PAPER

A Fault Model for Multiple-Valued PLA's and Its Equivalences

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SUMMARY In this paper, a fault model for multiple-valued programmable logic arrays (MV-PLAs) is proposed and the equivalences of faults of MV-PLA's are discussed. In a supposed multiple-valued NOR/TSUM PLA model, it is shown that multiple-valued stuck-at faults, multiple-valued bridging faults, multiple-valued threshold shift faults and other some faults in a literal generator circuit are equivalent or subequivalent to a multiple crosspoint fault in the NOR plane or a multiple fault of weights in the TSUM plane. These results lead the fact that multiple-valued test vector set which indicates all multiple crosspoint fault and all multiple fault of weights also detects above equivalent or subequivalent faults in a MV-PLA.

key words: *equivalences of faults, fault model, multiple-valued logic, programmable logic array, test compaction*

1. Introduction

Multiple-valued circuit and systems have been investigated actively and various implementation techniques are proposed [1]–[3]. It is known that multiple-valued logic solves the interconnection and pin-limitation problems and will achieve higher performances of arithmetic computations. Unfortunately, however, as multiple-valued circuit uses multi-level signals, they have inherent disadvantages in comparison with conventional binary logic systems. Namely, faults of devices which occures in manufacturing processes or errors during operations are considered as serious defects.

In this paper, a fault model for MV-PLA's (multiple-valued programmable logic arrays) is proposed. The based MV-PLA for the fault model is NOR-TSUM type on the ground that the TSUM operation is investigated successfully in the circuit and logic minimization fields lately. It should also be added that in the phisical implementation of the MV-PLA, the NOR-NOR structure is used with static CMOS and NAND-NAND structure is for n-channel devices even when the logical implementation is AND-OR [6]. The fault model discussed here is partly an extension of a fault model of binary PLA proposed by Lighart [4], and it also includes some new fault models of MV-PLA's.

In the literature [4], faults for binary PLAs are classified into following four: (1) multiple stuck-at faults (2) multiple bridging faults (3) multiple crosspoint faults (4) multiple breaks. As the result, due to equivalence of faults, it is shown that the four classes of faults are reduced into two: (3) and (4).

The aim of the analysis of faults' equivalences is the reduction of test set for MV-PLA's.

In the remaining part of this paper, multiple crosspoint faults, multiple fault of weights, multiple-valuled stuck-at faults, multiple-valued threshold shift faults and multiple-valued bridging faults are proposed and their equivalences with respect to MV-PLAs are discussed.

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2. Definitions

To discuss the equivalences of faults on multiple-valued PLA, a model of MV-PLA and some definitions are required.

Assume $(r+1)$ is the radix number of multiple-valued logic functions.

A literal X^S of a (r+1)-valued logic function can be denoted as

$$X^S = \begin{cases} r: & X \in S \\ 0: & X \notin S \end{cases} \tag{1}$$

where $X \in L = \{0, 1, \cdots, r\}$ is a multiple-valued variable and S is a subset of L. $\overline{S}$ expresses a differencial subset: $\overline{S} = L - S$. In a four-valued literal X^{02}, for instance, $X^{\overline{S}} = \overline{X^S} = X^{13}$. If X^S is a *uniliteral*, that is to say $|S| = 1$ (e.g. $X^0, X^1, \cdots, X^r$), it is denoted as X^a.

In this paper, *NOR/TSUM* $(r+1)$-valued PLA's are considered, especially four-valued NOR/TSUM PLA referring to Fig. 1 is used for explanations. It is assumed that the MV-PLA in Fig. 1 contains *Literal Generators* (L.G.) shown in Fig. 2 (a), literal-lines and product-lines in the *NOR plane* which take only two values 0 and r and output-lines in the *TSUM plane* which takes $(r+1)$-valued signals. The TSUM plane implements *TSUM operations*, $TSUM(X,Y) = MIN(r, X+Y)$, after weightings with $w_{jh} (\in \{0, 1, \cdots, r\})$ at crosspoints as shown in Fig. 3.

Definition 1: Multiple-valued function f_h is described as follows,

$$f_h \ (\ X_1, X_2, \cdots, X_n)$$

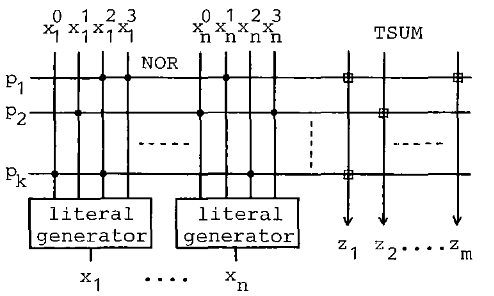

Fig. 1 Four-valued NOR/TSUM PLA structure.

