The Coupling of L-O Phonons and Plasmons in Polar Semiconductors

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Katsukuni YONASHIRO*

Abstract

The coupling between longitudinal optical phonons and plasmons in polar semiconductors has been investigated.

At the appropriate carrier density ($10^{17} \sim 10^{18}$ cm^{-3} in III·V semiconductor compounds) the frequency of carrier plasma oscillations is comparable with that of the lattice vibrations of the optical modes and remarkable coupling between them is to be expected.

In the long wavelength limit the coupled eigen frequencies of the coupled oscillations is obtained by relating the one particle Boltzmann-Vlasov equation as well as the single particle Liouville equation describing the motion of free carriers to the equations of Born and Huang for the optical lattice vibrations in ionic crystals.

1. Introduction

The electron-phonon interaction has given very important informations clarifying various properties of solids.

In this paper, we will obtain the coupled frequencies of plasma oscillations of longitudinal collective motion of free carriers and longitudinal optical phonons of collective motion of the lattice vibrations in polar semiconductors. As is well-known, the spectra of the lattice vibration in polar semiconductor have acoustic branch and optical branch. In acoustic branch the positive and the negative ions move in unison, but in the optical branch they oscillate in anti-phase. Consequently, the polarization field produced by electric polarization with the longitudinal wave of optical branch shakes free carriers. Since the polarization field propagates as a wave motion with lattice vibration in crystal, the carriers shaked by the polarization field are bunched. The carriers thus bunched produce electric field in themselves and then the electric field acting upon the ions provides reaction to the lattice vibrations.

I. Yokota[1] first predicted the existence of the frequencies of the coupled oscillations described above and dealt with this problem theoretically. The Yokota's idea was as follows: When carrier concentration in a polar semiconductor amounts to $10^{17} \sim 10^{18} cm^{-3}$, the frequency of carrier plasma oscillations becomes of the same order in magnitude as that of the lattice vibrations of the optical modes and

* Division of General Education, Univ. of the Ryukyus.

remarkable coupling between them is to be expected. For example, taking carrier effective mass, $m^* = 0.3m$, optical dielectric constant, $\epsilon_\infty = 10$, and carrier concentration, $n=10^{17}cm^{-3}$, the plasma oscillation frequency, $\omega_p = \left(\frac{4\pi n e^2}{\epsilon_\infty m^*}\right)^{\frac{1}{2}}$, results in 10^{13} sec^{-1}, which is of the order in magnitude of Reststrahlen frequency in typical ionic crystals. I. Yokota obtained the coupled freguencies by relating hydrodynamic equations of motion describing the plasma oscillations of carriers of long wavelengths to equations of Born and Huang[2] describing the lattice vibration of the optical modes of wavelengths in ionic crystals. His result is as follows;

$$\omega^2 = \frac{1}{2}\left\{(\omega_p^2+\omega_l^2) \pm \left[(\omega_p^2+\omega_l^2)^2 - 4\omega_p^2\omega_t^2\right]^{1/2}\right\},$$

where ω_t and ω_l are the transverse optical and longitudinal optical phonon frequencies, respectively.

B. B. Varga[3] has shown later that in the long-wavelength limit, the valence electrons, the polar lattice vibrations, and the conduction electrons make additive contributions to the total dielectric constant. Free longitudinal oscillations then occur in the system, whenever the conditions are such that the total dielectric constant equals zero. In this method he obtained the same result of Yokota's.

In the section 2, we will derive the coupled eigenfrequencies by relating the one particle Boltzmann-Vlasov equation and the single particle Liouville equation for the motion of free carriers to the equation of Born and Huang describing the optical lattice vibrations in ionic crystals.

In the section 3, the obtained result is discussed.

2. Calculation

In this section we will obtain the coupled eigenfrequencies of the coupled oscillations by relating the Boltzmann-Vlasov equation for free carrier plasma oscillations to the equations of Born and Huang describing the lattice vibrations of the optical modes in ionic crystals. Then by using the single particle Liouville equation instead of the Boltzmann-Vlasov equation, the same coupled frequencies above will be obtained.

