Preferential model for the evolution of pass networks in ball sports

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We propose a theoretical model to evaluate the temporally evolving ball-passing networks whose number of edges increases with time. The model incorporates a preferential selection of edges that chooses an edge based on its frequency of selection. Results are in good agreement with the corresponding ball-passing networks of real football, basketball, and rugby matches, enable the quantitative comparison of the passing activity among different teams or ball sports.

I. INTRODUCTION

Competitive team sports are recently being studied in physics. Researchers have successfully performed objective and quantitative research using digital data, such as player-tracking data [1]. For example, scoring events have been found to follow the Poissonian dynamics [2, 3]. However, anomalous diffusion has been discovered in a set of football (soccer) [4] and cricket [5] matches. Another example is the scaling laws regarding the fairness and efficiency of leagues and tournaments [6, 7].

Following are the reasons why sports are interesting in physics. A major question is how dynamics of individual players and their local interactions affect whole collective behavior and team performance [8]. The connection between microscopic and macroscopic properties is a common problem in statistical physics [9] and condensed matter physics [10]. An example of analogy between sports and physics systems is the comparison of the motion of a gas molecule with that of a soccer ball [11]. Moreover, the ideas and methods of statistical physics have been applied to the collective behavior of humans, as in econophysics [12] and social physics [13, 14]. Team sports may become the possible instances of such collective systems. Matches are played under fixed conditions using of standardized equipment, and certain rules restrict the actions of players'. Nevertheless, the flow and outcome of matches are extremely unpredictable [3, 15].

In ball sports, a series of passes among players forms a ball-passing network with the players acting as nodes. Complex network analysis of ball-passing networks has been performed intensively in football so far [16–20], and have also been applied to basketball [21], water polo [22], and rugby [23]. The ball-passing network elucidates statistical regularity in passing [18, 19], evaluating players' performance and role [16, 20] as well as team performance [17]. Although passing networks have been deeply investigated for individual sports, less attention has been paid to common network properties across different sports.

In thes present study, we investigate a dynamical property of ball-passing networks commonly observed in football, basketball, and rugby. We propose a mathematical model that incorporates the preferential selection of passes, indicating that a pass (directed edge in the network) is more likely to be selected if it has been selected frequently in the past. The team's passing activity can be characterized using a single parameter representing the preferential selection's strength. Thereafter, we show that the theoretical result fits real football matches and further compare with basketball and rugby. Different sports become comparable by introducing a scaling argument; thus, we aim to quantify ball sports' strategic differences using the preferential parameter. Previous studies adopted preferential mechanisms in various modeling systems, including scale-free networks [24] and innovation processes [25]. However, the preferential mechanism in the present study is different from the existing models and does not intend the power law.

II. DEFINITION AND ANALYSIS OF THE PROPOSED MODEL

For a team in a match, we let that $m(\tau)$ denotes the number of *different* passes when the number of passes is τ. The growth of $m(\tau)$ against τ slows down when the team tends to rely on a few certain passes. Therefore, $m(\tau)$ measures the variety or evenness of the passes executed by the entire team. For instance, Fig. 1 shows $m(\tau)$ for $\tau \leq 5$ in an NBA game of the Oklahoma City Thunder (game B2 in Table II). At $\tau = 4$, the pass from player 12 to player 0 was distinguished from the pass made by player 0 to 12 at $\tau = 3$, so $m(4) = m(3) + 1$. At $\tau = 5$, the pass from player 0 to 12 was already present; therefore, $m(\tau)$ did not increase [$m(4) = m(5)$]. When the number of players in a team

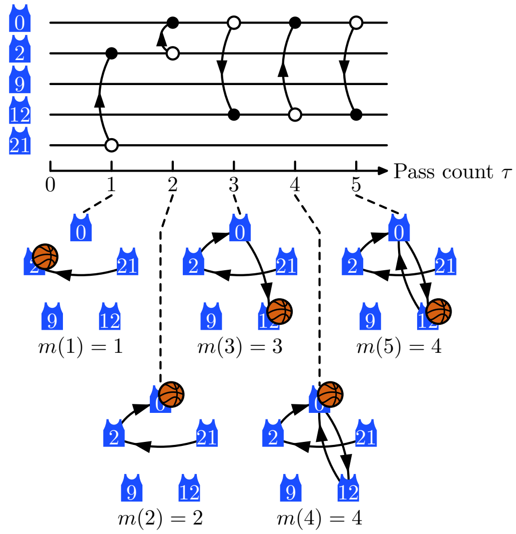

FIG. 1. Temporal network representation (upper) and the ball-passing network evolution (lower) in a real basketball team (Oklahoma City Thunder; game B2 in Table II). The arrows indicate the direction of a pass. The label of each node is the jersey number of the corresponding player.