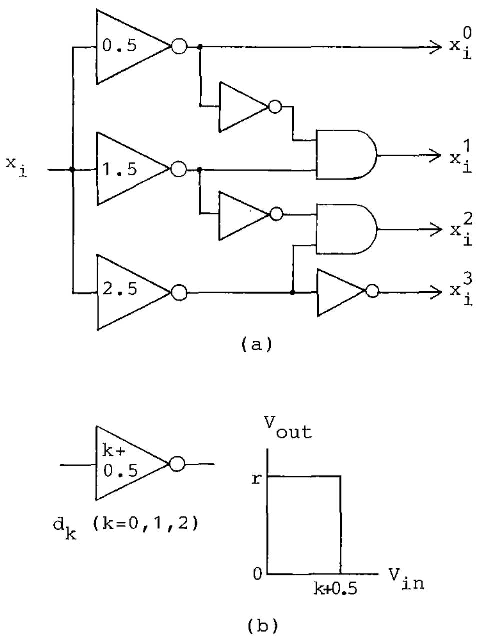

Fig. 2 Literal generator. (a) 4-valued literal generator. (b) Symbol and transfer curve of a threshold detector.

$$= \sum_{j=1}^{m}{}^{T} w_{jh} \cdot (X_1^{S_{1j}} \cdot X_2^{S_{2j}} \cdot \cdots \cdot X_n^{S_{nj}})$$
$$= \sum_{j=1}^{m}{}^{T} w_{jh} \cdot (\overline{X_1^{\overline{S_{1j}}} \vee X_2^{\overline{S_{2j}}} \vee \cdots \vee X_n^{\overline{S_{nj}}}})$$
$$= \sum_{j=1}^{m}{}^{T} w_{jh} \cdot (\overline{\bigvee_{i,g} X_i^{a_{ij}^g}}) \quad (2)$$

where $X_i^{\overline{S_{ij}}} = \bigvee_g X_i^{a_{ij}^g}$, (i.e. $a_{ij}^g \in \overline{S_{ij}}$), $i = 1, \cdots, n$. $\overline{\bigvee}$ and '$\cdot$' denote $(r+1)$-valued NOR operations and MIN operation, respectively. $\sum^T$ denotes $(r+1)$-valued truncated sum (TSUM) operations.

A four-valued literal generator in Fig. 2 (a) contains three kinds of *threshold detectors* d_0, d_1, d_2. Figure 2 (b) shows the symbol of threshold detectors and their behaviors.

Definition 2: The *threshold shift (TS) fault* of a threshold detector in a literal generator is defined as one-level logic shift at a time. A threshold shift fault to one-higher value said to be *up shift (US)* and to one-lower value said to be *down shift (DS)*.

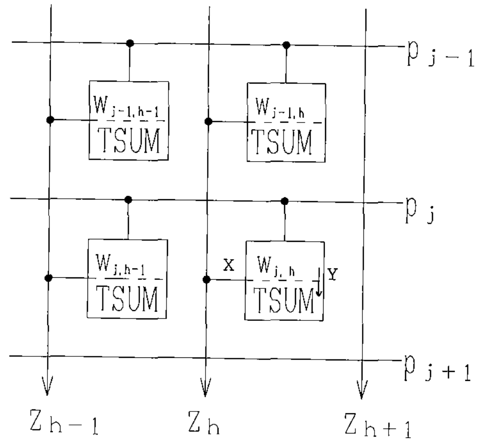

Fig. 3 Generalized bussed cellular expression of TSUM plane.

A generalized bussed cellular expression of TSUM plane is shown in Fig. 3. Programming of the TSUM plane is to assign a value of weight w_{jh} at each cross point. We assume that a fault of TSUM plane is a fault in w_{jh} or a fault in TSUM circuit. However, a fault in TSUM circuit is equivalent to a fault in w_{jh} regardless of the means of implementation for TSUM plane. Assume $X \boxplus Y = Z$ where $\boxplus$ denotes TSUM operation. For a fault of TSUM circuit, an errorneous value $Z^{(f)}$ will appear as $Z > Z^{(f)}$ or $Z < Z^{(f)}$. Then, in the TSUM plane, $Z^{(f)} = (X \boxplus Y) \pm \delta = X \boxplus (p_j \cdot w_{jh}^{(f)}) = X \boxplus \{p_j \cdot (w_{jh} \pm \delta)\}$, where $p_j \in \{0, r\}$ and '$\cdot$' denotes the MIN operator. Thus the faults in TSUM circuit are equivalent to the faults in w_{jh}.

Definition 3: In the TSUM plane, the value of w_{jh} is changed from e to $e^{(f)}$ by a certain fault, this is called *fault of weight* and is denoted as $w_{e^{(f)}}(j, h)$.

Fault of weights can be regarded as the extension of binary crosspoint faults. In fact, $w_0(j, h)$ and $w_r(j, h)$ correspond to binary missing crosspoint fault and binary extra crosspoint fault, respectively.

Definition 4: Two faults in a circuit are *equivalent* if the circuit has equal output values for each input pattern under both faults. Two faults are *subequivalent* if there exists at least one pattern such that the circuit has equal output values under both faults.

The following notations are used where x_i^a, p_i, z_i denote a literal line, a product line and an output line in the PLA, respectively. Note that each literal line x_i^a $(a = 0, 1, \cdots, r)$ implements a uniliteral X_i^a, respectively.

