule and the Electric Response Function of a Two-Component Quantum Plasma

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Abstract

The charge-density response function of a uniform two-component plasma is studied on the basis of the third frequency moment sum rule of its spectral function and the method of an equation of motion. An exact expression of the third sum rule is presented with emphasis on the existence of a singular term. It is found that available local-field theories for the response function do not satisfy this sum rule and any theory with frequency-independent local-field correction can not satisfy the third-and the perfect screening-sum rules, simultaneously. The response function$D^{r}{}_{ij}(\boldsymbol{q},\omega)$ and the corresponding frequency-dependent local-field correction G_{ij} $(\boldsymbol{q},\omega)$ are calculated on the basis of the equation of motion. The obtained result yields a well-known relation between the pair correlation function and its radial derivative at $r=0$ for large wave vectors and satisfies the third-and the perfect screening-sum rules.

§1. Introduction

There are a number of matters in nature whose electronic phenomena are viewed as plasma effect. An electron system imbeded in a uniform positive background is a well-known theoretical model in understanding the behaviours of conduction electrons in a real metal and a great deal of studies on this model have been done.[1] The central topics in the matters investigated by this model may be attributed to the correlation between electrons (same kinds of particles). On the other hand, plasma effects, for example, in a semimetal Bi and optically-excited semiconductors (Ge, GaAs, InSn, etc.) have been studied on the basis of a model for a multi-component system.[2]
The characteristic of the multi-component system, of course, lies in the existence of the interaction between components.

The charge-density response function describes microscopic mechanisms in the response of a plasma to a weak external electric field and plays a central role in studing

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the various plasma properties related to the charge density fluctuation.[1] The most commonly used calculation of this function is the random phase approximation (RPA).[3] RPA is adequate in the case of weak coupling where kinetic energy is dominant. In other cases, however, RPA involves some unphysical result.[4] This, needless to say, means that effects of the correlation between particles on the self-consistent field can not be neglected in the latter cases.[4]

In the uniform one-component electron system, approximate procedures to improve RPA have been proposed very early by Hubbard[5] and by Nozieres and Pines.[6] Hubbard corrected the RPA dielectric function by taking exchange-hole into account. This has been extended to include both exchange-and Coulomb-holes self-consistently by Singwi *et al.*,[7] where the dielectric function is expressed as a functional of the pair correlation function. A number of other approximate procedures to calculate the dielectric function have also been proposed.

Some of the theories developed[5),6),7)] above are generalized to the case of the multi-component system and have been used to calculate the annihilation rate and correlation energy of positron in metals,[8),9)] the thermodynamical properties of an electron-hole liquid in semiconductor,[10),11),12)] etc.

Now, calculating the third frequency moment of the charge-density response function of the multi-component system, we found the existence of a singular term in this moment which exists only in the multi-component system and becomes important in the long wavelength region. Goodman[13] noted the existence of a similar singular term in the third frequency moment of the spin-density response function of the uniform one-component electron system and later Goodman and Sjölander[14] analyzed in detail its physical content. However, in the theory for the multi-component plasma, the existence of the singular term and its significance have not been clarified yet, sufficiently. The existence of such a singular term awakes a caution in the generalization of the theories developed in the one-component system to the case of the multi-component system. For example, any frequency-independent local field correction does not satisfy the third-and the perfect screening-sum rules, simultaneously.

The aim of this paper is to investigate the charge-density response function in the multi-component plasma on the basis of the third sum rule and the equation of motion.

In §2, the general formulation of the theory will be given and the exact expression of the third sum rule will be presented with emphasis on the existence of a singular term. In §3, a local mean-field approach to the problem pointed out in §2 will be given and it will be shown that the frequency-dependence of the local field correction is essential in order that the third-and the perfect-screening-sum rules are satisfied, simultaneously. In §4, the response function and the corresponding frequency-dependent local field correction will be calculated on the basis of the equation of motion and it will be discussed that the results are satisfactory both for high-frequencies or large wave vectors and for low-frequencies and small wave vectors. In §5, some discussions will be given.

§2. General Formulation and Sum Rules

Let us consider a non-relativistic plasma contained in a large box of volume V with periodic boundary condition. Let average number-density, mass, charge, and magnitude of spin of i-species of particle be n_i, m_i, e_i, and σ_i, respectively. The Hamiltonian of this system (for convenience, let $V=1$ and $h=2\pi$), in a second quantized representation, is

$$H = H_0 + H_1, \tag{2.1}$$

where

$$H_0 = \sum_i \sum_{k\sigma} \varepsilon_i(k)\, C_i^+(\boldsymbol{k}\sigma)\, C_i(\boldsymbol{k}\sigma)$$

$$H_1 = \frac{1}{2} \sum_q v(q) [\rho(\boldsymbol{q})\rho(-\boldsymbol{q}) - \sum_i n_i e_i^2]$$

$$\varepsilon_i(k) = k^2/2m_i,$$

$$\rho(\boldsymbol{q}) = \sum_i \rho_i(\boldsymbol{q}),$$

$$\rho_i(\boldsymbol{q}) = e_i \sum_{k\sigma} C_i^+(\boldsymbol{k}\sigma)\, C_i(\boldsymbol{k}+\boldsymbol{q}\sigma)$$

and $\quad v(q) = 4\pi/q^2, \; q \neq 0$

$$\quad\quad = 0 \quad\quad , \; q = 0$$

The operators $C_i^+(\boldsymbol{k}\sigma)$ and $C_i(\boldsymbol{k}\sigma)$ are the creation-and the annihilation-operators for the i-species of particle with momentum $\boldsymbol{k}$ and spin σ, respectively.