is N, $m(\tau)$ can be defined in the range of $0 \leq m(\tau) \leq N(N-1)$. Here, $N(N-1)$ represents the total number of possible different passes and has been assumed as $M = N(N-1)$ for simplicity in recurrent use. [A similar analysis is possible for undirected networks by simply replacing $N(N-1)$ with $N(N-1)/2$.] Even for a small number N of nodes in a ball-passing network ($N \sim 10$), the regularity and commonality of passes can be studied through $m(\tau)$. A similar idea—seeking τ at which $m(\tau)$ reaches half the number of all possible edges—was introduced in a study of (undirected) temporal networks for a different purpose [26]. In this paper, the dynamical property of $m(\tau)$ for directed networks is studied.

The simplest and most intuitive technique for modeling for ball motion is based on Markov chain hopping on N nodes [18, 19]. However, the number of successive passes within a team is small in real scenarios [the average numbers are 3.37 in football (for nine matches in Table I), 2.65 in basketball (for two games in Table II), and 5.11 in rugby (for two matches in Table II)] owing to mistakes, interception by opponents, or fouls. Therefore, unlike a random walk, the receiver of the previous pass is not necessarily the initiator of the next pass.

In the present study, we adopted the preferential effect on pass selection instead of assuming the Markov chain. Although secured passes are preferred in competitive ball sports for maintaining ball possession, new passes need to be created to outwit the opponents. To simplify this consideration, we assumed that the probability $p_{i\to j}(\tau)$ at which the pass from player i to j is selected at τ is proportional to $1 + \alpha\mu_{i\to j}(\tau)$, where $\mu_{i\to j}(\tau)$ is the number of $i \to j$ passes from the first τ passes, and α is a non-negative constant. When $\alpha > 0$, the pass $i \to j$ having large $\mu_{i\to j}(\tau)$ is selected preferentially. The parameter α represents the strength of preferential selection. For instance, at $\tau = 0$, any of the M passes, say $i_0 \to j_0$, is selected with an equal probability of $1/M$. At $\tau = 1$, the pass $i_0 \to j_0$ has a statistical weight $1 + \alpha$, but the other $M - 1$ passes each have the weight of 1. Therefore, by normalizing the probabilities, the pass $i_0 \to j_0$ is selected again with $p_{i_0\to j_0}(1) = (1 + \alpha)/(M + \alpha)$, and each of the other passes is selected with the probability of $1/(M + \alpha)$ at $\tau = 1$.

By definition of $\mu_{i\to j}$, we can obtain $\sum\sum_{i\neq j} \mu_{i\to j}(\tau) = \tau$, so that $p_{i\to j}(\tau) = (1 + \alpha\mu_{i\to j}(\tau))/(M + \alpha\tau)$. $m(\tau)$ increases when one of the unused $M - m(\tau)$ passes, each with probability $1/(M + \alpha\tau)$, is selected for the first time. Thus, the difference equation can be described as follows:

$$m(\tau + 1) = m(\tau) + \frac{M - m(\tau)}{M + \alpha\tau}, \tag{1}$$

where the initial condition is $m(0) = 0$.

To solve Eq. (1), we subtract both sides from M:

$$M - m(\tau + 1) = \frac{M - 1 + \alpha\tau}{M + \alpha\tau}(M - m(\tau)).$$

For $\alpha = 0$,

$$M - m(\tau + 1) = \frac{M-1}{M}(M - m(\tau)).$$

Therefore,

$$M - m(\tau) = \left(\frac{M-1}{M}\right)^{\tau}(M - m(0)),$$

and the general solution can be expressed as

$$m(\tau) = M - \left(1 - \frac{1}{M}\right)^{\tau}(M - m(0)).$$

Under the initial condition of $m(0) = 0$, we obtain the solution

$$m(\tau) = M\left[1 - \left(1 - \frac{1}{M}\right)^{\tau}\right]. \tag{2}$$

Thereafter, considering the case of $\alpha > 0$,

$$M - m(\tau + 1) = \frac{(M-1)/\alpha + \tau}{M/\alpha + \tau}(M - m(\tau)).$$

Applying this equation recursively, we obtain

$$\begin{aligned} M - m(\tau) = & \frac{(M-1)/\alpha + \tau - 1}{M/\alpha + \tau - 1}\frac{(M-1)/\alpha + \tau - 2}{M/\alpha + \tau - 2} \\ & \cdots \frac{(M-1)/\alpha}{M/\alpha}(M - m(0)). \end{aligned}$$