- $\mathcal{P}(x_i^a)$ denotes a nonempty subset of the product lines connected to a literal line x_i^a.
- $\mathcal{X}(p_i)$ denotes a nonempty subset of the literal lines

connected to a product line p_i.

- $\mathcal{P}(z_i)$ and $\mathcal{Z}(p_i)$ are defined in a similar way.
- $NM(m,n)$ and $NE(m,n)$ denote a multiple missing crosspoint fault and extra crosspoint fault in the NOR plane, respectively, with $m,n \in \{x_i^a, p_i, z_i, \mathcal{X}(p_i), \mathcal{P}(z_i), \mathcal{Z}(p_i)\}$.
- $W_{e^{(f)}}(m,n)$ denotes a fault of weight in the TSUM plane with $m,n \in \{x_i, p_i, z_i, \mathcal{X}(p_i), \mathcal{P}(x_i^a), \mathcal{P}(z_i), \mathcal{Z}(p_i)\}$ where $e^{(f)}$ is an errorneous value of weight by a fault.
- $DS(d_k)$ and $US(d_k)$ are down-shift fault and up-shift fault of threshold detectors in the literal generators, respectively.
- $BA(m,n)$ and $BO(m,n)$ are AND-bridging fault and OR-bridging fault between m and n, respectively, with $m,n \in \{x_i^a, x_j^a, p_i, p_j, z_i, z_j\}$
- A sign '+' denotes the union of two faults.
- A sign '$*$' denotes a multiple-fault of two different kind of faults.

3. Stuck-at Fault and Fault on Literal Generator

3.1 Multiple-Valued Stuck-at Fault

In the MV-PLA presented in Sect. 2, stuck type faults appear as stuck-at-0 and stuck-at-r in the NOR plane since threshold detectors d_k in the literal generators change $(r+1)$-valued signals into (binary) control signals (refer Fig. 2). On the other hand, stuck-at-$0/r$ and stuck-at-q can occur in the TSUM plane, where q is one of the intermediate value of L (i.e. $q = 1, 2, \cdots, r-1$). We may note, in passing, that the control signals are changed to $(r+1)$-valued signals at the crosspoints between product lines and outputlines in the TSUM plane (refer Fig. 3). Note that '0' and 'r' are minimal and maximal elements in the ordered set of the logic values, respectively.

Six different stuck-at faults are considered in Lemmas 1–5.

Lemma 1: Literal line x_i^a stuck-at-0, denoted as s-a-0(x_i^a), is equivalent to a multiple missing crosspoint fault in the NOR plane. Literal line x_i^a stuck-at r, denoted as s-a-r (x_i^a), is equivalent to multiple faults of weights in the TSUM plane.

Proof: The output with stuck-at-0 fault in a certain literal line x_i^a completely corresponds to missing crosspoint faults at $(x_i^a, \mathcal{P}(x_i^a))$. For example, stuck-at-0 in Fig. 4 (b) is equivalent to missing crosspoint faults illustrated in Fig. 4 (c). Similar proof holds on literal line stuck-at-r (refer Figs. 4 (d),(e)). Thus the Eqs. (3),(4) are obtained.

$$s\!-\!a\!-\!0(x_i^a) = NM(x_i^a, \mathcal{P}(x_i^a)) \quad (3)$$
$$s\!-\!a\!-\!r(x_i^a) = W_0(\mathcal{P}(x_i^a), \mathcal{Z}(\mathcal{P}(x_i^a))) \quad (4)$$

Lemma 2: A product line stuck-at-0, s-a-0(p_j), is equivalent to a multiple fault of weights in the TSUM plane, and a stuck-at-r, s-a-r (p_j), is equivalent to a multiple missing crosspoint fault in the NOR plane.

Proof: Stuck-at-0 and stuck-at-r in the product lines are shown in Fig. 5 (a). Evidently, these faults are equivalent to a fault of weights and a multiple missing crosspoint fault shown in Figs. 5 (b),(c), respectively. Thus, the following equations can be stated.

$$s\!-\!a\!-\!0(p_j) = W_0(p_j, \mathcal{Z}(p_j)) \quad (5)$$
$$s\!-\!a\!-\!r(p_j) = NM(\mathcal{X}(p_j), p_j) \quad (6)$$

Lemma 3: The equivalence of the output line stuck-at faults, stuck-at 0, stuck-at r, stuck-at q $(q = 1, \cdots, r-1)$ is as follows.

(1-1) Stuck-at-0:

$$s\!-\!a\!-\!0(z_h) = W_0(z_h, \mathcal{P}(z_h)) \quad (7)$$

(1-2) Stuck-at-r

$$s\!-\!a\!-\!r(z_h) = W_{\Sigma w \geq r}(z_h, \mathcal{P}(z_h)) * NM(\mathcal{X}(\mathcal{P}(z_h)), \mathcal{P}(z_h)) \quad (8)$$

(1-3) Stuck-at-q ($q = 1, \cdots, r-1$: intermideate logic value)

$$s\!-\!a\!-\!q(z_h) = W_{\Sigma w = q}(z_h, \mathcal{P}(z_h)) * NM(\mathcal{X}(\mathcal{P}(z_h)), \mathcal{P}(z_h)) \quad (9)$$

In the Eq. (8), the first term of right means a fault of weights satisfying the restriction that total sum of weights on z_h is greater or equal to r. Similarly, the Eq. (9) satisfies the restriction that total sum of weights on z_h is just q.

