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Business Cycles and Unemployment with Growth Through Creative Destruction

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Abstract

This paper examines relationships among economic growth, business cycles, and unemployment in a general-equilibrium model. The model views creative destruction as a major source of business cycles, as well as of long-term growth. I show that the expected growth rate negatively relates to the amplitude of business cycles, but positively relates to the expected unemployment rate. I also present an alternative view to competitive-equilibrium business cycle models on the relationship between employment and productivity. Contrary to competitive-equilibrium business cycle models, this model shows that positive permanent shocks cause unemployment, but positive transitory shocks lead to full employment. These theoretical results are consistent with empirical evidence. Finally, I show that an increase in the expected growth rate has a negative connection with the frequency of recessions.

KEYWORDS: business cycles, economic growth, unemployment.

JEL classifications: E24, E32, O33

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1 Introduction

This paper explores relationships among economic growth, business cycles and unemployment by constructing a model that views the process of creative destruction as a major source of business cycles, as well as of economic growth. The main results are: first, the expected growth rate negatively relates to the amplitude of business cycles. Second, a permanent shock causes unemployment, but a transitory shock leads to full employment. Third, an increase in the expected growth rate reduces the frequency of recessions.

The model presented here displays a *negative* correlation between economic growth and the amplitude of business cycles. The relationship between economic growth and business cycles, both of which are traditionally treated as separate issues, has been studied in recent literature. Aghion and Saint-Paul (1998) construct an open economy model with exogenous demand shocks. They show a *positive* relationship between the long-term rate of economic growth and the amplitude of *firm-level* fluctuation when productivity shocks in the economy have negative effects on current production, even though the amplitude of aggregate fluctuation is irrelevant to economic growth. However, their result is not supported by the data, as they noted in their paper. For example, Ramey and Ramey (1995) find a strong negative relationship between volatility and growth, using data from 92 countries and a subset of OECD countries. Martin and Rogers (2000) also find the same result both for OECD countries and European regions with different sets of control variables and different specifications. Moreover, Martin and Rogers (2000) present a simple theoretical model in which human capital is accumulated through learning-by-doing and derive a negative relationship between economic growth and the amplitude of business cycles. However, positive exogenous productivity shocks in their model reduces unemployment since it is competitive-equilibrium one. Our model also displays a *negative* correlation between economic growth and the amplitude of business cycles but a *negative* correlation between permanent productivity shocks and employment.

I show that positive permanent shocks *reduce* employment, while positive transitory shocks *raise* it. One well-known paradox of the competitive-equilibrium business cycle models is that they display a strong and highly *positive* relationship between shocks and employment. In competitive-equilibrium business cycle models, productivity shocks raise labor demand, increasing the employment level. This result of competitive-equilibrium business cycle models is inconsistent with empirical evidence. For example, Hansen and Wright (1992) find evidence that the correlation between hours of labor and productivity is near zero or slightly negative. To reconcile this puzzle, much of the literature has introduced various kinds of transitory shocks. For example, Christiano and Eichenbaum (1992) add shocks to government purchases within a real business cycle model, to moderate the positive relationship between them. Benhabib, Rogerson, and Wright (1991) add shocks to household production. These modified versions of the models inevitably conclude that positive permanent shocks *raise* the employment level while positive transitory shocks *reduce* it. In their assumption, transitory shocks reduce labor demand, thereby reducing the employment level. However, recent empirical

literature suggests evidence to the contrary. Galí (1999) finds evidence that permanent shocks *reduce* employment and transitory shocks *raise* it, based on data for G7 countries and using a new Keynesian model. Many recent works find this evidence for a broader range of specifications. For example, Francis and V.A. Ramey (2005) observe this relationship using the U.S. and U.K. data for one century.¹

I also show that the frequency of recessions (the ratio of a recession period to a business cycle) is a factor through which a higher expected growth rate leads to a higher expected unemployment rate. Some recent works re-examine the relationship between long-term economic growth and unemployment. Pissarides (2000) presents a model in which higher growth rate leads to lower natural unemployment rate. In his model, exogenous productivity shocks raise the number of vacancies, thereby increasing the probability of matching vacancies with the unemployed to reduce unemployment. On the other hand, Aghion and Howitt (1994) introduce the matching process, developed by Pissarides, to an endogenous growth model. They emphasize that opposite results may occur from economic growth: either an increase or decrease in the unemployment rate. Since an increase in economic growth through creative destruction inclines the job-separation rate to an increase, it has a factor that raises the unemployment rate. Along these studies, our model shows a similar effect to Aghion and Howitt (1994). But it particularly emphasizes the frequency of recessions, not only with the size of unemployment in a recession.

The paper is organized as follows. Section 2 specifies the behavior of agents in the economy and determines the equilibrium of the model. Section 3 considers the properties of perfect foresight equilibria. Section 4 concludes.

2 Model

This section first specifies the behavior of households and firms in the intermediate good and research sectors. Next, I determine the wage rate in each sector and derive the amount of research labor, which characterizes the equilibrium of the economy.

2.1 Behavior of Households

Each household, which is distributed over the closed unit interval $[0, 1]$, is endowed with a unit amount of labor and land, earns income by the rent of land, the wage of labor devoted to the final good and intermediate good sectors, and dividend payments from the monopolist in the intermediate good sector, and consumes the final good.

Each household has identical risk-neutral preferences with a positive and constant subjective discount rate, denoted by r . There is no instantaneous disutility from supplying labor. The instantaneous utility

¹ For other recent literature, see, Wen (2001), Basu, Fernald, and Kimball (2006), Marchetti and Nucci (2005), etc. Some works challenge these results. See Chang and Hong (2003) and Christiano, Eichenbaum, and Vigfusson (2003).

function, denoted by $u(\cdot)$, takes linear form

$$u(c(\tau)) = c(\tau),$$

where $c(\tau)$ is consumption at time τ .

Each household allocates its labor between the intermediate good and research sectors to maximize its expected lifetime utility under its budget constraint;

$$w_I(\tau)l_I(\tau) + w_{Rs(\tau)}l_{Rs(\tau)} + \pi(\tau) + R = c(\tau)$$

and feasible condition;

$$l_I(\tau) + l_{Rs(\tau)} \leq 1.$$

Here $s(\tau)$ denotes a stochastic stage of technology at time τ (following a Poisson process as shown later)²; $w_I(\tau)$ and $w_{Rs(\tau)}$ are real wage rates in the intermediate good and research sectors respectively; $l_I(\tau)$ and $l_{Rs(\tau)}$ are fractions of labor provided for the intermediate good and research sectors respectively; $\pi(\tau)$ is dividend from the intermediate good sector; and R is rent of land. Since preferences are linear, the subjective discount rate is equal to the real interest rate at any time. Finally, the household consumes all the output of the final good.