According to Born and Huang the lattice vibrations of long wavelengths in ionic crystals are described by following equations:

$$\frac{\partial^2 \vec{W}}{\partial t^2} = -\omega_t^2 \vec{W} + b_{12}\vec{E}, \tag{2.1}$$

$$\vec{P} = b_{21}\vec{W} + \frac{\epsilon_\infty - 1}{4\pi}\vec{E}, \tag{2.2}$$

where $\vec{W}$ is the displacement of the positive ion relative to the negative, $\vec{u}_+ - \vec{u}_-$, multiplied by the square root of the reduced mass, $M = M_+M_-/(M_+ + M_-)$, per unit volume: $\vec{W} = \sqrt{\frac{M}{v_a}}\ (\vec{u}_+ - \vec{u}_-)$, v_a the volume of the unit cell, $\vec{P}$ the dielectric polarization, $\vec{E}$ the self-consistent electric field, $b_{12} = b_{21} = \omega_t \left(\frac{\epsilon_0 - \epsilon_\infty}{4\pi}\right)^{1/2}$ and ϵ_0 the static dielectric constant.

In equation (2.1) the forces acting on the lattice are the force obeying Hook's law, and the self-consistent electric field $\vec{E}$. Equation (2.2) relates the macroscopic polarization $\vec{P}$ in a given region to the relative displacements as well as to the field $\vec{E}$ polarizing the ions.

On the other hand, a carrier plasma collective motion is described by the Boltzmann-Vlasov equation in the gas plasma theory[4] with the collision term neglected. The neglection of the collision term means to neglect of short range fluctuation and is valid in the long wavelength limit.

We assume that we have one kind of free carriers, say electrons, with isotropic effective mass m* and charge $-e$. The distribution function of carriers is expressed as follows:

$$f(\vec{r}, \vec{v}, t) = f_0(\vec{v}) + f_1(\vec{r}, \vec{v}, t), \tag{2.3}$$

where $f_0(\vec{v})$ is the equilibrium distribution for the case without selfconsistent field and $f_1(\vec{r}, \vec{v}, t)$ is the deviation from the $f_0(\vec{v})$, which is considered to be of the same order as electric field. The linearized one particle Boltzmann-Vlasov equation[4] is given as follows:

$$\frac{\partial f_1}{\partial t} + (\vec{v} \cdot \vec{\nabla}_r) f_1 - \frac{e}{m^*}(\vec{E} \cdot \vec{\nabla}_v) f_0 = O \tag{2.4}$$

where $\vec{\nabla}_r = \frac{\partial}{\partial \vec{r}}$ and $\vec{\nabla}_v = \frac{\partial}{\partial \vec{v}}$. In obtaining this result we have used the fact that $\frac{\partial f_0}{\partial t} = \vec{\nabla}_v f_0 = O$. In equation (2.4) $\vec{E}$ is the self-consistent electric field acting on individual electrons and depends on the distribution function $f(\vec{r}, \vec{v}, t)$. The deviation from the equilibrium distribution produces the charge density;

$$\rho(\vec{r}, t) = -en \int f_1(\vec{r}, \vec{v}, t)\, d^3\vec{v}, \tag{2.5}$$

where n is the carrier density. This charge density $\rho(\vec{r}, t)$ in turn would produce the Coulomb field in vacuum : the vacuum electric field, $\vec{E}_{vac}$ is expressed as follows:

$$\vec{E}_{vac} = -\vec{\nabla}_r \int \frac{\rho(\vec{r}', t)}{|\vec{r} - \vec{r}'|} d^3\vec{r}' = -en \int \frac{\vec{r} - \vec{r}'}{|\vec{r} - \vec{r}'|^3} d^3\vec{r}' \int f_1(\vec{r}', \vec{v}', t)\, d^3\vec{v}'. \tag{2.6}$$

From the Poisson's equation in electrostatics

$$\vec{\nabla}(\vec{E}+4\pi\vec{P}) = 4\pi\rho,$$

the following relation holds between $\vec{E}_{vac}$, $\vec{E}$ and $\vec{P}$

$$\vec{E} = \vec{E}_{vac} - 4\pi\vec{P}. \tag{2.7}$$

Because of the linearity of the equations appeared above, we can consider a single Fourier component of $\vec{W}$,

$$\vec{W} = \frac{\vec{q}}{|\vec{q}|} w(\vec{q}, \omega) \exp(i\vec{q}\cdot\vec{r} - i\omega t). \tag{2.8}$$