Interestingly, from Lemma 3, the stuck-at fault occurrence factor in the output line can be regarded as s-a-0 > s-a-r > s-a-q, because this inequality also expresses the extents of the multiplexity of these faults.

Therefore, Theorem 1 is obtained from Lemmas 1–3 below.

Theorem 1: In the NOR/TSUM plane, any stuck-at fault is equivalent to a multiple missing crosspoint fault and/or a multiple faults of weights.

3.2 Threshold Shift Fault in L.G.

In Sects. 3.2 and 3.3, equivalences of threshold shift faults in 4-valued literal generators (L.G.) are considered.

Lemma 4: The fault pattern regarding literal generator depends on PLA personality. For a DS on d_k where $k = \{0, 1, 2\}$ (refer Figs. 2 (a),(b)), following results are obtained.

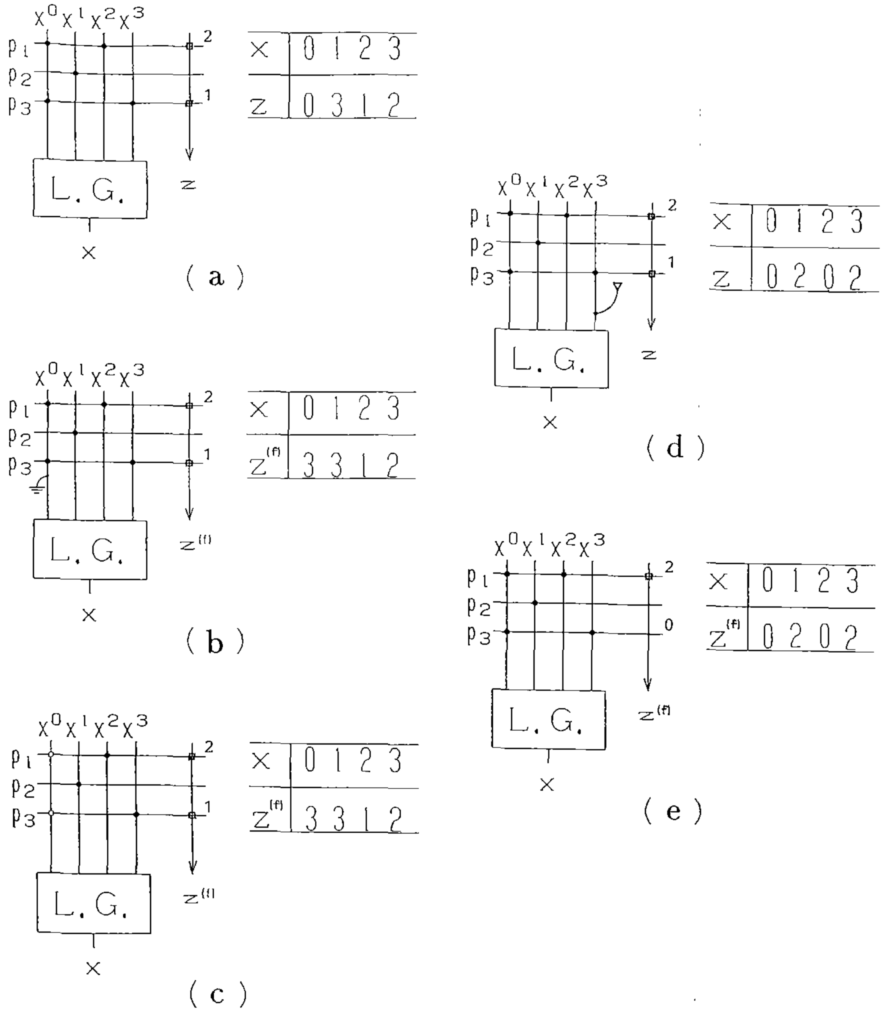

Fig. 4 (a) Diagram and normal output vector of a 4-valued PLA. (b) Same PLA with a literal line s-a-0. (c) Same PLA with a multiple crosspoint fault. (d) Same PLA with a literal line s-a-r. (e) Same PLA with a fault of weight.

(2-1) For X_i^k, X_i^{k+1} and $X_i^{S_{ij}}$, if $k \in X_{ij}$ and $k+1 \notin X_{ij}$), then $DS(d_k)$ is equivalent to a missing crosspoint fault at x_i^k in the NOR plane, where the literal line x_i^k implements the literal X_i^k.

(2-2) If $k \notin S_{ij}$ and $k+1 \in S_{ij}$, $DS(d_k)$ is equivalent to an extra crosspoint fault at x_i^k in the NOR plane.

(2-3) If $k \in S_{ij}$ and $k+1 \in S_{ij}$ then $DS(d_k)$ behaves as normal operations (i.e. the output string with the fault cannot be distinguished from normal pattern).