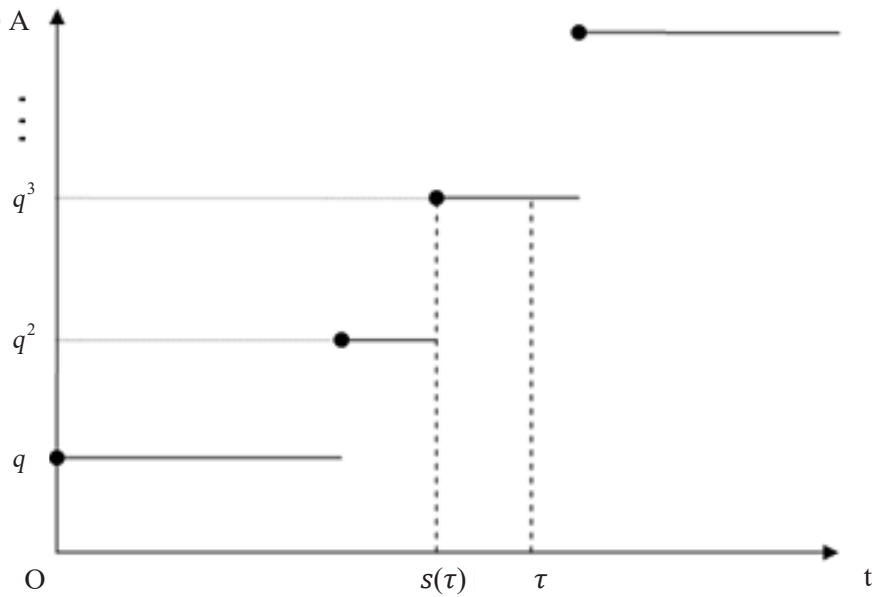


Figure 1: Innovation processes

² See Figure 1.

2.2 Behavior of Firms

Here I specify the behavior of firm in each sector; the final good, intermediate good, and research sectors. In the final good sector, firms competitively provide goods for households. In the intermediate good sector, the monopolist provides its output for firms in the final good sector by using labor and the current technology. During the implementation period, the monopolist incurs an extra cost, which shrinks the monopolist's labor demand and raises the price of the intermediate good under a given wage rate. These changes cause the employment rate and the amount of final goods to shrink during the implementation period, as shown later. Finally, firms in the research sector stochastically develop a state-of-the-art technology, which produces the more productive intermediate good.

2.2.1 Final Good Sector

Firms in the final good sector are assumed to have identical neoclassical technology, inputs for which are the intermediate good and a unit of the fixed resource such as land. The final good serves as numéraire.³

Firm i in the final good sector solves the following decision problem:

$$\begin{aligned} \max \quad & y(i, \tau) - p(\tau)x(i, \tau) \\ \text{s.t.} \quad & y(i, \tau) = A_{s(\tau)}f(x(i, \tau)), \end{aligned}$$

where $y(i, \tau)$ is flow of output of the final good produced by firm i at time τ ; $p(\tau)$ is the price of the intermediate good; $x(i, \tau)$ is the intermediate good employed by firm i ; and $A_{s(\tau)}$ is the indicator of productivity for the intermediate good with technology $s(\tau)$. The production function of the final good, $f(\cdot)$, is assumed to be three times continuously differentiable and satisfy the standard neoclassical properties.

Solving the profit-maximizing problem stated above, one finds the inverse demand function of firm i for the intermediate good:

$$p(\tau) = A_{s(\tau)} \frac{df(x(i, \tau))}{dx}. \quad (1)$$

Since the marginal productivity of the final good, the right-hand side of (1), is monotonically

³ If one takes explicitly a fixed resource as an input for production of final good into account, the production function can be written as

$$y(i, \tau) = A_{s(\tau)}F(x(\tau), L),$$

where $L(= 1)$ denotes a fixed resource households own and $F(\cdot)$ is a function of homogeneous of degree one. Hence, the objective of the firm can be rewritten as

$$\max y(i, \tau) - p(\tau)x(i, \tau) - RL.$$

It, therefore, follows from the homogeneity of the production function and perfect competitiveness of the final good market that the firm in the final good sector earns 0 profit. Since the fixed resource is less important to the implications of the model, it is not written explicitly in the context.

decreasing, demand for the intermediate good of any firm i , $x(i, \tau)$, is identically determined for all firms. It implies that aggregate demand for the intermediate good, $x(\tau)$, is equal to $x(i, \tau)$ for all i . Therefore, substituting $x(\tau) = x(i, \tau)$ into (1) and dividing both sides of (1) by $A_s(\tau)$ yield a productivity-adjusted aggregate inverse demand function,

$$\tilde{p}_D(x(\tau)) = \frac{df(x(\tau))}{dx}.$$

2.2.2 Intermediate Good Sector

The intermediate good is provided by a monopolist, employing labor and the state-of-the-art technology. The existing technology survives until a succeeding innovation occurs; that is, the paper concerns only the case of drastic innovation. Specifically, the k th technology survives during $[t_k, t_{k+1})$, where t_k denotes the stochastic instance of time at which the k th technology is developed; $t_k = \inf \{s^{-1}(k)\}$. A unit of the intermediate good is assumed to be produced by a unit of labor; $x(\tau) = l_I(\tau)$. Thus, demand for the intermediate good is equal to the amount of labor employed by a monopolist for producing the intermediate good. The monopolist producing the intermediate good with the k th technology has to pay $A_k\eta$ units of the extra cost per unit of labor it employs during the first δ time interval of the stage of the technology. This paper calls this extra cost an *implementation cost* and this first δ time interval of the stage of the technology an *implementation period*. The time length of the implementation period the monopolist in each stage of technology actually experiences may be shorter than δ , since there is a possibility that the succeeding innovation occurs within this first δ time; the current technology might be replaced by a succeeding one within this first δ time interval, after which the succeeding implementation period starts.