Here we have used the fact that since a carrier collective plasma oscillations are longitudinal, only longitudinal modes of optical lattice vibrations can be coupled with them. Accordingly, the deviation f_1 from the equlibrium distribution f_0 will have the form

$$f_1(\vec{r},\vec{v},t) = g(\vec{v})\exp(i\vec{q}\cdot\vec{r} - i\omega t). \tag{2.9}$$

Putting eq. (2.9) into eq. (2,6) and carrying out the integration with respect to $\vec{r}'$ we find

$$\vec{E}_{vac} = i\vec{q}\frac{4\pi ne}{|\vec{q}|^2}\exp(i\vec{q}\cdot\vec{r} - i\omega t)\cdot\int g(\vec{v}')\,d^3\vec{v}'. \tag{2.10}$$

Eliminating $\vec{P}$ from eq • (2.2), eq • (2.7) and substituting eq • (2.10), equation (2.7) leads to

$$\vec{E} = \frac{1}{\varepsilon_\infty}\vec{E}_{vac} - \frac{4\pi b_{21}}{\varepsilon_\infty}\vec{W}$$

$$= i\vec{q}\frac{4\pi ne}{\varepsilon_\infty|\vec{q}|^2}\exp(i\vec{q}\cdot\vec{r}-i\omega t)\cdot\int g(\vec{v}')d^3\vec{v}' - \frac{4\pi b_{21}}{\epsilon_\infty}w(\vec{q},\omega)\frac{\vec{q}}{q}\exp(i\vec{q}\cdot\vec{r}-i\omega t). \tag{2.11}$$

This $\vec{E}$ will be entered into equations of Born and Huang, and Boltzmann-Vlasov equation. By substituting this result into eq• (2.1), we obtain

$$\frac{\partial^2\vec{W}}{\partial t^2} = -\omega_1^2\vec{W} + \frac{b_{12}}{\epsilon_\infty}\vec{E}_{vac}$$

or

$$w(\vec{q},\omega) = -\frac{i4\pi b_{12}ne}{\epsilon_\infty|\vec{q}|}\cdot\frac{1}{\omega^2-\omega_l^2}\int g(\vec{v}')\,d^3\vec{v}', \tag{2.12}$$

where we used the relation of Lyddane-Sacks-Teller : $\omega_t^2 = \frac{\epsilon_\infty}{\epsilon_0}\omega_l^2$.

On the other hand substitutions of eq • (2.9) and eq • (2.11) into eq • (2.4) yield

$$\left[-i\omega+i(\vec{q}\cdot\vec{v})\right]g(\vec{v})-i\frac{\omega_p^2}{|\vec{q}|^2}(\vec{q}\cdot\vec{\nabla}_v)f_0\int g(\vec{v}')d^3\vec{v}'+\frac{4\pi b_{21}e}{m^*\epsilon_\infty q}w(\vec{q},\omega)(\vec{q}\cdot\vec{\nabla}_v)f_0=0. \tag{2.13}$$

Here we introduced plasma frequency: $\omega_p^2 = \frac{4\pi ne^2}{m^* \epsilon_\infty}$. Combining above equation with eq. (2.13) and eliminating $w(\vec{q},\omega)$, we arrived at the following equation :

$$\left[\omega - (\vec{q}\cdot\vec{v})\right] g(\vec{v}) + \frac{\omega_p^2}{|\vec{q}|^2}\left(1 + \frac{\omega_l^2 - \omega_t^2}{\omega^2 - \omega_l^2}\right)\cdot(\vec{q}\cdot\vec{\nabla}_v) f_o \int g(\vec{v}')\, d^3\vec{v}' = O,$$

therefore,

$$1 + \frac{\omega_p^2}{|\vec{q}|^2}\,\frac{\omega^2 - \omega_t^2}{\omega^2 - \omega_l^2}\int \frac{(\vec{q}\cdot\vec{\nabla}_v) f_o}{\omega - (\vec{q}\cdot\vec{v})}\, d^3\vec{v} = O. \qquad (2.14)$$

Assuming that $\vec{q} = (q, o, o)$ and $\omega \gg (\vec{q}\cdot\vec{v})$, we get

$$\int \frac{(\vec{q}\cdot\vec{\nabla}_v) f_o}{\omega - (\vec{q}\cdot\vec{v})}\, d^3\vec{v} = -\frac{q^2}{\omega^2}\left[1 + \frac{3}{\omega^2} < v^2 > q^2 + \cdots\cdots\right], \qquad (2.15)$$

where $< v^2 > = \int v \cos(\vec{q}\wedge\vec{v}) f_o\, d^3\vec{v}$.