Let us assume that the system described by the Hamiltonian (2.1) is in thermal equilibrium in the infinite past and then a ficticious weak external electric field is applied adiabatically.[15] Let $V_e^i(\boldsymbol{q},\omega)$ be the amplitude of the potential of the ficticious field with wave vector $\boldsymbol{q}$ and frequency ω which would couple only with the i-species of particles. According to the linear-response theory,[16] the Fourier transform of the response of the charge-density of i-species of particles is given by

$$\rho_i^{\text{ind}}(q,\omega) = \sum_j D_{ij}^r(q,\omega)\, V_e^j(q,\omega), \tag{2.2}$$

where the property of the translational invariance of the system is used and $D_{ij}^r(\boldsymbol{q},\omega)$ is the retarded density-response function defined as follows*:

$$D_{ij}^r(q,t-t') = -i\theta(t-t')\langle[\rho_i(q,t), \rho_j^+(q,t')]\rangle = \int \frac{d\omega}{2\pi} D_{ij}^r(q,\omega)\, e^{-i\omega(t-t')} \tag{2.3}$$

In eq. (2.3), $\langle\cdots\rangle = \mathrm{Tr}(e^{-\beta H}\cdots)/\mathrm{Tr}\,e^{-\beta H}$, $\beta = 1/k_B T$,

$$\rho_i(\boldsymbol{q},t) = e^{iHt}\rho_i(\boldsymbol{q})\, e^{-iHt}.$$

The generalized dielectric function $\varepsilon(\boldsymbol{q},\omega)$ defined by Nozières and Pines[3] is expressed as

$$1/\varepsilon(q,\omega) = 1 + v(q) \sum_{ij} D_{ij}^r(q,\omega). \tag{2.4}$$

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* We hope that the imaginary unit i and the suffix i specifying the species of the particle do not raise any confusion.

Let us define a function $\tau_{ij}(q,\omega)$ as

$$\langle[\rho_i(q,t), \rho_j^+(q,t')]\rangle = \int_{-\infty}^{+\infty} \frac{d\omega}{2\pi} \tau_{ij}(q,\omega) e^{-i\omega(t-t')} \tag{2.5}$$

This function, as is well-known, embodies all the properties of the system related to the charge-density fluctuation.[4] It can be shown directly from the definitions (2.3) and (2.5) that

$$D^r_{ij}(q,\omega) = \int_{-\infty}^{+\infty} \frac{d\omega'}{2\pi} \frac{\tau_{ij}(q,\omega')}{\omega-\omega'+i\varepsilon}, \qquad (\varepsilon \to +0) \tag{2.6}$$

$$\tau_{ij}(q,\omega) = i\,[D^r_{ij}(q,\omega) - D^r_{ji}(-q,-\omega)]. \tag{2.7}$$

There exist a number of exact sum rules which $\tau_{ij}(q,\omega)$ has to satisfy. These are very useful in checking the validity of a particular approximation in the calculation of D^r_{ij}. For the sake of the completeness, we shall derive some of them.

(i) Perfect screening sum rule:

$$\lim_{q\to 0} 1/\varepsilon\,(q,0) = 0 \tag{2.8}$$

(ii) Zeroth moment sum rule: the static form factor $S_{ij}(q)$ is defined by

$$S_{ij}(q) = \langle\rho_i(q)\rho_j^+(q)\rangle / n_i e_i e_j. \tag{2.9}$$

From the definitions (2.5) and (2.9), we get

$$\int_{-\infty}^{+\infty} \frac{d\omega}{2\pi} \frac{\tau_{ij}(q,\omega)}{1-e^{-\beta\omega}} = n_i e_i e_j S_{ij}(q). \tag{2.10}$$

(iii) Frequencey-moment sum rules: from the high frequency asymptotic expansion of eq.(2.6), we get

$$v(q)\,D^r_{ij}(q,\omega) = \sum_{\ell=1}^{\infty} M_{\ell,ij}(q)/\omega^{\ell+1}$$

where

$$M_{\ell,ij}(q) = v(q) \int_{-\infty}^{+\infty} \frac{d\omega}{2\pi} \omega^{\ell} \tau_{ij}(q,\omega).$$

With the use of eq.(2.5) as usual, $M_{\ell,ij}(q)$ can be calculated from the Hamiltonian (2.1). In particular,

$$M_{1,ij}(q) = \delta_{ij}\,\omega_i^2, \tag{2.11}$$

$$M_{3,ij}(q) = \omega_i^2\omega_j^2 + \delta_{ij} M^{(0)}_{3,i}(q) \tag{2.12}$$

$$+ (e_i e_j/m_i m_j) \sum_{q'} v(q) v(q')(q.q')^2 \langle \rho_i(q')\rho_j(-q') - \delta_{ij}\rho(q')\rho_j(-q')\rangle$$

$$+ (e_i e_j/m_i m_j) \sum_{q'(\neq q)} v(q) v(q')(q.q')^2 \langle \rho_i(q-q')\rho_j(q'-q) - \rho_i(q')\rho_j(-q')\rangle,$$

where

$$M_{3,i}^{(0)} = (8\pi e_i^2 K_i q^2/m_i^2 + \pi n_i e_i^2 q^4/m_i^3), \quad \omega_i^2 = 4\pi n_i e_i^2/m_i,$$

and $K_i = \sum_{k\sigma} \varepsilon_i(k) \langle C_i^{+}(k\sigma)\, C_i(k\sigma)\rangle$. The f-sum rule (2.11) is, as is well-known,[4] the direct consequence of the particle conservation in each component of the system. The third sum rule (2.12) explicitly involves the interaction between particles. Therefore, a characteristic in the multi-component system, which is essentially related to the interaction between components, should appear in this sum rule. Here, we should note the third-term on the right hand side of eq.(2.12). This term exists only in the multi-component system and does not vanish even in the long wavelength limit. In the following, we will refer to this term as the singular term.

Well, the first- and the second-terms of eq.(2.12), where the third frequency moment of D^{r}_{ij} in RPA consists of these two terms, are connected with the ordinary plasma oscillations and the single particle excitations, respectively. The response function D^{r}_{ij} in RPA, as is well known, describes the response of the free particles to self-consistent Hartree-field in the system under an external disturbance. The effect of correlation between particles, however, results the cumulation or depletion of particles around each particle (the correlation particle-hole). The third- and the last-terms of eq.(2.12) are connected with the response related to this correlation particle-hole. In particular, the appearance of the singular term means that the correlation particle-hole has a much more important effect on the response in the multi-component system than in the one-component system, particulary in the long wavelength limit. In view of this, an application of the theories developed in the one-component system to the case of the multi-component system should have to be done with a caution.