Since there exists the implementation cost, profit of the monopolist with the k th technology can be written as

$$\pi(\tau) = \begin{cases} \pi^-(\tau) & \text{if } t_k \leq \tau < \min[t_k + \delta, t_{k+1}] \\ \pi^+(\tau) & \text{if } \min[t_k + \delta, t_{k+1}] \leq \tau < t_{k+1}, \end{cases}$$

where

$$\pi^-(\tau) := A_k \tilde{p}_D(x(\tau)) x(\tau) - (w_I(\tau) + A_k \eta) x(\tau) \quad (2)$$

and

$$\pi^+(\tau) := A_k \tilde{p}_D(x(\tau)) x(\tau) - w_I(\tau) x(\tau). \quad (3)$$

Here, $\pi^-(\tau)$ and $\pi^+(\tau)$ denote profits during and after the implementation period, respectively. To simplify notation, the productivity-adjusted profit of the monopolist in both periods are defined as divisions of $\pi^-(\tau)$ and $\pi^+(\tau)$ by A_k respectively. Therefore, (2) and (3) can be rewritten as

$$\tilde{\pi}^-(x(\tau), \tilde{w}_I(\tau)) := \tilde{p}_D(x(\tau)) x(\tau) - (\tilde{w}_I(\tau) + \eta) x(\tau) \quad (4)$$

and

$$\tilde{\pi}^+(x(\tau), \tilde{w}_I(\tau)) := \tilde{p}_D(x(\tau))x(\tau) - \tilde{w}_I(\tau)x(\tau), \quad (5)$$

where $\tilde{\pi}^-(x(\tau), \tilde{w}_I(\tau))$ and $\tilde{\pi}^+(x(\tau), \tilde{w}_I(\tau))$ are productivity-adjusted profits during and after the implementation period, respectively; $\tilde{\pi}^-(x(\tau), \tilde{w}_I(\tau)) := \pi^-(\tau)/A_k$ and $\tilde{\pi}^+(x(\tau), \tilde{w}_I(\tau)) := \pi^+(\tau)/A_k$, and $\tilde{w}_I(\tau)$ is the productivity-adjusted real wage rate; $\tilde{w}_I(\tau) := w(\tau)/A_k$.

Finally, this paper denotes a productivity-adjusted marginal revenue function of the intermediate good sector $\mu(x)$ as

$$\mu(x) := \frac{df(x)}{dx} + x \frac{d^2f(x)}{dx^2},$$

which is assumed to satisfy

$$\mu(x) > 0 \text{ and } \frac{d\mu(x)}{dx} < 0 \text{ for all } x > 0$$

and

$$\lim_{x \rightarrow 0} \mu(x) = \infty \text{ and } \lim_{x \rightarrow \infty} \mu(x) = 0.$$

2.2.3 Innovation Processes

Research firms stochastically develop new technologies by employing labor, of which wages are paid through issuing shares. If a research firm succeeds in innovation, it becomes a monopolist, or monopolistically provides the state-of-the-art technology for the existing monopolist in the intermediate good sector in exchange for the patent of the technology instead. Each research firm assumes to innovate independently of other research firms. The probability that a research firm succeeds in innovation depends only on labor devoted to it. Each firm in the research sector determines the amount of labor when it starts a new research project (that is, the latest technology has just been developed). It is assumed that each firm does not change the amount of research labor until the research project ends, or unless one of the research firms succeeds in its innovation and the incumbent research project ends.⁴

Specifically, one of the research firms is assumed to succeed in the $k + 1$ th innovation with probability $\lambda l_{Rk} dt$ at each instance. Here l_{Rk} denotes the aggregate amount of labor employed by research firms in the k th stage of technology and λ does an indicator of the frequency of innovations. The current value of the $k + 1$ th technology, evaluated at t_{k+1} , is the expected present discounted value of the monopolist's profit accruing from the $k + 1$ th technology:

4 For example, an empirical study of Saint-Paul (1993) finds little evidence of any pro- or counter-cyclical behavior of the amount of research labor. The stage-wise contracts between research firms and workers in our model seem to encourage research workers to break their contracts and to reallocate their labor to the intermediate good sector if the wage in the intermediate good sector behaves procyclically. In this paper, however, it is assumed that if some of them break their contracts, they are never employed both in the intermediate good and research sector again. In this case, research workers have no incentive to break their contracts.

$$V_{k+1} := E_{t_{k+1}} \left[\int_{t_{k+1}}^{t_{k+2}} e^{-r(\tau-t_{k+1})} \pi(\tau) d\tau \right]. \quad (6)$$

Thus, by denoting the wage of research labor in the k th stage of technology by w_{Rk} , the zero-profit condition for the research firms is

$$E_{t_k} \left[e^{-r(t_{k+1}-t_k)} V_{k+1} \right] = E_{t_k} \left[\int_{t_k}^{t_{k+1}} e^{-r(\tau-t_k)} w_{Rk} d\tau \right],$$

where the left-hand side represents the expected current value of research activities for the $k + 1$ th innovation, evaluated at the beginning of the research projects t_k and the right-hand side does the expected overall cost.

2.3 Equilibrium

Here one finds the aggregate amount of research labor to characterize perfect foresight equilibria. In perfect foresight equilibria, the amount of research labor is constant over time. I derive the wage in the intermediate good sector and then, by arbitrage between the wages in the intermediate good and research sectors, determine the amount of research labor. The amount of research labor characterizes perfect foresight equilibria.

2.3.1 Determination of Wage in Intermediate Good Sector

The wage in the intermediate good sector is determined by Nash bargaining between the monopolist and workers. Formally, their purpose is to maximize the Nash products subject to the feasible constraint:

$$\begin{aligned} \max_{\{\tilde{w}_I(\tau), x(\tau)\}} & \left(A_{s(\tau)} \tilde{w}_I(\tau) x(\tau) \right)^\beta \left(A_{s(\tau)} \tilde{\pi}(x(\tau), \tilde{w}_I(\tau)) \right)^{1-\beta} \\ \text{s.t.} & \quad 1 - I_{Rs(\tau)} \geq x(\tau), \end{aligned}$$

where $\tilde{\pi}(x(\tau), \tilde{w}_I(\tau))$ is the productivity-adjusted profit; $\tilde{\pi}(x(\tau), \tilde{w}_I(\tau)) := \pi(\tau)/A_{s(\tau)}$, and β is the bargaining power of workers to the monopolist. This problem can be solved using the Kuhn-Tucker conditions. The solutions at the k th stage of technology are divided into two cases; during and after the implementation period.