Proof: For an input string $x_i =< 0\ 1\ 2\ 3 >$, the output strings of $DS(d_k)$ and the output strings of $US(d_k)$ in L.G. are shown in Table 1 and in Table 2, respectively. Thus the TS faults can be denoted as follows.

$$DS(d_k) = NM(x_i^k, \mathcal{P}(x_i^k)) + NE(x_i^k, \mathcal{P}(x_i^{k+1})) \tag{10}$$

Table 1 Output vectors of $DS(d_k)$ for input vector $< 0\ 1\ 2\ 3 >$ in a L.G.

	d_0	d_1	d_2
x^0	$< 0\ 0\ 0\ 0 >$	$< 3\ 0\ 0\ 0 >$	$< 3\ 0\ 0\ 0 >$
x^1	$< 3\ 3\ 0\ 0 >$	$< 0\ 0\ 0\ 0 >$	$< 0\ 3\ 0\ 0 >$
x^2	$< 0\ 0\ 3\ 0 >$	$< 0\ 3\ 3\ 0 >$	$< 0\ 0\ 0\ 0 >$
x^3	$< 0\ 0\ 0\ 3 >$	$< 0\ 0\ 0\ 3 >$	$< 0\ 0\ 3\ 3 >$

$$US(d_k) = NM(x_i^{k+1}, \mathcal{P}(x_i^{k+1})) + NE(x_i^{k+1}, \mathcal{P}(x_i^k)) \tag{11}$$

From Eqs. (10) and (11), Lemma 4 is obtained.

Thus, from Lemma 4, Theorem 2 can be obtained below.

Theorem 2: A single threshold shift fault in the literal generator is equivalent to a multiple missing crosspoint fault or multiple extra crosspoint fault in the NOR plane.

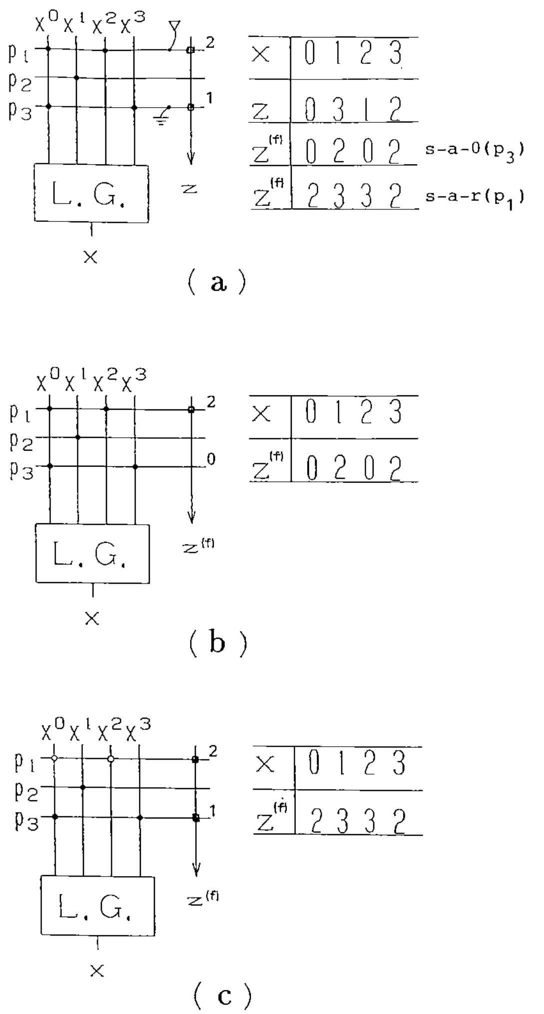

Fig. 5 (a) Diagram of a 4-valued PLA, normal output, s-a-0 (p_3) and s-a-r (p_1). (b) A fault of weight equivalent with s-a-0 (p_3). (c) Missing crosspoint faults equivalent with s-a-r (p_1).

Table 2 Output vectors of $US(d_k)$ for input vector < 0 1 2 3 > in a L.G.

	d_0	d_1	d_2
x^0	< 3 3 0 0 >	< 3 0 0 0 >	< 3 0 0 0 >
x^1	< 0 0 0 0 >	< 0 3 3 0 >	< 0 3 0 0 >
x^2	< 0 0 3 0 >	< 0 0 0 0 >	< 0 0 3 3 >
x^3	< 0 0 0 3 >	< 0 0 0 3 >	< 0 0 0 0 >

3.3 Other Stuck-Faults in L.G.

In the L.G. illustrated in Fig. 2, stuck-at faults of gates or interconnections are considered, and Lemmas 5–6 are obtained as follows.

Lemma 5: In a literal generator, a threshold detector stuck-at-0 is equivalent to a single missing crosspoint fault or single extra crosspoint fault in the NOR plane.