In the case of the two-component plasma, the singular term of eq.(2.12) can be expressed as

$$\omega_i^2\, \omega_j^2\, [g_{12}(r=0)-1]/3, \tag{2.13}$$

where we used the rotational invariance of the system and $n_1e_1+n_2e_2=0$, and $g_{ij}(r)$ is the pair correlation function of i-species of- and j-species of-particles defined as

$$g_{ij}(r) - 1 = \sum_{q(\neq 0)} g_{ij}(q)\, e^{iqr} \tag{2.14}$$

$$g_{ij}(q) = [S_{ij}(q) - \delta_{ij}]/n_j. \tag{2.15}$$

The third sum rule in the case of the one-component system has been very familiar. Historically, it was first derived and used by Yvon[17] for a classical liquid and by Puff[18] for an interacting Bose liquid. Later, it was extensively applied together with other sum rules to the classical liquid,[19] the interacting Bose liquid,[20] and the interacting electron liquid.[21),22),23] On the other hand, the third sum rule for a spin-density response function was first derived by Goodman[13] for the electron liquid and by Safir and Widom[24] for a Fermi liquid with the short-range interaction. They noted there that this sum rule contains a singular q-dependent term as compared with the number-density response function. Later, the physical content of the third sum rule, particularly, of this singular term has been analyzed in detail by Goodman and Sjölander.[14] They found that available local field theories for the electric-and the magnetic-response functions in the electron liquid can not be valid both for low and high frequencies. Though they were concerned about the electron liquid, their discussion may be valuable for another system which consists of some subsystems responding, respectively, in the different ways to an external disturbance.

§3. Local Mean-Field Theory

In various local field theories for the dielectric function,[7),25),26)] one makes the assumption that the particles respond as free particles to an effective field depending only on the induced local mean charge-density. In the case of the two-component system,[8),12)] such an assumption gives the expression for the response $\rho_i^{ind}(q,\omega)$, instead of eq. (2.2), as

$$\rho_i^{ind}(q,\omega) = D_i^{(0)}(q,\omega)\, V_{eff}^i(q,\omega), \tag{3.1a}$$

$$V_{eff}^i(q,\omega) = V_e^i(q,\omega) + \sum_j^2 \psi_{ij}(q,\omega)\, \rho_j^{ind}(q,\omega), \tag{3.1b}$$

where $D_i^{(0)}$ is the non-interacting response function of i-component given by

$$D_i^{(0)}(q,\omega) = e_i^2 \sum_{k\sigma} \frac{n_i(k\sigma) - n_i(k+q\sigma)}{\omega - \varepsilon_i(k+q) + \varepsilon_i(k) + i\varepsilon} \tag{3.2}$$

$n_i(k\sigma)$ being the occupation number of i-species of particles, while

$$\psi_{ij}(q,\omega) = v(q)\,[1 - G_{ij}(q,\omega)],$$

$G_{ij}(q,\omega)$ being the local field correction. The two quantities $n_i(k\sigma)$ and $G_{ij}(q,\omega)$ have to be given, consistently. From eqs. (2.2), (3.1a) and (3.1b), we can obtain the general expression for the response function D_{ij}^r in terms of G_{ij}:[12)]

$$D_{ij}^r = [\delta_{ij} D_i^{(0)} - D_1^{(0)} (\sum_{i'=1}^2 \psi_{ii'} \delta_{ij} - \psi_{ij})\, D_2^{(0)}]/\Delta, \tag{3.3a}$$

where

$$\Delta = \prod_{i=1}^2 [1 - \psi_{ii} D_i^{(0)}] - D_1^{(0)} \psi_{12} \psi_{21} D_2^{(0)} \tag{3.3b}$$

The asymptotic expansion for this D_{ij}^r is

$$v(q)\, D_{ij}^r(q,\omega) \sim \delta_{ij}\omega_i^2/\omega^2 + [\omega_i^2\omega_j^2 - \omega_i^2\omega_j^2 G_{ij}(q,\infty) + \delta_{ij} M_{3,i}^{(0)}(q)]/\omega^4 + \cdots \tag{3.4}$$

That the coefficient of ω^{-2} in the above expansion agrees with eq. (2.11) shows that any theory in the local field approach satisfies the f-sum rule, automatically. On the other hand, in order that the theory satisfies the third sum rule, the coefficient of ω^{-4} in eq. (3.4) has to be equal to eq. (2.12). Therefore, we get the exact local field correction in the high-frequency limit as

$$G_{ij}(q,\infty) = [1-g_{12}(0)]/3$$
$$+\sum_{q'(\neq 0,-q)}\left\{\frac{(qq')^2}{q^2q'^2}-\frac{[q(q+q')]^2}{q^2(q+q')^2}\right\}g_{ij}(q'), \tag{3.5}$$

where we used eq.s (2.9), (2.13) and (2.15). Note that the first- and second-terms of eq. (3.5) correspond to the third- and the last-terms of eq. (2.12), respectively; the first term of eq. (3.5) exists only in the multi-component system and becomes dominant in the long wavelength local field.

Now, let us assume a theory in which local field correction does not depend on the frequency. Let us denote this correction by $G_{ij}(\boldsymbol{q})$. If this theory satisfies the third sum rule, then $G_{ij}(\boldsymbol{q})$ is equal to $G_{ij}(\boldsymbol{q},\infty)$ given by (3.5). By the way, such a choice of $G_{ij}(\boldsymbol{q})$ is the generalization of the Pathak-Vashishta[22] approach in the electron liquid to the multi-component system. This, however, can not satisfy the perfect screening sum rule given by eq. (2.8); noticing $G_{11}(0)=G_{12}(0)=G_{21}(0)=G_{22}(0)$, we can show from eqs. (2.4), (3.2) and (3.3) that $1/\varepsilon(\boldsymbol{q},0)$ does not vanish in $q\to 0$. Therefore, we conclude that such a theory is qualitatively incorrect and the frequency dependence of the local field correction is essential for the proper description of the high-frequency response of the multi-component plasma.

In available local field theories for the two-component plasma[10),12)] which have been obtained by the generalization of the theories[5),7)] developed in the electron liquid, the local field correction G_{ij} is frequency-independent and $G_{ij}(\boldsymbol{q}) = O(q^2)$ in $q\to 0$. Therefore, these theories disagree with the third sum rule and are inadequate for the discription of the high-frequency phenomena. This has already been pointed out in the electron liquid.[27)] At present, there is no such a proper local field theory for the two-component plasma that satisfies the third sum rule and further is valid for low frequencies as well.