During Implementation Period There are two Nash bargaining solutions, depending on how much labor is employed in the research sector. One is when the amount of research labor employed is so small that there exists unemployment (i.e. (8) holds with strict inequality), and the other is when the amount of research labor is so large that there is no unemployment (i.e. (8) holds with equality). In the case where (8) holds with strict inequality, the necessary conditions for the Nash bargaining problem (7) and (8) are

$$\tilde{w}_I(\tau) = \frac{\beta}{1-\beta} [\tilde{p}_D(x(\tau)) - (\tilde{w}_I(\tau) + \eta)] \quad (9)$$

and

$$-[\mu(x(\tau)) - (\tilde{w}_I(\tau) + \eta)] = \frac{\beta}{1-\beta} [\tilde{p}_D(x(\tau)) - (\tilde{w}_I(\tau) + \eta)]. \quad (10)$$

Here (9) expresses a sharing rule between the monopolist and workers; the shares of monopolist and workers for the revenue from producing the intermediate goods, $\tilde{p}_D(x(\tau)) - \eta$, are $1 - \beta$ and β , respectively. On the other hand, (10) suggests a contracting rule; the left-hand side of (10) divided by $x(\tau)$ gives the marginal rate of substitution of employment for wage (i.e. the ratio of the marginal profit of the monopolist's to the marginal cost), and the right-hand side of (10) divided by $x(\tau)$ expresses the worker's marginal rate of substitution. Hence, the efficient labor contracts equate the marginal rate of substitution of employment for wage with the worker's marginal rate of substitution. It follows from (9) and (10) that the wage rate and amount of employment are

$$x(\tau) = x^- := \mu^{-1}(\eta) \quad (11)$$

$$\tilde{w}_I(\tau) = \tilde{w}_I^- := \beta \left(\tilde{p}_D(\mu^{-1}(\eta)) - \eta \right). \quad (12)$$

By substituting (11) into (8), the feasible condition (8) can be rewritten as

$$1 - l_{Rs(\tau)} \geq \mu^{-1}(\eta). \quad (13)$$

Therefore, if (13) holds with strong inequality, the wage and employment are determined by (11) and (12). The equilibrium amount of the monopolist's profit during the implementation period is obtained by substituting (11) and (12) into the productivity-adjusted profit of the monopolist during the implementation period (4):

$$\tilde{\pi}^{*-} := \frac{1-\beta}{\beta} \tilde{w}_I^- x^-. \quad (14)$$

Figure 2 depicts the labor market in the intermediate good sector in the case (13) holds with strict inequality. The iso-profit curves of the monopolist are hump-shaped in the figure, and a lower wage implies a higher profit. The indifference curves of workers are convex, and a higher wage suggests a higher utility for workers in the intermediate good sector. The amount of labor in the intermediate good sector is determined on a vertical line tracing points of tangency between iso-profit and indifference curves. Therefore, the unemployment rate is shown by the gap between $\mu^{-1}(\eta)$ and $1 - l_r$. The wage, \tilde{w}_I^- , is determined by dividing the gap between the monopolist's threat point, $\tilde{p}_D(x^-) - \eta$, where the monopolist earns zero profit, and the workers' threat point, 0, where their benefits are indifferent between working and retiring, in the ratio of $1 - \beta : \beta$. On the other hand, if the feasible condition (13) holds with equality, the solution for the Nash bargaining problem (7) and (13) is given by

$$\begin{aligned} x(\tau) &= 1 - l_{Rs(\tau)} \\ \tilde{w}_I(\tau) &= \beta \left(\tilde{p}_D(1 - l_{Rs(\tau)}) - \eta \right). \end{aligned}$$

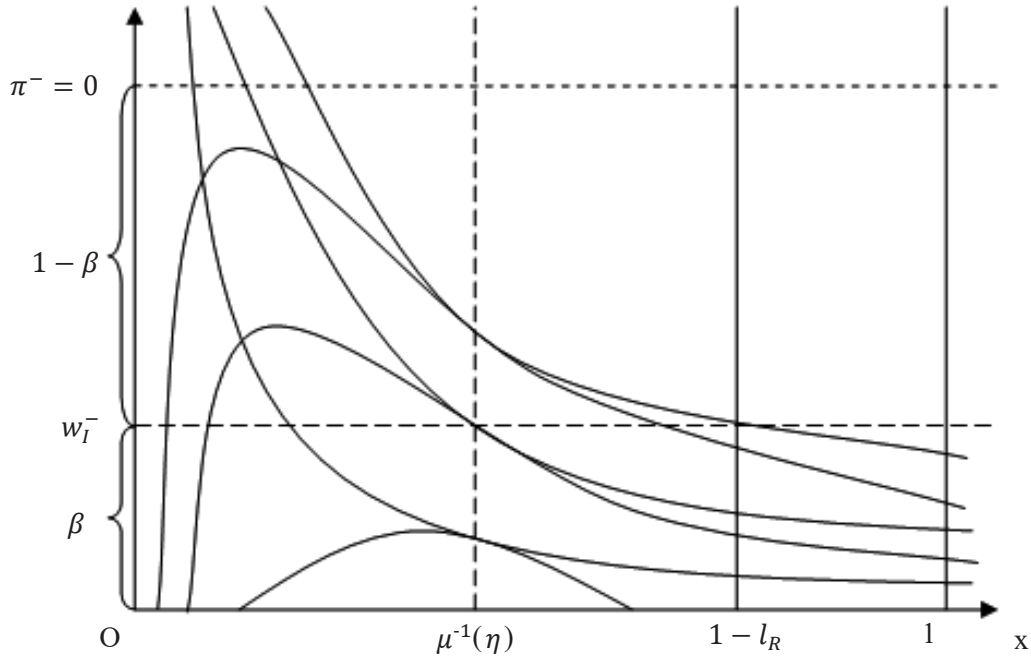


Figure 2: Labor market in recession

Since this paper focuses on the situation with unemployment, in what follows, I pay attention only to the case where the feasible condition (13) holds with strict inequality.

After Implementation Period The feasible condition (8) always holds with equality. Therefore, the Nash bargaining solution after the implementation period is given by

$$x(\tau) = 1 - l_{Rs(\tau)} \tag{15}$$

$$\tilde{w}_I(\tau) = \tilde{w}_I^+(l_{Rs(\tau)}) := \beta \tilde{p}_D (1 - l_{Rs(\tau)}) . \tag{16}$$

Similarly to the case during the implementation period, substitution of (15) and (16) into the productivity-adjusted profit of the monopolist's after the implementation period (5) yields the profit for the monopolist in equilibrium:

$$\tilde{\pi}^{*+}(l_{Rs(\tau)}) := \frac{1 - \beta}{\beta} \tilde{w}_I^+(l_{Rs(\tau)}) (1 - l_{Rs(\tau)}) . \tag{17}$$

Figure 3 depicts the labor market in the intermediate good sector after the implementation period. Differently from the case during the implementation period, there is no point of tangency between the iso-profit curve of the monopolist and the indifference curve of the workers. Therefore, the amount of employment in the intermediate good sector is determined on its maximum, $1 - l_{Rs(\tau)}$. Again, the wage $\tilde{w}_I^+(l_{Rs(\tau)})$ is determined so as to divide the gap between the monopolist's threat point, $\tilde{p}_D (1 - l_{Rs(\tau)})$, and the workers' threat point, 0, in the ratio of $1 - \beta : \beta$.