The denominator and numerator are expressed by the gamma function as

$$x(x+1)\cdots(x+\tau-1) = \frac{\Gamma(x+\tau)}{\Gamma(x)}.$$

Therefore, the general solution can be expressed as

$$m(\tau) = M - \frac{\Gamma(M/\alpha)}{\Gamma((M-1)/\alpha)}\frac{\Gamma((M-1)/\alpha + \tau)}{\Gamma(M/\alpha + \tau)}(M - m(0)).$$

Under the initial condition of $m(0) = 0$, we obtain the solution:

$$m(\tau) = M\left[1 - \frac{\Gamma(M/\alpha)}{\Gamma((M-1)/\alpha)}\frac{\Gamma((M-1)/\alpha + \tau)}{\Gamma(M/\alpha + \tau)}\right]. \tag{3}$$

Figure 2 shows the graph of $m(\tau)$ for $M = 90$ (corresponding to $N = 10$) and $\alpha = 0, 1,$ and 2. New passes are less likely to be selected for large α (strong preferential effect), and the increase in m slows down as α becomes large.

III. DATA ANALYSIS

A. Analysis of football matches

In this subsection, we apply the theoretical result of $m(\tau)$ to the real football data, comprising nine matches of the Japan Professional Football League held in 2016, as shown in Table I. The matches F1-F8 were played on February 27, 2016, and the match F9 was played on February 28, 2016. We used the player-tracking data with information on ball possession. Both were provided by DataStadium Inc., Japan, and automatically generated the sequence of the passes.

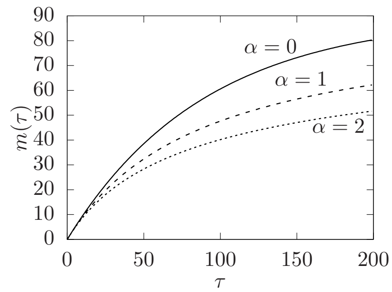

FIG. 2. Graph of $m(\tau)$ for $M = 90$ and $\alpha = 0$ (solid curve), 1 (dashed curve), and 2 (dotted curve). The increase becomes slow for large α.

TABLE I. List of the nine football matches.

Match ID	Home team	Away team	α_{home}	α_{away}
F1	Júbilo Iwata	Nagoya Grampus Eight	0.75	0.34
F2	Tokyo Verdy	Omiya Ardija	0.90	0.58
F3	Sanfrecce Hiroshima	Kawasaki Frontale	1.31	1.39
F4	Sagan Tosu	Avispa Fukuoka	0.89	0.28
F5	Kashiwa Reysol	Urawa Red Diamonds	1.22	0.42
F6	Shonan Bellmare	Albirex Niigata	0.63	0.88
F7	Vissel Kobe	Ventforet Kofu	0.84	0.95
F8	Yokohama FC	Vegalta Sendai	0.69	0.59
F9	Gamba Osaka	Kashima Antlers	0.86	0.51

In our analysis, we excluded the goalkeeper, because of its exceptional position. Therefore, the number of nodes in a team is $N = 10$ (namely, $M = 90$). We counted the throw-ins as a pass if a teammate received the ball successfully. In substituting of players, we regarded the passes made by the new player as those made by the substituted player. That is, if the player i is substituted for i', we counted $m(\tau)$ by regarding the passes of $i' \to j$ and $j \to i'$ to be identical with those made by $i \to j$ and $j \to i$, respectively. In other words, a node in our network represents a position, such as center back and left side half in football, rather than a player. Although other treatments are possible, we considered this to be the simplest.

Figure 3 shows the graphs of $m(\tau)$ in the football matches F1–F9 in the first half including its additional time. Circular and square plots represent the home and away teams, respectively. Solid and dashed curves represent the fitting curves of Eq. (3) for home and away teams. The points represent only the increase in $m(\tau)$ to avoid overlapping. Therefore, an interval between adjacent points indicates that existing passes were used and no new pass was made. The real data are in good agreement with the theoretical curves, except for Reysol in match F5, as Reysol might have changed its passing strategy in the middle of the first half. Table I shows the α values, whose average value is 0.78 and standard deviation is 0.31.