§ 4. The Calculation of the Response Function

The response function $D^r_{ij}(\boldsymbol{q},t)$ defined by eq. (2.3) can be written as

$$D^r_{ij}(q,t) = \sum_{k\sigma} e_i R_{i:j}(\boldsymbol{k}\sigma;q,t), \tag{4.1}$$

$$R_{i:j}(\boldsymbol{k}\sigma;q,t) \equiv \langle\langle C_i^+(\boldsymbol{k}-q/2\sigma)\,C_i(\boldsymbol{k}+q/2\sigma):\rho_j^+(q)\rangle\rangle^t, \tag{4.2}$$

where

$$\langle\langle A:B\rangle\rangle^t = -i\theta(t)\langle[A(t),B(0)]\rangle$$
$$=\int_{-\infty}^{+\infty}\frac{d\omega}{2\pi}\langle\langle A:B\rangle\rangle^{\omega}e^{-i\omega t},$$

and $A(t)$ is the Heisenberg representation of an operator A. The equation of motion for $R_{i:j}(\boldsymbol{k}\sigma;\boldsymbol{q},t)$ is obtained with the use of the Hamiltonian (2.1) as

$$(i\partial/\partial t-\boldsymbol{q}\boldsymbol{k}/m_i)\,R_{i:j}(\boldsymbol{k}\sigma;\boldsymbol{q},t) \tag{4.3}$$

$$=\delta(t)\,\delta_{ij}\,e_i[n_i(\boldsymbol{k}-\boldsymbol{q}/2\sigma)-n_i(\boldsymbol{k}+\boldsymbol{q}/2\sigma)]$$

$$+\sum_{i'}\sum_{q'}\sum_{k'\sigma'}e_i e_{i'}v(q')\langle\!\langle C_i^{+}(\boldsymbol{k}-\boldsymbol{q}'/2-(\boldsymbol{q}-\boldsymbol{q}')/2\sigma)\,C_{i'}^{+}(\boldsymbol{k}'-\boldsymbol{q}'/2\sigma')\,C_{i'}(\boldsymbol{k}'+\boldsymbol{q}'/2\sigma')\,C_i(\boldsymbol{k}-\boldsymbol{q}'/2+(\boldsymbol{q}-\boldsymbol{q}')/2\sigma)$$

$$-C_i^{+}(\boldsymbol{k}+\boldsymbol{q}'/2-(\boldsymbol{q}-\boldsymbol{q}')/2\sigma)\,C_{i'}^{+}(\boldsymbol{k}'-\boldsymbol{q}'/2\sigma')\,C_{i'}(\boldsymbol{k}'+\boldsymbol{q}'/2\sigma')\,C_i(\boldsymbol{k}+\boldsymbol{q}'/2+(\boldsymbol{q}-\boldsymbol{q}')/2\sigma):\rho_j^{+}(\boldsymbol{q})\rangle\!\rangle'.$$

Let us define another function as follows:

$$R_{ii':j}(\boldsymbol{k}\sigma,\boldsymbol{k}'\sigma';\boldsymbol{q},\boldsymbol{q}',t)\equiv\langle\!\langle C_i^{+}(\boldsymbol{k}-\boldsymbol{q}/2\sigma)\,C_{i'}^{+}(\boldsymbol{k}'-\boldsymbol{q}'/2\sigma')\,C_{i'}(\boldsymbol{k}'+\boldsymbol{q}'/2\sigma')\,C_i(\boldsymbol{k}+\boldsymbol{q}/2\sigma);\rho_j^{+}(\boldsymbol{q}+\boldsymbol{q}')\rangle\!\rangle$$

$$-\delta_{q,0}n_i(\boldsymbol{k}\sigma)\,R_{i':j}(\boldsymbol{k}'\sigma';\boldsymbol{q}'t)-\delta_{q'0}n_{i'}(\boldsymbol{k}'\sigma')\,R_{i:j}(\boldsymbol{k}\sigma;\boldsymbol{q},t). \tag{4.4}$$

With the use of this definition, we get from eqs.(4.1) and (4.3)

$$D^{r}_{ij}(\boldsymbol{q},\omega)=\delta_{ij}D_i^{(0)}(\boldsymbol{q},\omega)+v(q)\sum_{i'}[D_i^{(0)}(\boldsymbol{q},\omega)\,D^{r}_{i'j}(\boldsymbol{q},\omega)+D^{r}_{ii'}(\boldsymbol{q},\omega)\,D_j^{(0)}(\boldsymbol{q},\omega)]/2$$

$$+\sum_{i'}[\widetilde{R}_{ii':j}(\boldsymbol{q},\omega)+\widetilde{R}^{*}_{ji':i}(-\boldsymbol{q},-\omega)]/2, \tag{4.5}$$

where $D_i^{(0)}(\boldsymbol{q},\omega)$ is defined by eq.(3.2),

$$\widetilde{R}_{ii':j}(\boldsymbol{q},\omega)=\sum_{q'}\sum_{k\sigma}\sum_{k'\sigma'}e_i^2 e_{i'}\,v(q')\Big[(\omega-\boldsymbol{q}\boldsymbol{k}/m_i-\boldsymbol{q}\boldsymbol{q}'/2m_i+i\varepsilon)^{-1}$$

$$-(\omega-\boldsymbol{q}\boldsymbol{k}/m_i+\boldsymbol{q}\boldsymbol{q}'/2m_i+i\varepsilon)^{-1}\Big]R_{ii':j}(\boldsymbol{k}\sigma,\boldsymbol{k}'\sigma';\boldsymbol{q}-\boldsymbol{q}',\boldsymbol{q}',\omega), \tag{4.6}$$

and the star on the function means the replacement of $+i\varepsilon$ to $-i\varepsilon$. If we neglect the last term in eq.(4.5), we obtain the RPA response function. Therefore, we must calculate this term in order to proceed beyond RPA. In general, $D^{r}_{ij}(\boldsymbol{q},\omega)=D^{r*}_{ji}(-\boldsymbol{q},-\omega)$. Equation (4.5) is written in order that an approximation may satisfy this symmetry property automatically.