Moreover, substituting productivity-adjusted profits both during and after the implementation period, (14) and (17), into the current value of the technology (6) and replacing $s(\tau)$ with the $k + 1$ give the productivity-adjusted value of the $k + 1$ th technology:

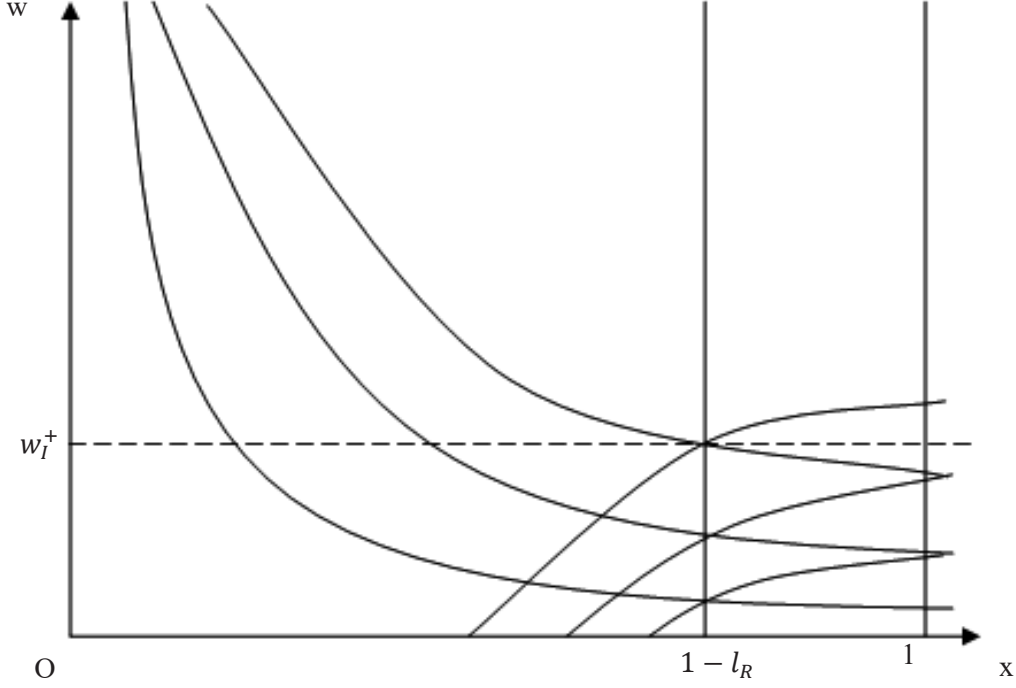


Figure 3: Labor market in boom

$$v(l_{Rk+1}) := \frac{V_{k+1}}{A_{k+1}} = \frac{1}{r + \lambda l_{Rk+1}} \frac{1 - \beta}{\beta} \left[\left(1 - e^{-(r + \lambda l_{Rk+1})\delta}\right) \tilde{w}_I^- x^- + e^{-(r + \lambda l_{Rk+1})\delta} \tilde{w}_I^+ (l_{Rk+1}) (1 - l_{Rk+1}) \right], \quad (18)$$

where the multiplied term with square parentheses on the right-hand side represents a weighted average of the monopolists' profits during and after the implementation period. By using (18), the zero-profit condition (6) is simplified to⁵

$$\lambda q v(l_{Rk+1}) = \tilde{w}_{Rk}. \quad (19)$$

Here \tilde{w}_{Rk} denotes the productivity-adjusted real wage rate in the research sector; $\tilde{w}_{Rk} := w_{Rk}/A_k$, and q is the size of innovation; $q := A_{s(\tau)+1}/A_{s(\tau)}$. In this paper, q is assumed to be large enough so that innovation is drastic.

2.3.2 Arbitrage Between Incomes from Two Sectors

Each household equates between the expected present discounted value of income from the intermediate good and research sectors by each stage of the technology since each firm in the research sector does not change the amount of labor during each research project. Because of the possibility of unemployment in the intermediate good sector, households take this possibility into account. Therefore, from the risk-neutrality of households' preferences, they allocate their labor between two sectors to equate expected

⁵ See Appendix B for detail calculation of the productivity-adjusted value of the technology.

present discounted values of incomes earned from each sector. The no arbitrage condition, therefore, can be written as

$$W_{Ik}(l_{Rk}) = W_{Rk}, \quad (20)$$

where $W_{Ik}(l_{Rk})$ and W_{Rk} are expected present discounted values of productivity-adjusted incomes from the intermediate good and research sector at the stage of k th technology, respectively:⁶

$$W_{Ik}(l_{Rk}) := E_{t_k} \left[\frac{x^-}{1 - l_{Rk}} \int_{t_k}^{\min[t_k + \delta, t_{k+1}]} e^{-r(\tau - t_k)} \tilde{w}_I^- d\tau + \int_{\min[t_k + \delta, t_{k+1}]}^{t_{k+1}} e^{-r(\tau - t_k)} \tilde{w}_I^+(l_{Rk}) d\tau \right] \quad (21)$$

and

$$W_{Rk} := E_{t_k} \left[\int_{t_k}^{t_{k+1}} e^{-r(\tau - t_k)} \tilde{w}_{Rk} d\tau \right]. \quad (22)$$

The multiplier of the first integrated term on the right-hand side of (21), $x^-/(1 - l_{Rk})$, represents the probability of unemployment, which is uniformly given to each household. Therefore, using (14), (17), (21) and (22), a bit of manipulation simplifies (20) to

$$\tilde{w}_{Rk} = \omega(l_{Rk}), \quad (23)$$

where

$$\omega(l_{Rk}) := \left(1 - e^{-(r + \lambda l_{Rk})\delta}\right) \frac{x^-}{1 - l_{Rk}} \tilde{w}_I^- + e^{-(r + \lambda l_{Rk})\delta} \tilde{w}_I^+(l_{Rk}).$$

Moreover, substituting (23) into the zero-profit condition (19) yields

$$\lambda q v(l_{Rk+1}) = \omega(l_{Rk}). \quad (24)$$

Further substitution of (18) into (24) simplifies the zero-profit condition:

$$\lambda q \frac{1 - \beta}{\beta} \frac{1 - l_{Rk+1}}{r + \lambda l_{Rk+1}} \omega(l_{Rk+1}) = \omega(l_{Rk}),$$

which determines uniquely the amount of research labor in perfect foresight equilibria:

⁶ The unemployed may have an incentive to reallocate their resources devoted to the intermediate good sector to the research sector if $\tilde{w}_{Ik}^+(l_{Rk}) < \tilde{w}_{Rk}$. However, this paper assumes that the unemployed has to pay some fraction of search cost, denoted by K , when they reallocate her resource to the research sector. If K satisfies

$$E_{t_k} \left[\int_{\min[t_k + \delta, t_{k+1}]}^{t_{k+1}} e^{-r(\tau - t_k)} \tilde{w}_I^+(l_{Rk}) d\tau \right] \geq E_{t_k} \left[\int_{t_k}^{t_{k+1}} e^{-r(\tau - t_k)} \tilde{w}_{Rk} d\tau \right] - K,$$

they have no incentive to reallocate their resource. This paper assumes this condition in the following.

$$l_R := l_{Rk+1} = l_{Rk} = \frac{\lambda q^{\frac{1-\beta}{\beta}} - r}{\lambda \left(q^{\frac{1-\beta}{\beta}} + 1 \right)}. \quad (25)$$

Note that l_R is decreasing in the bargaining power for workers, β ; increasing in the size of innovation, q , and the arrival rate of innovation, λ ; and independent of implementation cost, η , and the length of implementation period, δ . The following sections focus only on perfect foresight equilibria.

3 Properties of Perfect Foresight Equilibria

This section is devoted to showing the main results; the relationships between economic growth and the amplitude of business cycles, business cycles and shocks, and economic growth and unemployment. The relationship between the expected rate of economic growth and the amplitude of business cycles is negative since the amount of labor in the intermediate good sector in a recession is determined independently of the amount of research labor. Permanent shocks reduce the expected rate of unemployment, while transitory shocks lead to full employment since a permanent shock accompanies the implementation cost, a factor reducing the demand for labor in the intermediate good sector. Finally, higher economic growth raises the frequency of recessions, a factor raising the expected unemployment rate.

3.1 Economic Growth, Business Cycles, and Unemployment

This section specifies the expected growth rate, the frequencies of booms and recessions, the amplitude of business cycles, and the expected unemployment rate.

Expected Growth Rate The expected growth rate takes the same form as of the standard model.⁷

$$g = \lambda l_R \ln q. \quad (26)$$

Note that by substituting the amount of research labor in perfect foresight equilibria (26), the expected growth rate is increasing in the arrival rate λ , the size of innovation q , and the amount of labor in the research sector l_R .

Frequencies of Booms and Recessions *The frequencies of booms and recessions are defined by ratios of the expected periods of booms and recessions to the expected periods of business cycles, respectively. A recession period is defined by a successive series of implementation periods, of which the recession period consists. This definition of the recession is intuitive, since the economy experiences unemployment during implementation periods. Finally, a boom period is defined by a period when the*

⁷ See Aghion and Howitt (1992). For calculation, see appendix A.

economy is outside implementation periods since the economy attains full employment outside implementation periods.

Specifically, I first calculate the expected periods of boom, recession, and business cycle, and then derive the frequencies of booms and recessions. The expected boom period, denoted by τ_+ , can be obtained by straightforward calculation, noting that the probability density of occurrence of innovation τ time after a current time is $\lambda l_R e^{-\lambda l_R \tau}$:

$$\begin{aligned} E[\tau_+] &:= \int_0^{\infty} \lambda l_R e^{-\lambda l_R \tau_+} \tau_+ d\tau_+ \\ &= \lambda l_R \left[\tau_+ \int_0^{\infty} e^{-\lambda l_R \tau_+} d\tau_+ - \int_0^{\infty} \int_0^{\tau_+} e^{-\lambda l_R \tau_+} d\tau_+ d\tau_+ \right]_0^{\infty} \\ &= \frac{1}{\lambda l_R}. \end{aligned} \quad (27)$$

To calculate the expected period of recessions, I first decompose a recession period into a series of implementation periods, and then calculate an expectation of each implementation period. Since the probability density that a new technology develops within an implementation period is $1 - e^{-\lambda l_R \delta}$ and the probability density that there is no innovation within an implementation period is $e^{-\lambda l_R \delta}$, a probability density that implementation periods continue n times is written as $(1 - e^{-\lambda l_R \delta})^{n-1} e^{-\lambda l_R \delta}$. It follows that the expected recession period can be calculated to

$$\begin{aligned} E[\tau_-] &= E[n\tau_I] \\ &= \sum_{n=1}^{\infty} (1 - e^{-\lambda l_R \delta})^{n-1} e^{-\lambda l_R \delta} n E[\tau_I] \\ &= e^{\lambda l_R \delta} E[\tau_I], \end{aligned} \quad (28)$$

where τ_- and τ_I denote a recession period and an implementation period of which the recession period consists, respectively. Since the probability density that a new technology develops τ units of time after the last technology developed is $\lambda l_R e^{-\lambda l_R \tau}$ and the probability that any new technology does not developed during the implementation period is $e^{-\lambda l_R \delta}$, the expected time length of an implementation period can be written as

$$\begin{aligned} E[\tau_I] &= \int_0^{\delta} \lambda l_R e^{-\lambda l_R \tau_I} \tau_I d\tau_I + e^{-\lambda l_R \delta} \delta \\ &= \frac{1}{\lambda l_R} (1 - e^{-\lambda l_R \delta}). \end{aligned} \quad (29)$$

By substituting (29) into (28), the expected period of a recession can be written as

$$E[\tau_-] = \frac{1}{\lambda l_R} (e^{\lambda l_R \delta} - 1). \quad (30)$$

Therefore, it follows from the expected period of boom (27) and the expected period of recession (30)

that the expected period of a business cycle C can be written as

$$C := E[\tau_+] + E[\tau_-] = \frac{1}{\lambda l_R} e^{\lambda l_R \delta}. \quad (31)$$

Finally, dividing the expected periods of boom and recession by (31) yields the frequency of boom and recession:

$$B := \frac{E[\tau_+]}{C} = e^{-\lambda l_R \delta} \quad (32)$$

and

$$R := \frac{E[\tau_-]}{C} = 1 - e^{-\lambda l_R \delta}, \quad (33)$$

where B and R denote the frequencies of boom and recession, respectively.

Combining the comparative statics of the amount of research labor, which is stated below (25), with (27), (30), (31), (32) and (33), one finds that an increase of the time length of the implementation period, δ , reduces the expected period of booms, $E[\tau_+]$, and the frequency of booms, B . On the other hand, it raises the expected recession period, $E[\tau_-]$, the frequency of recessions, R , and the expected period of business cycles, C . Increases of other parameters that raise the amount of research labor raise the frequency of recessions but reduce the expected period and the frequency of booms. They have ambiguous effects on the expected periods of recessions and business cycles.