Hence in the long wavelength limit $q \to o$ according to eq. (2.14) we thus obtain coupled eigen frequencies of the coupled oscillations:

$$\omega^2 = \frac{1}{2}\left\{(\omega_l^2 + \omega_p^2) \pm \left[(\omega_l^2 + \omega_p^2)^2 - 4\omega_p^2\, \omega_t^2\right]^{1/2}\right\} \qquad (2.16)$$

This result coincides with the Yokota's.

Next we shall examine the derivation by using the Liouville equation for the single particle density matrix. Let's consider the following single particle Hamiltonian

$$H = H_o + V(\vec{r}, t),$$

where H_o is the free elctron Hamiltonian: $H_o = p^2/2m$ and $V(\vec{r}, t)$ is the self-consistent potential regarded as perturbation describing the interaction of the lattice vibrations and density fluctuations. The eigenfunctions $\Psi(\vec{r}, t)$ of H may be expanded in terms of eigen functions of H_o:

$$\Psi(\vec{r}, t) = \sum_n a_n(t)\, u_n(\vec{r}),$$

here $u_k(\vec{r}) = \Omega^{\frac{1}{2}} \exp(i\vec{k}\cdot\vec{r})$, Ω is volume of the system. Then the elements of the density matrix ρ are given by

$$< n \mid \rho \mid m > = \rho_{nm} = \overline{a_m^* a_n},$$

where the bar denotes the ensemble average[5]. The time dependence of the density matrix obeys the following single particle Liouville equation, which is obtained from the Schrödinger equation :

$$i\hbar \frac{\partial \rho}{\partial t} = \left[H, \rho\right], \qquad (2.17)$$

here $[H, \rho] = H\rho - \rho H$. We separate the density matrix into unperturbed and perturbed parts: $\rho = \rho_0 + \rho_1$. The operator ρ_0 describes an ensemble which does not change with time. The k-th diagonal element of ρ_0 is

$$<\vec{k} \mid \rho_0 \mid \vec{k}> = \overline{a_{\vec{k}}^* a_{\vec{k}}} = \overline{\mid <\Psi \mid \vec{k}> \mid^2} = f(\vec{k}), \tag{2.18}$$

where the function $f(\vec{k})$ is the ensemble average of the probability that the particle described by Ψ be in state $\vec{k}$, that is, the distribution function. Therefore,

$$\rho_0 \mid \vec{k}> = f(\vec{k}) \mid \vec{k}>.$$

Noting that $\left[H_0, \rho_0\right] = \frac{\partial \rho_0}{\partial t} = 0$,, and linearizing to the first order in V, we get equation (2.19) as follows

$$i\hbar \frac{\partial \rho_1}{\partial t} = \left[H_0, \rho_1\right] + \left[V, \rho_0\right]. \tag{2.19}$$

By taking the matrix element of above equation between $\vec{k}$ and $\vec{k}+\vec{q}$, we obtain as follows·

$$i\hbar \frac{\partial}{\partial t} <\vec{k} \mid \rho_1 \mid \vec{k}+\vec{q}> = <\vec{k} \mid [H_0, \rho_1] \mid \vec{k}+\vec{q}> + <\vec{k} \mid [V, \rho_0] \mid \vec{k}+\vec{q}>$$

$$= \{E(\vec{k}) - E(\vec{k}+\vec{q})\} <\vec{k} \mid \rho_1 \mid \vec{k}+\vec{q}> + \{f(\vec{k}+\vec{q}) - f(\vec{k})\} <\vec{k} \mid V \mid \vec{k}+\vec{q}>, \tag{2.20}$$

where $H_0 \mid \vec{k}> = E(\vec{k}) \mid \vec{k}>$.