The equation of motion for $R_{ii':j}$ is obtained from eqs.(2.1) and (4.4) with the use of eq.(4.3). The calculation is lengthy but straightforward. After rearranging a number of terms according to the aspects of the interaction,[28] we obtain

$$(i\partial/\partial t-\boldsymbol{k}\boldsymbol{q}/m_i-\boldsymbol{k}'\boldsymbol{q}'/m_{i'})\,R_{ii':j}(\boldsymbol{k}\sigma,\boldsymbol{k}'\sigma';\boldsymbol{q},\boldsymbol{q}',t)$$

$$=F^{(0)}_{ii':j}(\boldsymbol{k}\sigma,\boldsymbol{k}'\sigma';\boldsymbol{q},\boldsymbol{q}',t)+F^{(0)}_{i'i:j}(\boldsymbol{k}'\sigma',\boldsymbol{k}\sigma;\boldsymbol{q}',\boldsymbol{q},t)$$

$$+F^{(2)}_{ii':j}(\boldsymbol{k}\sigma,\boldsymbol{k}'\sigma';\boldsymbol{q},\boldsymbol{q}',t)+F^{(m)}_{ii':j}(\boldsymbol{k}\sigma,\boldsymbol{k}'\sigma';\boldsymbol{q},\boldsymbol{q}',t)$$

$$+F^{(m)}_{i'i:j}(\boldsymbol{k}'\sigma',\boldsymbol{k}\sigma;\boldsymbol{q}',\boldsymbol{q},t), \tag{4.7}$$

where

$$F^{(0)}_{ii'}(k\sigma,k'\sigma';q,q',t)=\delta(t)\delta_{ij}e_i\Big[f_{ii'}(k-(q+q')/2\sigma,k'\sigma';-q')$$

$$-f_{ii'}(k+(q+q')/2\sigma,k'\sigma';-q')\Big], \tag{4.7a}$$

$$F^{(2)}_{ii':j}(k\sigma,k'\sigma';q,q',t) \tag{4.7b}$$

$$=e_ie_{i'}v(q)\Big[n_i(k-q/2\sigma)\,R_{i':j}(k'+q/2\sigma';q+q',t)-n_i(k+q/2\sigma)\,R_{i':j}(k'-q/2\sigma':q+q',t)\Big]$$

$$+e_ie_{i'}v(q')\Big[n_{i'}(k'-q'/2\sigma')\,R_{i:j}(k+q'/2\sigma;q'+q,t)-n_{i'}(k'+q'/2\sigma')\,R_{i:j}(k-q'/2\sigma:q'+q,t)\Big]$$

$$+e_ie_{i'}\sum_{q''}v(q'')\Big[R_{ii':j}(k+q''/2\sigma,k'-q''/2\sigma';q+q'',q'-q'',t)-R_{ii':j}(k-q''/2\sigma,k'+q''/2\sigma';q+q'',q'-q'',t)\Big],$$

$$F^{(m)}_{ii':j}(k\sigma,k'\sigma';q,q',t) \tag{4.7c}$$

$$=e_i\Big[f_{ii'}(k-(q+q')/2\sigma,k'\sigma';-q')-f_{ii'}(k+(q+q')/2\sigma,k'\sigma';-q')\Big]\;v(q+q')\;\sum_{i'}D^{r}_{i'j}(q+q',t)$$

$$+\sum_{i''k''\sigma''}e_ie_{i''}v(q')\,f_{i'i''}(k'\sigma',k''\sigma'';q')\Big[R_{i:j}(k+q'/2\sigma;q+q',t)-R_{i:j}(k-q'/2\sigma;q+q',t)\Big]$$

$$+\sum_{i''k''\sigma''}e_ie_{i''}v(q)\Big[n_i(k-q/2\sigma)-n_i(k+q/2\sigma)\Big]\;R_{i'i'':j}(k'\sigma,k''\sigma'';q',q,t)$$

$$+\sum_{q''i''k''\sigma''}e_ie_{i''}v(q'')\Big[R_{ii'i'':j}(k-q''/2\sigma,k'\sigma',k''\sigma'';q-q'',q',q'',t)-R_{ii'i'':j}(k+q''/2\sigma,k'\sigma',k''\sigma'';q-q'',q',q'',t)\Big],$$

In eqs.(4.7), $f_{ii'}(k\sigma,k'\sigma';q)$ is the two-particle correlation function between $ik\sigma$-particle (i-species of particle with momentum k and spin σ) and $i'k'\sigma'$-particle defined as follows:

$$f_{ii'}(k\sigma,k'\sigma';q)=\langle C_i^{+}(k-q/2\sigma)\,C_{i'}^{+}(k'+q/2\sigma')\,C_{i'}(k'-q/2\sigma')\,C_i(k+q/2\sigma)\rangle$$

$$-\delta_{q,0}\,n_i(k\sigma)\,n_{i'}(k'\sigma').$$

Also, $R_{ii'i'':j}(k\sigma,k'\sigma',k''\sigma'';q,q',q'',t)$in eq.(4.7c) can be defined in analogy with eqs.(4.2) and (4.4) and this function describes the response of the three-particle correlation among $ik\sigma$-, $i'k'\sigma'$-, and $i''k''\sigma''$-particles to the external field introduced in §2. However, since we will not be concerned about its expression in the following discussion and since it is very lengthy, we do not write down its explicit definition.

Well, $R_{ii':j}(k\sigma,k'\sigma';q,q',t)$ is the response function of the correlation between $ik\sigma$- and $i'k'\sigma'$-particles to the external field and eq. (4.7) describes its time-variation. The source of the time-variation of the correlation between $ik\sigma$- and $i'k'\sigma'$-particles in the system under the influence of the external field has three kinds of aspects[28] : (i) the instantaneous action of the external field on $ik\sigma$- or $i'k'\sigma'$-particle in the presence of the other particle denoted by $F^{(0)}_{ii':j}(k\sigma,k'\sigma':q,q',t)$ or $F^{(0)}_{i'i:j}(k'\sigma',k\sigma;q',q,t)$, respectively,

(ii) the action of the interaction between two particles in consideration denoted by $F^{(2)}_{ii':j}(k\sigma,k'\sigma';q,q',t)$, and (iii) the action of the surrounding particles to $ik\sigma$- or $i'k'\sigma'$-particle in the presence of the other particle denoted by $F^{(m)}_{ii':j}(k\sigma,k'\sigma';q,q',t)$ or $F^{(m)}_{i'i:j}(k'\sigma',k\sigma;q',q,t)$, respectively. The action (iii) leads to a screening of (i) and (ii); the first term of eq.(4.7c) screens the action of eq.(4.7a), the second-and third-terms screen the action of eq.(4.7b), and the last term of (4.7c), which is concerned with the three particle correlation among $ik\sigma$-, $i'k'\sigma'$-, and $i''k''\sigma''$ (sorrounding particle)-particles, modifies the screenings described above.