The Amplitude of Business Cycles and Unemployment I now define the amplitude of business cycles and calculate the expected unemployment rate. The *amplitude* of business cycles in this paper is defined as

$$a := 1 - (l_R + x^-). \quad (34)$$

The first term on the right-hand side of (34) is the employment level in boom, and the rest of the term is that in recessions. Therefore, the amplitude of business cycles is the difference between the amounts of employment in booms and recessions. Used the frequency of recessions (33) and unemployment rate during recessions, $1 - l_R - x^-$, which is equal to the amplitude of business cycles, the expected unemployment rate can be written as

$$E[u(\tau)] = Ra. \quad (35)$$

3.2 Economic Growth and Amplitude of Business Cycles

This subsection shows a negative relationship between the expected growth rate and the amplitude of business cycles. Total derivatives of (25) and (26) are respectively written as

$$dg = l_R \ln q d\lambda + \frac{\lambda l_R}{q} dq + \lambda \ln q dl_R, \quad (36)$$

and

$$da = -dl_R + dx^-, \quad (37)$$

where

$$\frac{dl_R}{d\lambda} > 0 \quad \text{and} \quad \frac{dl_R}{dq} > 0. \quad (38)$$

From (36) to (38), one finds that any factor raising the expected growth rate reduces the amplitude of business cycles.

Intuitively, this result follows from the fact that economic growth affects the amount of labor in the intermediate good sector in booms and recessions asymmetrically. For example, suppose that a reduction of the expected growth rate is introduced by a decrease in the amount of research labor. A decrease in the amount of research labor, on the one hand, does not affect the aggregate amount of labor in booms because it raises the same amount of labor in the intermediate good sector through (15). A decrease in the amount of research labor, on the other hand, affects the aggregate amount of labor in recessions because the amount of labor in the intermediate good sector in recessions is determined by (11), independently of the research labor l_R . It is this asymmetry that results in the reduction of the amplitude of business cycles.

3.3 Business Cycles and Shocks

Moreover, the model shows that permanent productivity shocks reduce the total amount of employment while transitory productivity shocks raise it. First, I show that there is a positive relationship between transitory shocks and changes in employment. It follows from the production function in the final good sector (2) that changes in the logarithmic amount of output is

$$d\ln y(t) = d\ln A_{s(t)} + d\ln f(x(t)),$$

where the first term on the right-hand side of the expression represents contributions of permanent productivity improvements on changes in the amount of output, and the second term does that in employment levels in logarithmic scales, respectively. Therefore, the covariance between changes in output and employment levels can be decomposed as

$$\begin{aligned} \text{Cov}[d\ln y(t), d\ln x(t)] \\ = \text{Cov}[d\ln A_{s(t)}, d\ln x(t)] + \text{Cov}[d\ln f(x(t)), d\ln x(t)], \end{aligned} \quad (39)$$

where the first term on the right-hand side of the expression gives the relationship between permanent productivity-shocks and changes in employment levels, and thus the rest of the term implies the relationship between transitory productivity shocks and the changes in employment levels. Immediately from (39), one finds a positive relationship between transitory shock and the changes in employment levels. On the contrary, there is a negative relationship between permanent shocks and the changes in employment levels. The arrival time when a research firm succeeds in innovation follows the

exponential distribution whose probability density is $e^{-\lambda_R t}$ on perfect foresight equilibria. Therefore, the arrival number of innovations follows a Poisson distribution as is well-known⁸. It follows that changes in productivity can be expressed by

$$d \ln A_{s(t)} = \ln q d\psi(t),$$

where $\psi(t)$ follows a Poisson process, with

$$d\psi(t) = \begin{cases} 1 & \text{if innovation occurs at time } t, \\ 0 & \text{if innovation does not occur at time } t; \end{cases}$$

with the probability of success in innovation, $\lambda_R dt$. Likewise, the processes of logarithmic employment levels can be written as

$$d \ln x(t) = -\ln \frac{x^+}{x^-} \varphi(t) d\psi(t) + \ln \frac{x^+}{x^-} \varphi(t) d\psi(t - \delta), \quad (40)$$

where

$$\varphi = \begin{cases} 1 & \text{if innovation does not occur during } \delta \text{ time interval,} \\ 0 & \text{if innovation occurs during } \delta \text{ time interval} \end{cases}$$

with the probability that $\varphi = 1$, $e^{-\lambda_R \delta}$. The first term on the right-hand side of (40) represents the effects of permanent productivity shocks on employment levels in the intermediate good sector; an amount of labor employed at time t is x^+ if $\varphi = 1$, since innovation does not occur within the last δ time interval, and thus the economy is in boom. And there occurs a permanent productivity shock at time t if $d\psi = 1$ that leads $x(t)$ to x^- . Therefore, if $\varphi = 1$ and $d\psi = 1$, the logarithmic amount of employment changes by $\ln(x^-/x^+)$.

Similarly, the second term of the right-hand side of (40) represents the effects of transitory shocks on employment levels. Therefore, the covariance between permanent shocks and changes in employment levels in logarithmic scales can be calculated as

$$\begin{aligned} & \frac{\text{Cov} [d \ln A_{s(t)}, \ln x(t)]}{dt} \\ &= \frac{1}{dt} \text{Cov} \left[\ln q d\psi(t), -\ln \frac{x^+}{x^-} \varphi d\psi(t) + \ln \frac{x^+}{x^-} \varphi d\psi(t - \delta) \right] \\ &= -\ln q \ln \frac{x^+}{x^-} \frac{\text{Cov} [d\psi(t), \varphi d\psi(t)]}{dt} + \ln q \ln \frac{x^+}{x^-} \frac{\text{Cov} [d\psi(t), \varphi d\psi(t - \delta)]}{dt} \\ &= -\lambda_R \ln q \ln \frac{x^+}{x^-} e^{-\lambda_R \delta} < 0, \end{aligned} \quad (41)$$

where additivity of Poisson processes is used from the third to fourth lines. Since the sign of (41) is negative, permanent productivity shocks and changes in employment levels correlate negatively.

⁸ See, for example, Section 23 of Billingsley (1995) for the details of Poisson processes and exponential distribution.

3.4 Economic Growth and Unemployment

Finally, this subsection shows that the frequency of recessions can be a factor through which the expected growth rate raises the expected unemployment rate. Taking total differentiation of the frequency of recessions (33) and a few manipulations yield

$$dR = (1 - R)(l_R \delta d\lambda + \lambda \delta dl_R + l_R \delta d\delta). \quad (42)$$

From (36) and (42), one finds that any factor raising the expected growth rate raises the frequency of recessions.