Proof: If a stuck-at-0 fault of a threshold detector d_k $(k = 0, 1, 2)$ exists in a L.G., elements of output vectors $(x_i^0 x_i^1 x_i^2 x_i^3)$ of the L.G.'s are

$$\begin{cases} x_i^k = 0 \\ x_i^{k+1} = \bigvee_{a \le k+1} x_i^a \end{cases} \quad (12)$$

where $a = 0,\ 1,\ 2,\ 3.$

If a d_k stuck-at-r exists, then

$$\begin{cases} x_i^{k+1} = 0 \\ x_i^k = \bigvee_{a \ge k} x_i^a \end{cases} \quad (13)$$

holds, where $\vee$ denotes OR operations with 0 and r. Thus, the following results are obtained.

$$s-a-0(d_k) = NM(x_i^k, \mathcal{P}(x_i^k)) + \bigvee_{a \le k} NE(x_i^a, \mathcal{P}(x_i^{a+1})) \quad (14)$$

$$s-a-r(d_k) = NM(x_i^{k+1}, \mathcal{P}(x_i^{k+1})) + \bigvee_{a \ge k} NE(x_i^a, \mathcal{P}(x_i^a)) \quad (15)$$

Lemma 6: In a literal generator, single stuck-at faults in device elements: NOT gates, AND gates and interconnections in a L.G. are equivalent to a multiple crosspoint fault in the NOR plane.

Proof: In a literal generator, single stuck-at faults in device elements (NOT gates, AND gates: refer Fig. 2) or of their interconnections are classified into the following three cases (summarized in Table 3). Note that the symbols a, b, $\cdots$, e in the Table 3 corresponds to the each lead-line in Fig. 2 (a), respectively.

(3-1) The fault equals to a threshold shift fault. Particularly, it equals to $DS(x_i^0)$ or $US(x_i^r)$.

(3-2) The faults equals to a literal line single stuck-at-0, or equals to a literal line single stuck-at and single another adjacent line's stuck-at r fault.

(3-3) The fault equals to a multiple extra crosspoint fault in the NOR plane, or equals to multiple crosspoint fault in the NOR plane and literal line single stuck-at-0 fault.

From Theorem 2 and Lemma 1, faults (3-1), (3-2) and (3-3) are equivalent to a multiple crosspoint fault in the NOR plane.

From Lemma 1–6, any single stuck-at fault is equivalent to a multiple crosspoint fault in the NOR plane and/or a multiple fault of weights in the TSUM plane of the MV-PLA.

4. Bridging Fault

In a PLA, a bridging fault is short between two interconections: literal lines, product lines and output lines.

Table 3 Equivalences of faults in a L.G.

	s-a-0	s-a-r
a	$= DS(d_0)$	$=$ s-a-$r(x_i^0)$ + s-a-0(x_i^1)
b	$=$ s-a-0(x_i^1)	$= NE(x_i^1, \mathcal{P}(x_i^0)$
c	$=$ s-a-0$(x_i^1) + NE(x_i^2), \mathcal{P}(x_i^0)$ $+NE(x_i^2, \mathcal{P}(x_i^1))$	$=$ s-a-0$(x_i^2) + NE(x_i^1, \mathcal{P}(x_i^2))$ $+NE(x_x^1, \mathcal{P}(x_i^3))$
d	s-a-0(x_i^2)	$= NE(x_i^2, \mathcal{P}(x_i^0)) + NE(x_i^2, \mathcal{P}(x_i^1))$
e	$=$ s-a-0(x_i^2) + s-a-$r(x_i^3)$	$= US(d_2)$

Six different kinds of bridging faults are considered in Lemmas 7–12.

Lemma 7: A bridging fault between two literal lines is subequivalent to a multiple missing crosspoint fault or a multiple extra crosspoint fault in the NOR plane.

Proof: Bridging fault between two literal lines can be classified into the following two types of faults.

(4-1) Consider a bridging fault between two literal lines x_i^k, x_i^{k+1} $(k = 0, 1, 2)$ from the same literal generator.

- AND bridging fault between x_i^k and x_i^{k+1}

  As literal generator output patterns are 1-hot codes of $(r + 1)$-bits, $x_i^k \cdot x_i^{k+1}$ never pulls up to r. Thus, the AND bridging fault between $x_i^k - x_i^{k+1}$, $BA(x_i^k, x_i^{k+1})$, is equivalent to multiple missing crosspoint fault in the NOR plane as follows.

$$BA(x_i^k, x_i^{k+1}) = NM(x_i^k, \mathcal{P}(x_i^k)) + NM(x_i^{k+1}, \mathcal{P}(x_i^{k+1})) \quad (16)$$

- OR bridging fault between x_i^k and x_i^{k+1}

  For literals X_i^k, X_i^{k+1} and $X_i^{S_i}$, if $k \in S_i$ and $k+1 \in S_i$ then $BO(x_i^k, x_i^{k+1})$ behaves as regular operations, where the literal line x_i^k implements the literal X_i^k.

  If $k \in S_i$ and $k+1 \notin S_i$ (or $k \in S_i$ and $k \notin S_i$), $BO(x_i^k, x_i^{k+1})$ is equivalent to extra crosspoint fault in the NOR plane as follows.

$$BO(x_i^k, x_i^{k+1}) = NE(x_i^k, \mathcal{P}(x_i^{k+1}) + NE(x_i^{k+1}, \mathcal{P}(x_i^k)) \quad (17)$$

(4-2) Consider a bridging fault between two literal lines $x_i^r - x_{i+1}^0$ from adjoining two literal generators. Only when $X_i^0 \in X_i^{S_i}$ and $X_{i+1}^r \in X_{i+1}^{S_{i+1}}$, error strings will appear in the product lines.