Now, the particles in the system should respond as free-particles to the high-frequency or large wave vector external-field. In such a case, in the right hand side of eq.(4.7), the first- and second-terms are dominant in comparison with the remaining terms containing the interaction between particles. On the other hand, for the external field with low frequencies and small wave vectors, the retarded action of the system, particularly, the screening effect on the above two terms should become important. Then, in order to get the response function valid both for high-frequencies or large wave vectors and for low-frequencies and small wave vectors, we shall take this screening effect into account and neglect the effective action (ii) screened by (iii) and the effect of the three particle correlation:

$$F^{(2)}_{ii':j}(k\sigma,k'\sigma';q,q',t) \simeq 0, \tag{4.8a}$$

$$F^{(m)}_{ii':j}(k\sigma,k'\sigma';q,q',t) \simeq e_i\Big[f_{ii'}(k-(q+q')/2\sigma,k'\sigma';-q')$$
$$-f_{ii'}(k+(q+q')/2\sigma,k'\sigma';-q')\Big]\ v(q+q')\sum_{i''}D^{\mathrm{r}}_{i''j}(q+q',t), \tag{4.8b}$$

$$F^{(m)}_{i'i:j}(k'\sigma',k\sigma;q',q,t) \simeq e_{i'}\Big[f_{i'i}(k'-(q'+q)/2\sigma',k\sigma;-q)$$
$$-f_{i'i}(k'+(q'+q)/2\sigma',k\sigma;-q)\Big]\ v(q+q')\sum_{i''}D^{\mathrm{r}}_{i''j}(q+q',t). \tag{4.8c}$$

It is easy to solve eq.(4.7) with the approximation of eqs. (4.8a~c). The substitution of the obtained solution into eq.(4.6) yields

$$\widetilde{R}_{ii':j}(q,\omega) = \Big[\delta_{ij}+v(q)\sum_{i''}D^{\mathrm{r}}_{i''j}(q,\omega)\Big]\ D^{\mathrm{p}}_{ii'}(q,\omega)$$
$$+\Big[\delta_{i'j}+v(q)\sum_{i''}D^{\mathrm{r}}_{i''j}(q,\omega)\Big]\ D^{\mathrm{c}}_{ii'}(q,\omega), \tag{4.9}$$

where

$$D^{\mathrm{p}}_{ii'}(q,\omega) = \sum_{q'}\sum_{k\sigma}\sum_{k'\sigma'}e_i^3 e_{i'}v(q')f_{ii'}(k\sigma,k'\sigma';-q') \tag{4.10a}$$

$$\times\Big\{\frac{1}{\omega-q^2/2m_i-(k-q'/2)q/m_i+(k/m_i-k'/m_{i'})q'+i\varepsilon}\Big[\frac{1}{\omega-q^2/2m_i-(k+q'/2)q/m_i+i\varepsilon}$$
$$-\frac{1}{\omega-q^2/2m_i-(k-q'/2)q/m_i+i\varepsilon}\Big]-\frac{1}{\omega+q^2/2m_i-(k+q'/2)q/m_i+(k/m_i-k'/m_{i'})q'+i\varepsilon}$$
$$\times\Big[\frac{1}{\omega+q^2/2m_i-(k+q'/2)q/m_i+i\varepsilon}-\frac{1}{\omega+q^2/2m_i-(k-q'/2)q/m_i+i\varepsilon}\Big]\Big\},$$

$$D^{c}_{ii'}(q,\omega)=\sum_{q'}\sum_{k\sigma}\sum_{k'\sigma'}e_i^2e_{i'}^2v(q+q')f_{ii'}(k\sigma,k'\sigma';-q') \tag{4.10b}$$

$$\times\left(\frac{1}{\omega-q^2/2m_i-(k+q'/2)q/m_i+i\varepsilon}-\frac{1}{\omega+q^2/2m_i-(k-q'/2)q/m_i+i\varepsilon}\right)$$

$$\times\left[\frac{1}{\omega-q^2/2m_{i'}-(k'+q'/2)q/m_{i'}+(k/m_i-k'/m_{i'})q'+i\varepsilon}\right.$$

$$\left.-\frac{1}{\omega+q^2/2m_{i'}-(k'-q'/2)q/m_{i'}+(k/m_i-k'/m_{i'})q'+i\varepsilon}\right]$$

The $ik\sigma$- and $i'k'\sigma'$-particles are treated symmetrically in eq. (4.7). However, when we consider the roles of these particles in eq.(4.6), we see that $i'k'\sigma'$-particle is one of the correlation particle (or hole) around $ik\sigma$-particle in consideration. Therefore, the first term of eq.(4.7) raises up the motion of $ik\sigma$-particle relative to its correlation particle (or hole), while the second term is concerned with the dynamics of the correlation particle (or hole) around $ik\sigma$-particle.[28] The functions $D^{p}_{ii'}$ and $D^{c}_{ii'}$ in eq.(4.9) correspond to the response of the particle motion in its correlation particle (or hole) and the dynamics of the correlation particle (or hole) around the particular particle, respectively.