In the above equation, $<\vec{k} \mid V \mid \vec{k}+\vec{q}>$ is the q-th Fourier coefficient in the expansions,

$$V(\vec{r},t) = \sum_{\vec{q},\omega} V(\vec{q},\omega) \exp(i\vec{q}\cdot\vec{r} - i\omega t).$$

Assuming that ρ_1 and V has the same time dependence, equation (2.23) yields

$$<\vec{k} \mid \rho_1 \mid \vec{k}+\vec{q}> = \frac{f(\vec{k}+\vec{q}) - f(\vec{k})}{E(\vec{k}+\vec{q}) - E(\vec{k}) - \hbar\omega} \cdot V(\vec{q}, \omega). \tag{2.21}$$

Since the ensemble average of the expectation value of any single particle operator O is

$$\overline{<O>} = \mathrm{T_r}(\rho O),$$

the density increment $n(\vec{r})$ is

$$n(\vec{r}) = \mathrm{T_r}(\rho_1 \, \delta(\vec{r}_{op} - \vec{r})).$$

Here $\vec{r}_{op}$ is the position operator and $\delta(\vec{r}_{op} - \vec{r})$ is the density operator. The trace may be written as

$$n(\vec{r})=\sum_{\vec{k},\vec{k}'} <\vec{k}' \mid \delta(\vec{r}_{op}-\vec{r}) \mid \vec{k}> <k \mid \rho_1 \mid \vec{k}'>.$$

By introducing the complete set of eigenstates $\mid\vec{x}>$ of the position operator we have the following:

$$n(\vec{r}) = \int d\vec{x} \sum_{\vec{k},\vec{k}'} <\vec{k}' \mid \vec{x}> \delta(\vec{x}-\vec{r}) <\vec{x} \mid \vec{k}> <\vec{k} \mid \rho_1 \mid \vec{k}'>$$

$$= \Omega^{-1} \sum_{\vec{k},\vec{k}'} \exp\left\{i(\vec{k}-\vec{k}')\cdot\vec{r}\right\} <\vec{k} \mid \rho_1 \mid \vec{k}'>,$$

where

$$<\vec{x} \mid \vec{k}> = \Omega^{-\frac{1}{2}} \exp(i\vec{k}\cdot\vec{x}).$$

Setting $\vec{k}'=\vec{k}+\vec{q}$, the above equation takes the form

$$n(\vec{r}) = \sum_{\vec{q}} \exp(-i\vec{q}\cdot\vec{r})\,\Omega^{-1} \sum_{\vec{k}} <\vec{k} \mid \rho_1 \mid \vec{k}+\vec{q}>.$$

On the other hand if $n(\vec{r})$ is expanded in Fourier series, we have

$$n(\vec{r}) = \sum_{\vec{q}} n(\vec{q}) \exp(-i\vec{q}\cdot\vec{r}).$$

By comparing these results and using equation (2.21) we obtain

$$n(\vec{q}) = \frac{V(\vec{q},\omega)}{\Omega} \sum_{\vec{k}} \frac{f(\vec{k}+\vec{q})-f(\vec{k})}{E(\vec{k}+\vec{q})-E(\vec{k})-\hbar\omega}. \qquad (2.22)$$

The induced density change of free carriers is related to $\vec{E}_{vac}$ in equation (2.6) by the Poisson's equation:

$$\nabla\cdot\vec{E}_{vac} = -4\pi en(r). \qquad (2.23)$$

By introducing the potential $\phi(\vec{r},t)$ as $V(\vec{r},t) = -e\phi(\vec{r},t)$, the selfconsistent electric field $\vec{E}$ is given by

$$\vec{E} = -\vec{\nabla}\phi \qquad (2.24)$$

We now expand $\vec{W}$, ϕ, and $\vec{E}_{vac}$ in the Fourier series:

$$\vec{W} = \sum_{\vec{q},\omega} \frac{\vec{q}}{q} w(\vec{q},\omega)\exp(i\vec{q}\cdot\vec{r}-i\omega t),$$

$$\phi = \sum_{\vec{q},\omega} \phi(\vec{q},\omega)\exp(i\vec{q}\cdot\vec{r}-i\omega t), \qquad (2.25)$$

$$\vec{E}_{vac} = \sum_{\vec{q},\omega} \vec{E}_{vac}(\vec{q},\omega)\exp(i\vec{q}\cdot\vec{r}-i\omega t).$$

