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Ryukyu Math. J., 22 (2009), 115-117

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As noticed and kindly pointed out by Alain Valette MR 2493871 (2010a: 19004), Mathematical Reviews on the Web for the paper [2] of Sudo, the following statement is not correct:

Lemma 1.1.2. *Let $\mathfrak{A} * \mathfrak{B}$ be the full free product of C^*-algebras $\mathfrak{A}$ and $\mathfrak{B}$. Suppose that there exists a common C^*-subalgebra $\mathfrak{C}$ of $\mathfrak{A}$ and $\mathfrak{B}$. Then the following is exact:*

$$0 \to K_*(\mathfrak{C}) \to K_*(\mathfrak{A}) \oplus K_*(\mathfrak{B}) \to (K_*(\mathfrak{A}) \oplus K_*(\mathfrak{B}))/K_*(\mathfrak{C}) \to 0$$

for $* = 0, 1$, where a K-theory class $[p]$ of $K_0(\mathfrak{C})$ is mapped to $([p], -[p]) \in K_0(\mathfrak{A}) \oplus K_0(\mathfrak{B})$ and a K-theory class $[u]$ of $K_1(\mathfrak{C})$ is mapped to $([u], [u^{-1}]) \in K_1(\mathfrak{A}) \oplus K_1(\mathfrak{B})$.

Proof for being not correct. Provided by Alain Valette is a counterexample as follows. Let $\mathfrak{A} = C^*(F_2)$ and $\mathfrak{B} = C^*(F_2)$ the full group C^*-algebras of the free groups of two generators a, b and c, d respectively. Let $\mathfrak{C}$ be the C^*-subalgebra of $\mathfrak{A}$ generated by $aba^{-1}b^{-1}$. Identify $\mathfrak{C}$ with the C^*-subalgebra of $\mathfrak{B}$ generated by $cdc^{-1}d^{-1}$. In this case, the map from $K_1(\mathfrak{C})$ to $K_1(\mathfrak{A}) \oplus K_1(\mathfrak{B})$ is the zero map, but $K_1(\mathfrak{C}) \cong K_1(C^*(\mathbb{Z})) \cong K_1(C(\mathbb{T})) \cong \mathbb{Z}$. Hence, the map is not injective. Indeed, the class $[aba^{-1}b^{-1}]$ in $K_1(\mathfrak{C})$ is just the generator, not equal to $[1]$, but $[aba^{-1}b^{-1}] = [a][b][a^{-1}][b^{-1}] = [1]$ in $K_1(\mathfrak{A})$. □

To remedy it to be correct, we have the following:

Received November 30, 2009.

Lemma 1.1.2 revised. *Let $\mathfrak{A} * \mathfrak{B}$ be the full free product of C^*-algebras $\mathfrak{A}$ and $\mathfrak{B}$. Suppose that there exists a common C^*-subalgebra $\mathfrak{C}$ of $\mathfrak{A}$ and $\mathfrak{B}$. Then the following is exact:*

$$0 \to \iota_* K_*(\mathfrak{C}) \to K_*(\mathfrak{A}) \oplus K_*(\mathfrak{B}) \to (K_*(\mathfrak{A}) \oplus K_*(\mathfrak{B}))/\iota_* K_*(\mathfrak{C}) \to 0$$

for $* = 0, 1$, where a K-theory class $[p]$ of $K_0(\mathfrak{C})$ is mapped by ι_* to $([p], -[p]) \in K_0(\mathfrak{A}) \oplus K_0(\mathfrak{B})$ and a K-theory class $[u]$ of $K_1(\mathfrak{C})$ is mapped by ι_* to $([u], [u^{-1}]) \in K_1(\mathfrak{A}) \oplus K_1(\mathfrak{B})$, and $\iota_* K_*(\mathfrak{C})$ means the image of $K_*(\mathfrak{C})$ under ι_*.

The rest of the proofs in [2] depending on that wrong lemma will work on this correct basement (if no other mistakes), by replacing several places applied by the lemma.

As for the computation in the paper [2] on the K-theory of the full group C^*-algebra $C^*(SL_n(\mathbb{Z}))$ of $SL_n(\mathbb{Z})$ to yield a finitely generated group for K_0 and the trivial group for K_1, Alan Valette also points out that the computation contradicts to a known fact (of Wang [3]) that for G a residually finite group with Kazhdan's property T, the K_0-group $K_0(C^*(G))$ contains a copy of the free abelian group on countably many generators. Indeed, if π is an n-dimensional irreducible representation of $C^*(G)$, one has the direct sum decomposition: $C^*(G) = \ker(\pi) \oplus M_n(\mathbb{C})$, so that $K_0(M_n(\mathbb{C})) \cong \mathbb{Z}$ is a free summand of $K_0(C^*(G))$. In other words, the equivalence class of π is an isolated point in the spectrum of $C^*(G)$. Hence, $K_0(C^*(SL_n(\mathbb{Z})))$ contains a countably infinite direct sum of $\mathbb{Z}$.

This error comes from interpreting incorrectly Soulé's description [1] on $SL_n(\mathbb{Z})$ $(n \geq 3)$ as a multi-amalgam of its subgroups. The point is that in the paper [2], $SL_n(\mathbb{Z})$ is viewed as a finite multi-amalgam of its subgroups. This is not correct. To remedy it to be correct, $SL_n(\mathbb{Z})$ should be viewed as an infinite multi-amalgam of its subgroups. That is, $SL_n(\mathbb{Z})$ is an inductive limit of finite multi-amalgams of its subgroups, so that the K-theory of $C^*(SL_n(\mathbb{Z}))$ is computed as an inductive limit of that of the full group C^*-algebras of finite multi-amalgams of its subgroups, where the connecting maps are not necessarily injective. Since the case of finite multi-amalgams can be done in the paper [2] with those corrections, it should follow that K_0 of $C^*(SL_n(\mathbb{Z}))$ is an inductive limit of finitely generated groups and K_1 trivial.

Note. Since the dead line for submitting papers to this volume was fortunately postponed, the author could read timely the review item on the web, and quickly revised it to be correct (probably).

References

[1] C. Soulé, *The cohomology of $SL_3(\mathbb{Z})$*, Topology, **17** (1978), 1-22.

[2] T. Sudo, *K-theory for amalgams and multi-ones of C^*-algebras*, Ryukyu Math. J. **21** (2008), 57-139.

[3] P. S. Wang, *On isolated points in the dual spaces of locally compact groups*, Math. Ann. **218** (1975), 19-34.

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