Now, we shall assume in the following

$$f_{ii'}(k\sigma,k'\sigma';q)=f_{ii'}(k\sigma,k'\sigma';-q).$$

Then we can show $D^{p}_{ii'}(q,\omega)=D^{p*}_{ii'}(-q,-\omega)$ and $D^{c}_{ii'}(q,\omega)=D^{c*}_{ii'}(-q,-\omega)$. These functions, however, are not invariant under the interchange of i and i'. Let us define

$$D^{c\pm}_{ii'}(q,\omega)=\left[D^{c}_{ii'}(q,\omega)\pm D^{c}_{i'i}(q,\omega)\right]/2$$

From eqs.(2.4), (4.5) and (4.9), we get the dielectric function and the response function as

$$\varepsilon(q,\omega)=1-v(q)\sum_i I_i(q,\omega), \tag{4.11}$$

$$D^{r}_{ij}(q,\omega)=\delta_{ij}D^{(0)}_i+\delta_{ij}\sum_{i'}D^{p}_{ii'}+D^{c-}_{ij}$$

$$+v[I_i+L_i][I_j+L_j]/\varepsilon-v\varepsilon L_iL_j, \tag{4.12}$$

where

$$I_i(q,\omega)=D^{(0)}_i(q,\omega)+\sum_{i'}[D^{p}_{ii'}(q,\omega)+D^{c+}_{ii'}(q,\omega)],$$

$$L_i(q,\omega)=\sum_{i'}D^{c-}_{ii'}(q,\omega)/(1+\varepsilon).$$

In order to reexpress our result (4.12) in terms of the local field correction appearing in eq.(3.1), we express G_{ij} in terms of D^{r}_{ij} by solving eq.(3.3) inversely as

$$G_{ij}(q,\omega)=1-\delta_{ij}/\left[v(q)D^{(0)}_i(q,\omega)\right]+\mathcal{D}_{ij}(q,\omega)/v(q), \tag{4.13}$$

where

$$\mathcal{D}_{ij}(q,\omega)=\left[\delta_{ij}\sum_{i=1}^{2}D^{r}_{ii}(q,\omega)-D^{r}_{ij}(q,\omega)\right]/\left[D^{r}_{11}(q,\omega)D^{r}_{22}(q,\omega)-D^{r}_{12}(q,\omega)D^{r}_{21}(q,\omega)\right]. \tag{4.14}$$

With somewhat lengthy but strainhtforward calculations, we get from eqs.(4.12), (4.13), and (4.14)

$$G_{ij}=\frac{\delta_{ij}I'_i-D^{c+}_{12}+v[I_iL'_i+I'_jL'_i+2L^2_i(1-\varepsilon)]}{v[I_1I_2-D^{c+}_{12}(I_1+I_2)+(1-\varepsilon)^2L^2_1]}-\frac{\delta_{ij}}{vD^{(0)}_i} \tag{4.15}$$

where $I'_1=I_2$, $I'_2=I_1$, $L'_1=L_2$, and $L'_2=L_1$

Now, let us investigate the limiting behaviors of the results (4.11), (4.12) and (4.15). First, we shall study the case of large wave vectors q or high frequencies ω. When we expand eqs.(3.2), (4.10a) and (4.10b) in inverse powers of$(\omega\pm q^2/2m_i)$we get

$$D^{(0)}_i(q,\omega)=\frac{\omega^2_i/v(q)}{\omega^2-(q^2/2m_i)^2} \tag{4.16a}$$

$$D^{c}_{ij}(q,\omega)=\frac{\omega^2_i\,\omega^2_j/v(q)}{[\omega^2-(q^2/2m_i)^2][\omega^2-(q^2/2m_j)^2]}\sum_{q'}\frac{[q(q+q')]^2}{q^2(q+q')^2}g_{ij}(q'), \tag{4.16b}$$

and

$$D^{p}_{ij}(q,\omega)=\frac{-\omega^2_i\,\omega^2_j\,m_je_i}{2v(q)m_ie_j}\left[(\omega-q^2/2m_i)^{-4}+(\omega+q^2/2m_i)^{-4}\right]\sum_{q'}\frac{(qq')^2}{q^2q'^2}g_{ij}(q'), \tag{4.16c}$$

where we used

$$\sum_{k\sigma}\sum_{k'\sigma'}f_{ii'}(k\sigma,k'\sigma';-q)=n_in_{i'}g_{ii'}(q),$$

From these asymptotic expressions, we see that $D^{c}_{ij}/D^{(0)}_i$ and $D^{p}_{ij}/D^{(0)}_i$are of order of $(\omega\pm q^2/2m_i)^{-2}$. Then, we get the asymptotic expansions of eqs.(4.12) and (4.15) for large q or high ω as

$$D^{r}_{ij}(q,\omega)=\delta_{ij}D^{(0)}_i(q,\omega)+\delta_{ij}\sum_{i'}D^{p}_{ii'}(q,\omega)+D^{c}_{ij}(q,\omega)+v(q)D^{(0)}_i(q,\omega)D^{(0)}_j(q,\omega), \tag{4.17}$$

$$G_{ij}(q,\omega)=\delta_{ij}\alpha_i(q,\omega)\sum_{i'}\frac{n_{i'}e_{i'}}{n_ie_i}\sum_{q'}\frac{(qq')^2}{q^2q'^2}g_{ii'}(q')$$

$$-\sum_{q'}\frac{[q(q+q')]^2}{q^2(q+q')^2}g_{ij}(q'), \tag{4.18}$$

where

$$\alpha_i(q,\omega)=\frac{1}{2}\left\{\frac{(\omega+q^2/2m_i)^2}{(\omega-q^2/2m_i)^2}+\frac{(\omega-q^2/2m_i)^2}{(\omega+q^2/2m_i)^2}\right\}$$

and $D^{(0)}_i$, $D^{c}_{ii'}$ and $D^{p}_{ii'}$ in eq.(4.17) are given by eqs.(4.16a), (4.16b), and (4.16c), respectively. The counterpart in the one-component electron liquid of the result (4.18) has been first derived by Niklasson.[28] This result is the generalization of the Niklasson's one to the case of the two component system. When q is finite but ω tends to infinity, the result agrees with the exact result (3.5). This guarantees that our results (4.11), (4.12), and (4.15) satisfy the third sum rule (2.12).