Substitutions of eq. (2.24) and eq. (2.25) into eq. (2.1) yield

$$(\omega^2-\omega_t^2)w(\vec{q},\omega) = -ib_{12}q\phi(\vec{q},\omega). \qquad (2.26)$$

Alternatively substitutions of eq. (2.22), eq. (2.23), eq. (2.24) and eq. (2.25) into eq. (2.7) leads to

$$\left\{1-\frac{4\pi e^2}{\epsilon_\infty |\vec{q}|^2} g(\vec{q},\omega)\right\}\phi(\vec{q},\omega) = \frac{i4\pi b_{21}}{\epsilon_\infty |\vec{q}|} w(\vec{q},\omega) \qquad (2.27)$$

In order for ϕ and w to have non-trivial simultaneous solutions of the eq. (2.26) and eq. (2.27), following equations must hold:

$$\begin{vmatrix} \omega^2-\omega_t^2 & i\,b_{12}\,|\vec{q}\,| \\ -\dfrac{i4\pi b_{21}}{\varepsilon_\infty\,|\vec{q}\,|} & 1-\dfrac{4\pi e^2}{\epsilon_\infty\,|\vec{q}\,|^2}\,g(\vec{q},\omega) \end{vmatrix} = 0\ ,$$

or

$$\left(\omega^2-\omega_t^2\right)\left(1-\frac{4\pi e^2}{\varepsilon_\infty\,|\vec{q}\,|^2}\,g(\vec{q},\omega)\right)-\frac{4\pi b_{21}^2}{\epsilon_\infty}=0\ . \qquad (2.28)$$

In the long wavelength limit we will show in Appendix that,

$$g(\vec{q},\omega)\simeq\frac{n\,|\vec{q}\,|^2}{m^*\omega^2}\ .$$

By substituting this result into eq. (2.28), we would be able to derive identical result as the one given in eq. (2.16).

3. Conclusions

The coupled eigen frequencies of the coupling of L-O phonons and plasmons are obtained by relating the equations of Born and Huang to the single particle Liouville equation as well as the Boltzmann-Vlasov equation in the long wavelength limit. The obtained result completely agrees with Yokota's and is given as follows:

$$\omega_\pm^2=\frac{1}{2}\left\{(\omega_l^2+\omega_p^2)\pm\left[(\omega_l^2+\omega_p^2)^2-4\omega_p^2\,\omega_t^2\right]^{\frac{1}{2}}\right\}.$$

In the ω_+ mode the lattice and electronic polarizations point in the same direction, whereas in the ω_- mode they point in opposite directions. The ω^2/ω_l^2 is illustrated in the figure below in terms of ω_p^2/ω_l^2, which is proportional to carrier concentration.

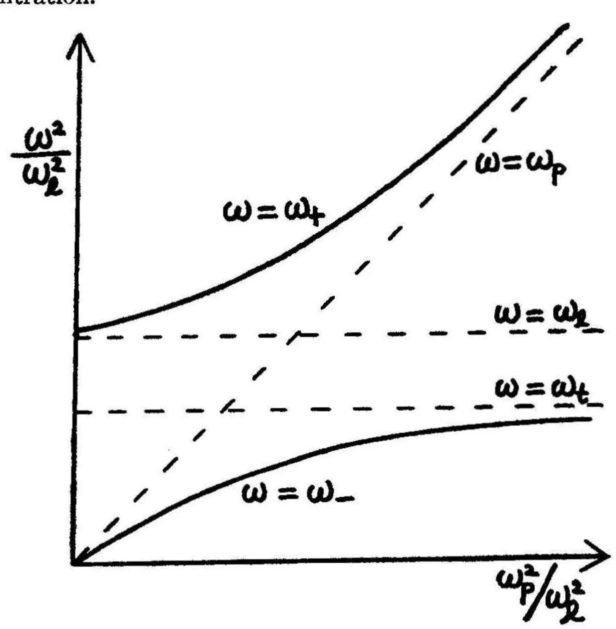

Figure. The eigen frequencies of the coupled mode ω^2/ω_l^2 is plotted against $\omega_p^2\ \omega_l^2 \propto n$.