The passing network in the second half can also be expressed well by Eq. (3). However, Fig. 4 shows that we did not find a correlation or tendency between α in the first and second halves (the correlation coefficient is 0.33). We surmise that this result reflects randomness in football, and further analysis requires a large data set.

B. Comparison among football, basketball, and rugby

In this subsection, we present additional results for basketball and rugby matches: two for each sport (namely, four networks), as shown in Table II. The number N of players on the field is $N = 5$ (basketball) and $N = 15$ (rugby) per team. We manually marked the passes for the basketball games by visualizing player-tracking data and referring event data [27]. For the rugby matches, we manually marked the passes from video footages. We focused only on the first quarter (basketball) or half (rugby) of the playing match including its additional time. We counted the succeeded throw-in (basketball) and lineout (rugby) as a pass, as well as football matches. A rebound was not counted as a pass in basketball because the shooter did not intend to pass the ball. The player substitution was treated similarly in the football matches.

Figure 5 presents the graphs of $m(\tau)$ in the real matches listed in Table II. Circular and square plots represent home

TABLE II. List of basketball and rugby matches for the analysis.

Sport	N	M	League	Date	Home team	Away team	ID	α_{home}	α_{away}
Basketball	5	20	NBA	2016.01.03	Denver Nuggets	Portland Trail Blazers	B1	0.33	0.26
				2016.01.04	Oklahoma City Thunder	Sacramento Kings	B2	0.56	0.40
Rugby	15	210	Super Rugby	2019.02.16	Sunwolves	Sharks	R1	2.31	3.60
				2019.06.01	Sunwolves	ACT Brumbies	R2	4.32	3.26

and away teams, respectively. Solid and dashed curves represent the fitting curves of Eq. (3) for home and dashed away teams, respectively. The empirical basketball graphs show the rough step-like plots caused by the relatively large fluctuation imparted by the small N and M values. Nevertheless, all empirical data are broadly consistent with the theoretical results.

From the comparison of the three ball sports, the α value in rugby ($\alpha \approx 3$) is the largest and that in basketball ($\alpha \approx 0.4$) is the smallest. This difference in α values implies that the passing event in basketball is relatively close to random (the preferential effect is weak), whereas the passing in rugby tends to concentrate on a few specific passes. In rugby, each position's role is thoroughly separated [28], and it is strategically important to whom each player passes the ball. On the contrary, a pass made to an unguarded teammate is safe in basketball. The passes in basketball become closer to random as various teammates can get openings in an instant. The ability of the movement of players and spatial constraints can also affect the passing strategy. Thus, the α value can reflect such strategic differences in ball sports. However, another possible hypothesis may attribute this difference in α values to the difference in N; the data of basketball, football, and rugby indicate a small α value for a sport having a large N. This hypothesis is suggested to be incorrect through the following analysis of $m(\tau)$.

By taking the continuum limit $m(\tau+1)-m(\tau) \simeq dm/d\tau$, the difference equation (1) turns into differential equation

$$\frac{dm}{d\tau} = \frac{M-m}{M+\alpha\tau}.$$

Under the initial condition $m(0) = 0$ as above, the solution becomes

$$m(\tau) = \begin{cases} M\left[1-\left(1+\frac{\alpha\tau}{M}\right)^{-1/\alpha}\right], & \alpha > 0, \\ M(1-e^{-\tau/M}), & \alpha = 0. \end{cases}$$

These solutions are obtained by applying Stirling's approximation to Eqs. (2) and (3). The effect of M can be eliminated by focusing on m/M and τ/M, instead of m and τ. This simple scaling property emerges as a result of considering the continuum limit. However, to a certain extent, it holds in the original discrete system when M/α is large enough to apply Stirling's approximation to Eq. (3).

As an empirical instance, we compared Denver Nuggets in game B1 ($M = 20$ and $\alpha = 0.33$) with Nagoya Grampus Eight in match F1 ($M = 90$ and $\alpha = 0.34$). Although these two teams had different M, and their (τ, m) plots differed greatly [Fig. 6(a)], these two graphs overlapped well in the $(\tau/M, m/M)$ plot [Fig. 6(b)]. Therefore, the effect of M can be absorbed in the scalable factors of τ/M and m/M, and the strategic differences are more appropriately reflected by the change in α.