- AND-bridging fault between x_i^r and x_{i+1}^0

  If both literal lines are r or both are 0, $BA(x_i^r, x_{i+1}^0)$ behaves as regular operations.

  If one of the literal lines is 0 and another is r, both x_i^r and x_{i+1}^0 are pulled down. Thus, $BA(x_i^r, x_{i+1}^0)$ is subequivalent to missing crosspoint fault in the NOR plane as follows.

$$BA(x_i^r, x_{i+1}^0) \cong NM(x_i^r, \mathcal{P}(x_i^r)) + NM(x_{i+1}^0, \mathcal{P}(x_{i+1}^0)) \quad (18)$$

- OR-bridging fault between x_i^r and x_{i+1}^0

  Both x_i^r and x_{i+1}^0 are r, if one of these literal lines is r. Thus $BO(x_i^r, x_{i+1}^0)$ is equivalent to an extra crosspoint fault in the NOR plane as follows.

$$BO(x_i^r, x_{i+1}^0) = NE(x_i^r, \mathcal{P}(x_{i+1}^0)) + NE(x_{i+1}^0, \mathcal{P}(x_i^r)) \quad (19)$$

From (4-1) and (4-2), Lemma 7 is derived.

Lemma 8: An OR-bridging fault between two product lines p_i and p_j is equivalent to a multiple extra crosspoint fault in the NOR plane. An AND-bridging fault between p_i and p_j is subequivalent to a multiple missing crosspoint fault in the NOR plane.

Proof: If p_i is pulled up then p_j is also r. The same results can be obtained when p_j has the same PLA personality with p_i and vice versa, where the p_i and the p_j are adjacent product lines each other. This can be described as follows.

$$BO(p_i, p_j) = NE(\mathcal{X}(p_i), p_j) + NE(\mathcal{X}(p_j), p_i) \quad (20)$$

If there exists an AND-bridging fault between p_i and p_j,

$$BA(p_i, p_j) \cong NM(\mathcal{X}(p_j), p_j) + NM(\mathcal{X}(p_j), p_j) \quad (21)$$

can be stated (refer Eqs. (16) and (18)).

Lemma 9: A bridging fault between two output lines z_i and z_j is subequivalent to a multiple fault of weights in the TSUM plane.

Proof: Multiple-valued signals appear on output lines as defined MV-PLA previously. And note that the bridging faults of output lines depend on the way of TSUM implementation. In case of undefined output value $z' \notin L$, consideration of AND bridging fault and OR bridging fault between two output lines must be difficult even if z' be quantized into L. It may be needed to apply the technic of error detecting or error correcting codes. Then, here we treated just the case of AND/OR bridging fault between two neighbor output lines.

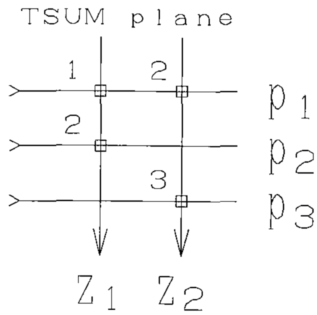

Fig. 6 A personality in the TSUM plane.

In case of AND bridging fault between z_i and z_j, z_i and z_j becomes $MIN(z_i, z_j)$. In case of OR bridging fault, z_i and z_j becomes $MAX(z_i, z_j)$. Therefore, AND/OR bridging faults are stated as follows.

$$BA(z_i, z_j) \cong W_{w_j}(z_i, \mathcal{P}(z_j)) + W_{w_i}(z_j, \mathcal{P}(z_i)) \quad (22)$$

$$BO(z_i, z_j) \cong W_{w_i}(z_j, \mathcal{P}(z_i)) + W_{w_j}(z_i, \mathcal{P}(z_j)) \quad (23)$$

These two formulas can be rewritten as follows.

$$BF(z_i, z_j) \cong W_{w_i}(z_j, \mathcal{P}(z_i)) + W_{w_j}(z_i, \mathcal{P}(z_j)) \quad (24)$$

For example, consider a bridging fault between z_1 and z_2 in Fig. 6. When p_1 is pulled up to r, normal output $< z_1, z_2 >=< 1, 2 >$ is changed to $< z_1, z_2 >^{(f)}=< 1, 1 >$ by AND bridging fault and $< z_1, z_2 >^{(f)}=< 2, 2 >$ by OR bridging fault.

From Eq. (23), Lemma 9 can be obtained.

Lemma 10: An OR-bridging fault between a literal line x_i and a product line p_j is equivalent to a multiple missing and a extra crosspoint fault in the NOR plane.

