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1. Let k be an integer ≥ 2 and fix it. Let N be any positive integer and let $N=DX^k$. Then D is uniquely determined and called the k -th power free kernel of N . Let $\{a_n\}$ be any sequence of positive integers and $a_n \rightarrow \infty$ as $n \rightarrow \infty$. Then we prove following

THEOREM 1. *If $k \geq 5$, then the sequence $\{a_n^n+1\}$ has infinitely many distinct k -th power free kernels.*

THEOREM 2. *Let $\{a_n\}$ be any sequence of even integers. Then the sequence $\{a_n^n+1\}$ has infinitely many distinct k -th power free kernels.*

In particular, for $k=2$ Theorem 2 shows that the odd sequence a_n^n+1 generates infinitely many real quadratic fields $\mathbf{Q}(\sqrt{a_n^n+1})$.

2. PROOF.

Proof of Theorem 1: Assume that there are a finite number of k -th power free kernels of a_n^n+1 when n runs all natural numbers. We denote them D_1, D_2, \dots, D_s . Let p_1, \dots, p_t be all distinct prime divisors of D_1, \dots, D_s and m any positive integer. Put

$$n = (p_1 - 1) \dots (p_t - 1) km.$$

If we take m sufficiently large, then by Tijdeman's result [2, Theorem 1] and our assumption we can write

$$a_n^n + 1 = D_i X^k$$

for some $D_i > 1$ and we may assume $D_i \neq 2$ by the result of Domar [1]. Taking any prime divisor $p_j \neq 2$ of D_i , by Fermat's theorem we have

$$-1 \equiv a_n^n \equiv (a_n^{n(p_j-1)})^{p_j-1} \equiv 0 \text{ or } 1 \pmod{p_j},$$

which is a contradiction since $p_j \neq 2$.

Proof of Theorem 2 is similar to one of Theorem 1. In this case every

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p_j is not 2.

References

- [1] Domar, Y., On the diophantine equation $|Ax^n - By^n| = 1$, $n \geq 5$. Math. Scand., **2** (1954) 29—32 (Mordell, Diophantine Equations § 28 Theorem 12, p. 274)
- [2] Tijdeman, R., On the equation of Catalan, Acta Arith. **29** (1976) 179—209.