IV. DISCUSSION

In this study, we propose and analyze a stochastic model for the growth of the ball-passing network based on the preferential selection approach of passes. The solution $m(\tau)$ is characterized by a parameter α, representing the strength of the preferential selection. Although this model does not incorporate the direction of attack or spatial configuration of the players (passing the ball to distant players is difficult), the theoretical result of $m(\tau)$ in Eq. (2) is consistent with real networks in football, basketball, and rugby. We believe that the preferential selection indirectly includes these spatial factors. In conclusion, the proposed model is simple but effectively captures the essential features of passing events in various ball sports.

However, nine matches for football and two matches for basketball and rugby are insufficient to assess the validity and reliability of the model. Moreover, applying the proposed model to various sports other than football, basketball, and rugby can confirm its generality and applicability. These studies will require large-scale empirical data sets. Future studies in this field depend highly on the quantity and quality of accessible empirical data.

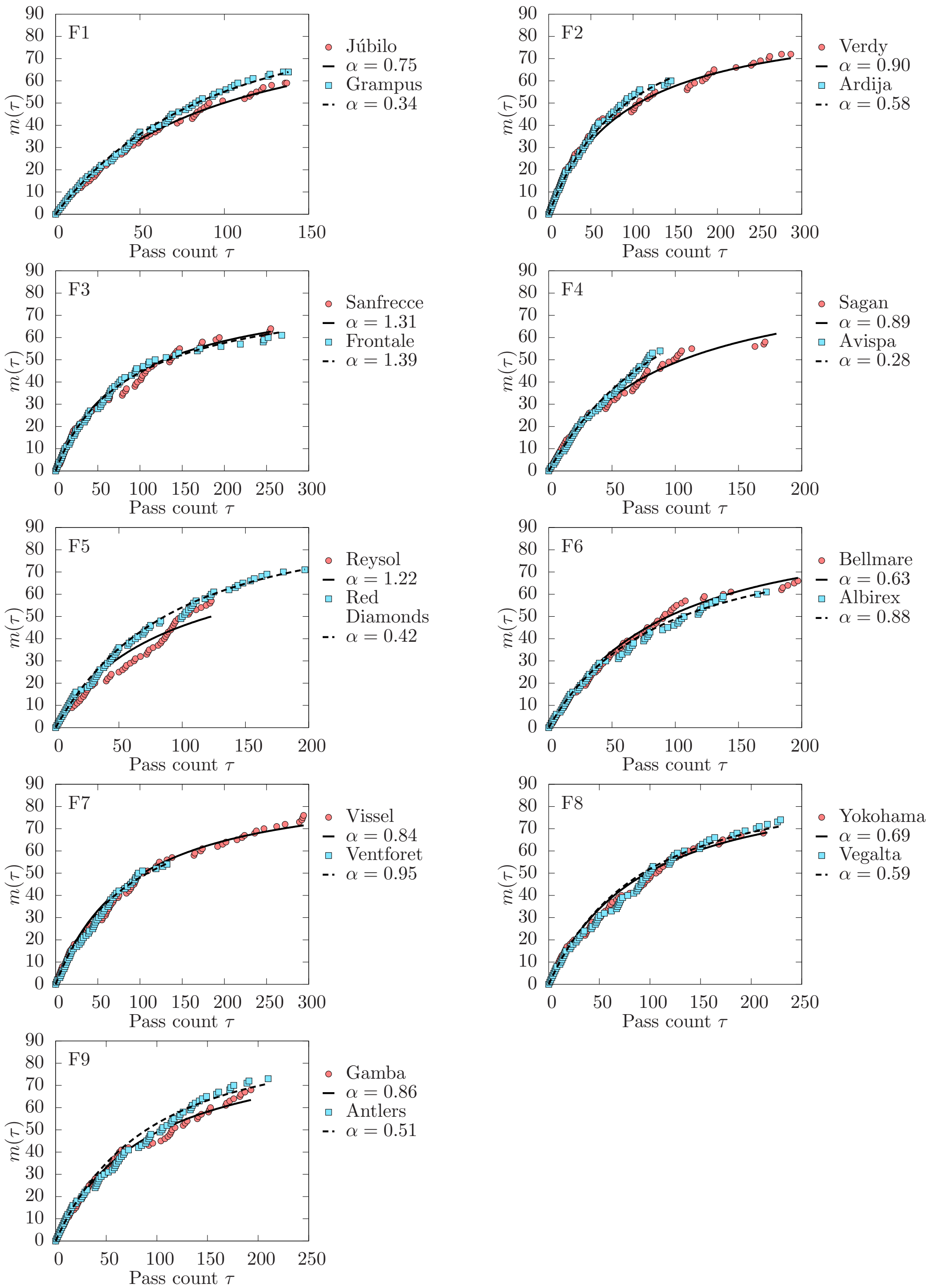

FIG. 3. Plots of $m(\tau)$ for the first half of the football matches F1–F9. Circular and square points represent home and away teams, respectively. The points are drawn only where $m(\tau)$ increases. Solid and dashed curves represent the fitting curves of Eq. (2) for home and away teams, respectively The team names and their corresponding α values are written on the right of each graph.

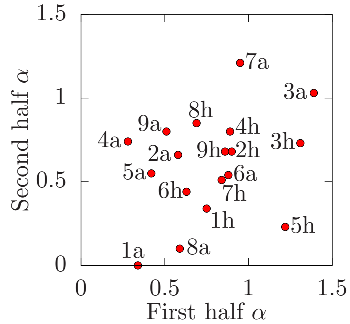

FIG. 4. Scatter plot of α in the first and second halves. The label near each point represents the team. For example, "1h" indicates the home team of match F1, and "2a" indicates the away team of match F2. The correlation coefficient is 0.33.