The expression (4.17), when we replace ω to $\omega+i\varepsilon$, can be used to study the static form factor for large q.[28] Niklasson[28] discussed the validity of this in detail in Appendix of his paper. From eq.(2.7), (2.10) and (4.17), we get

$$\lim_{q\to\infty}\left[S_{ij}(q)-\delta_{ij}\right]=\frac{-8\pi n_j}{a_{ij}q^4}g_{ij}(r)\Big|_{r=0}$$

where $a_{ij}=(1/m_i+1/m_j)/2e_ie_j$. On the other hand, the pair correlation function has the general property as

$$\lim_{q\to\infty}\left[S_{ij}(q)-\delta_{ij}\right]=\frac{-8\pi n_j}{q^4}\frac{\partial}{\partial r}g_{ij}(r)\Big|_{r=0}$$

Therefore, we obtain

$$\left[\partial g_{ij}(r)/\partial r\right]_{r=0}=g_{ij}(r=0)/a_{ij}$$

It is well-known that such a relation like this is derived from the solution of the two-particle Schrödinger equation. This exact relation has been derived by Kimball[29] and Niklasson[28] in the electron liquid. The method employed here is that of Niklasson.[28]

As for the static behavior of the results, we can show from eqs.(3.2), (4.10a), and (4.10b) that $D^{(0)}_i(q,0)$ and $D^{\mathrm{p}}_{ij}(q,0)$ are of order of q^0 and $D^{\mathrm{c}+}_{ij}(q,0)$ and $D^{\mathrm{c}-}_{ij}(q,0)$ are of order of q^2, when q tends to zero. Therefore, the dielectric function (4.11) satisfies the perfect screening sum rule (2.8).

§ 5 . Discussion

The dielectric function (4.11), the response function (4.12), and the corresponding local field correction (4.15) have been calculated by solving the equation of motion (4.7) with the approximation (8.8a,b,c). As is seen from the fact that the effect of two particle excitation to the response is explicitely involved in eqs.(4.10a) and (4.10b), the damping width of plasmon is taken into account in our result.
In fact, since

$$\lim_{q\to 0}\left[D^{\mathrm{p}}_{ij}(q,\omega)+D^{\mathrm{c}+}_{ij}(q,\omega)\right]=(e_i/m_i-e_j/m_j)O(q^2),$$

the dielectric function (4.11) gives finite damping width of plasmon even in $q\to 0$. This may be one of the characteristics of multi-component system.[30]

The obtained results are functionals of the occupation number $n_i(\boldsymbol{k}\sigma)$ and the two-partice correlation function $f_{ii'}(\boldsymbol{k}\sigma,\boldsymbol{k}'\sigma';\boldsymbol{q})$. Speaking from the completeness of the theory, these functions should also be calculated with the approximation which is consistent with (4.8a,b,c). We may, however, assume

$$n_i(\boldsymbol{k}\sigma)\cong n_i^{(0)}(\boldsymbol{k}\sigma),$$

$$f_{ii'}(\boldsymbol{k}\sigma,\boldsymbol{k}'\sigma';\boldsymbol{q})\cong n_i^{(0)}(\boldsymbol{k}\sigma)n_{i'}^{(0)}(\boldsymbol{k}'\sigma')g_{ii'}(\boldsymbol{q}),$$

where $n^{(0)}_i(\boldsymbol{k}\sigma)$ is the occupation number in the non-interacting system and $g_{ii'}(\boldsymbol{q})$ is the Fourier transform of the pair correlation function $g_{ii'}(\boldsymbol{r})$. Such a assumption conserves all the limiting behaviors investigated in the text and has been usually done in the available local field theories. In such theories, $g_{ii'}(\boldsymbol{q})$ is given self-consistently from the zeroth-moment sum rule (2.10). Though we have not specified explicitely the statistics obeyed by i-species of particle, this statistics is introduced when we give $n_i^{(0)}(\boldsymbol{k}\sigma)$.

The results (4.15) for the some limiting cases are perhaps worth remarking. When ω is fixed but q tends to infinity, we get from eq.(4.18)

$$\lim_{q\to\infty} G_{ij}(q,\omega)=[1-g_{12}(0)]/3+2[1-g_{ij}(0)]/3, \tag{5.1}$$

where we used eqs.(2.14). On the other hand, when q tends to zero, we get from eqs. (4.15) and (3.5)

$$\lim_{q\to 0} G_{ij}(q,\infty)=[1-g_{12}(0)]/3, \tag{5.2}$$

$$\lim_{q\to 0} G_{ij}(q,\omega)=O(q^0), \tag{5.3}$$

and

$$\lim_{q\to 0} G_{ij}(q,0)=O(q^2), \tag{5.4}$$

where we used that (a) $D_i^{(0)}(\boldsymbol{q},0)$ and $D_{ij}^{\mathrm{p}}(\boldsymbol{q},0)$ are of order of q^0 and $D_{ij}^{\mathrm{c}+}(\boldsymbol{q},0)$ and $D_{ij}^{\mathrm{c}-}(\boldsymbol{q},0)$ are of order of q^2, when q tends to zero and (b) $D_i^{(0)}(\boldsymbol{q},\omega)$, $D_{ij}^{\mathrm{p}}(\boldsymbol{q},\omega)$, $D_{ij}^{\mathrm{c}+}(\boldsymbol{q},\omega)$ and $D_{ij}^{\mathrm{c}-}(\boldsymbol{q},\omega)$ are of order of q^2, when q tends to zero but ω is finite. Equations (5.1) and (5.2) are independent of our approximation, because the result (4.15) are exact in such limiting cases.

Now, since the local field correction of the uniform one-component plasma vanishes independently of ω in $q\to 0$, the nonvanishing of $G_{ij}(\boldsymbol{q},\omega)$ in $q\to 0$ may be a characteristic of the multi-component plasma. As pointed out in §2 and §3, eq.(5.2) is related to the internal structure of quasiparticle. Since such a response corresponds to multi-particle excitations, eq.(5.2) is due to the non-vanishing of multi-particle excitations due to the external field with large ω but infinitely small q. As for the case of finite ω, Pine and Nozières[4] show that multi-particle excitation does compete with plasmon in all the frequency moment sum rules for a dynamical form factor in $q\to 0$, when the total current operator does not commute with the Hamiltonian. In the multi-component plasma in consideration, even if the total momentum commutes with the Hamiltonian, the total current operator does not do so. Therefore, in the multi-component plasma, multi-particle excitations due to the external field does not vanish even in $q\to 0$, independently of frequency of the field. This is the reason of eq.(5.3). In other word, since the local field described by $G_{ij}(\boldsymbol{q},\omega)$ does relax even in $q\to 0$ through the decay into the multi-particle excitations and its relaxation in $q\to 0$, according to the local field theory in §3, is closely related to Im $G_{ij}(\boldsymbol{q},\omega)$, then $\lim_{q\to 0}$ Im $G_{ij}(\boldsymbol{q},\omega)\neq 0$. The response described above, however, is essentially dynamical. The static local field may have the property of eq. (5.4), as usual.

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