Proof: When there is a bridging fault between a literal line x_i and product line p_j, if x_i is $0/r$ then product line p_j is also $0/r$, because x_i dominates p_j, regardress of the values of the other literal line: $x_i^{a'}$ where $a' \neq a$ or x_j^b where $j \neq i$, $b = 0, 1, \cdots, r$. On the other hand, if all crosspoint devices between p_j and $x_i^{\overline{a}}$ are removed, and the crosspoints between x_i^a and p_j are connected with extra devices, then the same result is obtained where $\overline{a} = s_i - a$. Thus, the bridging fault $BF(x_i^a, p_j)$ is as follows.

$$BF(x_i^a, p_j) = EM(x_i^a, p_j) + NM(x_i^{\overline{a}}, p_j) \quad (25)$$

Lemma 11: A bridging fault between a product line p_j and a output line z_j is subequivalent to a multiple fault of weights in the TSUM plane.

Proof: When there is a bridging fault between product line p_i and output line z_j, if p_i is $0/r$ then z_j is also $0/r$, and vice versas. Thus, if both p_i and z_j are pulled up to r, then it is a redundant fault. If p_i is pulled down to 0, it is the same result of stuck-at 0 with $\mathcal{Z}(p_i)$ in the TSUM plane. Also, if p_i is pulled up to r when z_j has an intermideate value, it is the same result of stuck-at-r with $\mathcal{Z}(p_j)$ in the TSUM plane.

$$BF(p_i, z_j) \cong W_0(\mathcal{P}(z_j), z_j) + W_r(p_i, z_j) \quad (26)$$

Lemma 12: A bridging fault between a literal line x_n^r and an output line z_1 is subequivalent with a missing crosspoint faults in the NOR plane, or multiple faults of extra crosspoint faults in NOR plane and fault of weights in TSUM plane.

Proof: For the above bridging fault, three cases can be considered.

(i) The literal line dominates the output line, i.e., if the literal line x_n^r is 0 (low), the output line is also 0, and if the literal line is r (high), the output line is also r. Therefore, this case equivalent with $NM(\mathcal{X}(\mathcal{P}(z_1)), \mathcal{P}(z_1)) + NE(x_i, \mathcal{P}(x_i)) * W_r(z_1, \mathcal{P}(x_n^r))$.

(ii) The short between both lines behaves as logical AND bridging. If literal line x_n^r is r (high), there is no faulty behavior. If literal line is 0 (low) then the output line z_1 is pulled down. Therefore, this case is subequivalent to $NM(\mathcal{X}(\mathcal{P}(z_1)), \mathcal{P}(z_1)) + NM(\mathcal{X}_1, \mathcal{P}(x_1))$.

(iii) The short between both lines behaves as logical OR bridging. If literal line x_n^r is 0 (low), there is no faulty behavior. If literal line is r (high) then the output line z_1 is pulled up to higher value r. Thus, this case is subequivalent to $NE(x_i, \mathcal{P}(x_i)) * W_r(z_1, \mathcal{P}(x_n^r))$.

From (i), (ii) and (iii), we find

$$\begin{aligned} &BF(x_n^r, z_1) \\ &= NM(\mathcal{X}(\mathcal{P}(z_1)), \mathcal{P}(z_1)) + NE(x_i, \mathcal{P}(x_i)) \\ &\quad * W_r(z_1, \mathcal{P}(x_n^r)). \end{aligned} \quad (27)$$

Therefore, Theorem 3 can be derived from Lemmas 6–12.

Theorem 3: In the NOR/TSUM plane of the MV-PLA, bridging fault is equivalent to or subequivalent to a multiple crosspoint fault in the NOR plane or a multiple fault of weights in the TSUM plane.

Finally, from Theorems 1–3, the Theorem 4 can be stated as follows.

Theorem 4: The test vector set which can detect all crosspoint faults and faults of weights also detects all multiple-valued stuck-at faults, all bridging faults, all threshold shift faults and stuck-at faults of gates or interconnections in a L.G..

Theorem 4 is the conclusion of the above whole discussions. In the literature [7], for instance, a method of test generation for MV-PLA which detects crosspoint faults in NOR plane and faults of weights in TSUM plane is presented.

5. Conclusions

In this paper, a generalized NOR/TSUM multiple-valued PLA and its fault models are defined, and the equivalencies of the faults are discussed. Consequently, the faults in multiple-valued PLAs: multivalued stuck-at fault, multiple-valued threshold shift fault, multiple-valued bridging fault and other faults in L.G. are equivalent or subequivalent to multiple crosspoint fault in the NOR plane or multiple-valued multiple fault of weights in the TSUM plane. From this result, the test vector set which can detect all crosspoint faults and faults of weights also detects above equivalent or subequivalent faults. Relevant to this point can be found in literature[7].

Moreover, some characteristics of multiple-valued faults are obtained: (1) s-a-0 occurs more than s-a-*r* or s-a-*q*, (2) multiple-valued bridging faults can be stated by some simple expressions.

The derived results and analyses also hold properly other types of MV-PLA's[4],[6].

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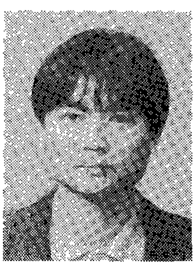

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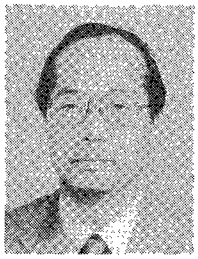

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