Moreover, the total differentiation of the expected unemployment rate (35) gives

$$E[du(\tau)] = Rda + adR. \quad (43)$$

Since (37) and (42) implies that any factor raising the growth rate reduces the amplitude of business cycles and raises the frequency of recessions, (43) shows that it has two opposite effects on the expected unemployment rate; one tends to reduce it by decreasing the amplitude of business cycles, and the other tends to raise it by increasing the frequency of recessions.

4 Conclusion

This paper investigated the relationship among economic growth, business cycles, and unemployment, driven by the implementation cost. First, the expected growth rate negatively relates to the amplitude of business cycles. Second, a permanent shock reduces the amount of employment, while the transitory shock increases the amount of employment. Finally, the expected growth rate has a negative connection with the frequency of recessions, which is a factor raising the expected rate of unemployment.

Appendix

A Expected Rate of Economic Growth

The expected amount of the output at time t ($\geq t_0 + \delta$) conditioned by time t_0 can be written as

$$\begin{aligned}
 E_{t_0} [\ln A_{s(t)} f(x_t)] &= E_{t_0} [\ln A_{s(t)}] + E_{t_0} [\ln f(x_t)] \\
 &= \sum_{i=0}^{\infty} e^{-\lambda l_R (t-t_0)} \frac{[\lambda l_R (t-t_0)]^i}{i!} \ln q^i A_{s(t_0)} \\
 &\quad + (1 - e^{-\lambda l_R \delta}) \ln f(x^+) + e^{-\lambda l_R \delta} \ln f(x^-) \\
 &= \lambda l_R (t-t_0) \ln q + \ln A_{s(t_0)} \\
 &\quad + (1 - e^{-\lambda l_R \delta}) \ln f(x^+) + e^{-\lambda l_R \delta} \ln f(x^-).
 \end{aligned}$$

Therefore, the expected rate of economic growth is written as

$$\begin{aligned}
 g &= \frac{E [d \ln A_{s(t)} f(x_t)]}{dt} \\
 &= \lim_{\Delta t \rightarrow 0} \frac{E [E_{t_0} [\ln A_{s(t+\Delta t)} f(x_{t+\Delta t})] - E_{t_0} [\ln A_{s(t)} f(x_t)]]}{\Delta t} \\
 &= \lambda l_R \ln q.
 \end{aligned}$$

B Expected Value of Monopolist

Substituting (14) and (17) into (6) yields

$$\begin{aligned}
v(l_{Rk+1}) &:= \frac{V_{k+1}}{A_{k+1}} \\
&= \frac{1-\beta}{\beta} \mathbb{E}_{t_{k+1}} \left[\int_{t_{k+1}}^{\min[t_{k+1}+\delta, t_{k+2}]} e^{-r(\tau-t_{k+1})} \tilde{w}_I^- x^- d\tau \right. \\
&\quad \left. + \int_{\min[t_{k+1}+\delta]}^{t_{k+2}} e^{-r(\tau-t_{k+1})} \tilde{w}_I^+ (l_{Rk+1})(1-l_{Rk}) d\tau \right] \\
&= \frac{1-\beta}{\beta} \frac{\tilde{w}_I^- x^-}{r} \mathbb{E}_{t_{k+1}} \left[1 - e^{-r \min[\delta, t_{k+2}-t_{k+1}]} \right] \\
&\quad + \frac{1-\beta}{\beta} \frac{\tilde{w}_I^+ (l_{Rk+1})(1-l_{Rk})}{r} \mathbb{E}_{t_{k+1}} \left[e^{-r \min[\delta, t_{k+2}-t_{k+1}]} - e^{-r(t_{k+2}-t_{k+1})} \right] \\
&= \frac{1-\beta}{\beta} \frac{\tilde{w}_I^- x^-}{r} \left(\int_{t_{k+1}}^{\infty} \lambda l_{Rk+1} e^{-\lambda l_{Rk+1}(t_{k+1}-t_{k+2})} dt_{k+2} \right. \\
&\quad \left. - \int_{t_{k+1}}^{\infty} \lambda l_{Rk+1} e^{-\lambda l_{Rk+1}(t_{k+2}-t_{k+1})} e^{-r \min[\delta, t_{k+2}-t_{k+1}]} dt_{k+2} \right) \\
&\quad + \frac{1-\beta}{\beta} \frac{\tilde{w}_I^+ (l_{Rk+1})(1-l_{Rk})}{r} \left(\int_{t_{k+1}}^{\infty} \lambda l_{Rk+1} e^{-\lambda l_{Rk+1}(t_{k+2}-t_{k+1})} e^{-r \min[\delta, t_{k+2}-t_{k+1}]} dt_{k+2} \right. \\
&\quad \left. - \int_{t_{k+1}}^{\infty} \lambda l_{Rk+1} e^{-(r+\lambda l_{Rk+1})(t_{k+2}-t_{k+1})} dt_{k+2} \right) \\
&= \frac{1-\beta}{\beta} \frac{\tilde{w}_I^- x^-}{r} \left[1 - \left(\int_{t_{k+1}}^{t_{k+1}+\delta} \lambda l_{Rk+1} e^{-(r+\lambda l_{Rk+1})(t_{k+2}-t_{k+1})} dt_{k+2} \right. \right. \\
&\quad \left. \left. + \int_{t_{k+1}+\delta}^{\infty} \lambda l_{Rk+1} e^{-\lambda l_{Rk+1}(t_{k+2}-t_{k+1})-r\delta} dt_{k+2} \right) \right] \\
&\quad + \frac{1-\beta}{\beta} \frac{\tilde{w}_I^+ (l_{Rk+1})(1-l_{Rk})}{r} \left[\int_{t_{k+1}}^{t_{k+1}+\delta} \lambda l_{Rk+1} e^{-(r+\lambda l_{Rk+1})(t_{k+2}-t_{k+1})} dt_{k+2} \right. \\
&\quad \left. + \int_{t_{k+1}+\delta}^{\infty} \lambda l_{Rk+1} e^{-\lambda l_{Rk+1}(t_{k+2}-t_{k+1})-r\delta} dt_{k+2} - \frac{\lambda l_{Rk+1}}{r+\lambda l_{Rk+1}} \right] \\
&= \frac{1-\beta}{\beta} \frac{1}{r+\lambda l_{Rk+1}} \left[\left(1 - e^{-(r+\lambda l_{Rk+1})\delta} \right) \tilde{w}_I^- x^- + e^{-(r+\lambda l_{Rk+1})\delta} \tilde{w}_I^+ (l_{Rk+1})(1-l_{Rk}) \right].
\end{aligned}$$

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