For low carrier concentrations ($\omega_p \ll \omega_t, \omega_l$) the ω_+ mode corresponds to the lattice vibrations mixed slightly with plasma oscillations, whereas the ω_- mode corresponds to the plasma oscillations, slightly mixed with the lattice vibrations. In other words we may say that ω_+ mode is the phonons dressed in plasmons, and ω_- mode is the plasmons dressed in the phonons.

On the other hand for high carrier concentrations ($\omega_p \gg \omega_t$, ω_l) ω_+ mode corresponds to the plasmons dressed in the phonons, and ω_- mode corresponds to the phonons dressed in the plasmons.

In the neighborhood of the carrier concentration $\omega_p = \omega_l$, since the both of ω_+ and ω_- are equally mixed with phonons and plasmons, we cannot say which dress which. This circumstances are happening right on at the order of carrier concentration $10^{17}\mathrm{cm}^{-3}$ as pointed out previously.

The experimental proof of the existence of the coupled frequencies thus predicted theoretically has been established by A. Moordian and G. B. Wright[6] in 1966. They have succeeded in observing the Raman scattering by plasmons in n-type GaAs which are coupled with longitudinal optical phonons.

Finally we will show in Appendix that we can derive the same result as the one derived by D. Pines et al[7),8)], if we deal only induced electron density.

Recently much work has been reported on the coupling of longitudinal optical and free carrier collective excitations in polar semiconductors in the presence of a magnetic field[9)].

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Appendix.

We will examine that the derivation of eq. (2.29) and the plasma frequency in the long wavelength limit by dealing only induced electron density.

By neglecting the lattice polarization and using eq. (2.7), eq. (2.23) and eq. (2.24), we obtain

$$\nabla^2 \phi = 4\pi en. \tag{A.1}$$

By expanding ϕ and n into Fourier series and substituting eq. (A.1), eq. (2.22) yields

$$1 = \frac{4\pi e^2}{|\vec{q}|^2 \Omega} \sum_{\vec{k}} \frac{f(\vec{k}+\vec{q}) - f(\vec{k})}{E(\vec{k}+\vec{q}) - E(\vec{k}) - \hbar\omega} \tag{A.2}$$

This result corresponds to that of Pines et al. derived from the dielectric response function, $\epsilon(\vec{q},\omega)=0$, which is the equation to determine the self-excitation plasma frequency.

By replacing $\vec{k}+\vec{q}$ by $\vec{k}$ in the first term in the right hand side of eq. (A.2), we get the following:

$$g(\vec{q},\omega)=\Omega^{-1}\sum_{\vec{k}}\frac{f(\vec{k}+\vec{q})-f(\vec{k})}{E(\vec{k}+\vec{q})-E(\vec{k})-\hbar\omega}$$

$$=\Omega^{-1}\sum_{\vec{k}}f(\vec{k})\left\{\frac{-1}{\hbar\omega+E(\vec{k}+\vec{q})-E(\vec{k})}+\frac{1}{\hbar\omega-E(\vec{k}+\vec{q})+E(\vec{k})}\right\}. \quad (A.3)$$

In the long wavelength limit $q\to o$, we have

$$E(\vec{k}+\vec{q})-E(\vec{k})=\frac{\hbar^2}{2m}(\vec{k}+\vec{q})^2-\frac{\hbar^2}{2m}k^2\simeq\frac{\hbar^2\vec{k}\cdot\vec{q}}{m}. \quad (A.4)$$

By expanding eq. (A.3) with respect to $(E(\vec{k}+\vec{q})-E(\vec{k}))/\hbar\omega$, and using the eq. (A.4), we get the following:

$$g(\vec{q},\omega)=\Omega^{-1}\sum_{\vec{k}}f(\vec{k})\frac{2}{\hbar\omega}\left\{\frac{E(\vec{k}+\vec{q})-E(\vec{k})}{\hbar\omega}+\left(\frac{E(\vec{k}+\vec{q})-E(\vec{k})}{\hbar\omega}\right)^3+\cdots\cdots\right\}$$

$$\simeq\frac{nq^2}{\omega^2 m}.$$

Therefore, eq (A.2) yields

$$\omega^2=\frac{4\pi ne^2}{m}.$$

This is the plasma frequency ω_p in the long wavelength limit.

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