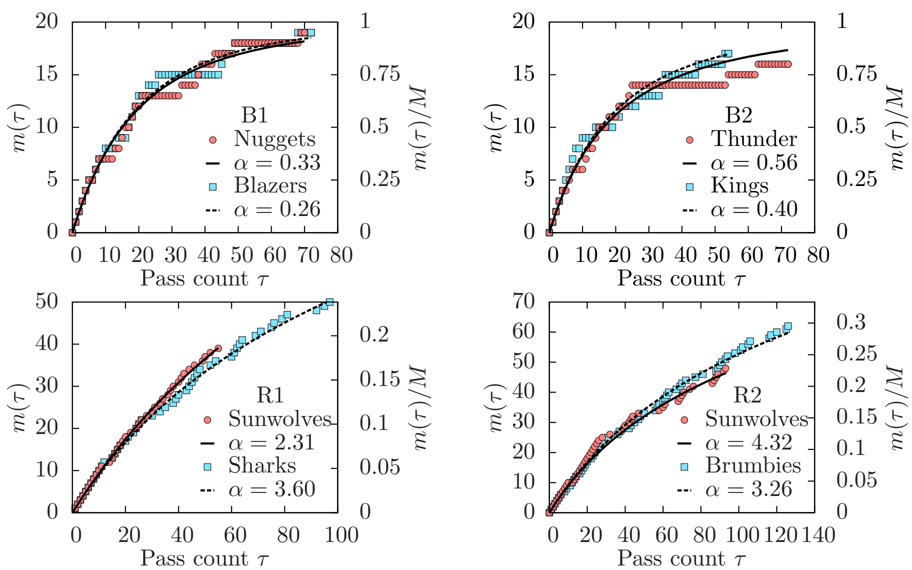

FIG. 5. Graphs of $m(\tau)$ in basketball (upper row) and rugby (lower row). Circular and square points represent home and away teams, respectively. Solid and dashed curves represent the fitting curves of Eq. (3) for home and away teams, respectively. The corresponding α values are shown in the graphs. The ratio $m(\tau)/M$ is shown on the right vertical axis. In the graphs for rugby, the points are drawn only where $m(\tau)$ increases.

In this study, a node in the network is a player. The pitch network [29], another type of passing network, is possible. The pitch is partitioned into areas, and the nodes of the pitch network are these areas. When a player in area 1 passes the ball to a player in area 2, an edge is drawn from node 1 to 2. Moreover, a network whose node is identified by both player and area has been studied [18]. The investigation and comparison of $m(\tau)$ on these networks are the natural and interesting extensions of the present study.

Lastly, we see a brief application of the proposed model on a social system other than sports. Previously, Duch, Waitzman, and Amaral [16] applied their analysis method for ball-passing networks to the patterns of email exchanges among laboratory members. An email exchange is a mode of communication between people, and the set of emails among the members of a certain project corresponds to the set of passes in a game of ball sports. We use the "Enron email dataset" [30] as an empirical dataset for the analysis. This dataset is an email log of 150 employees of the Enron Corporation. We can create a network by representing an email address as a node and an email as a directed edge (from the sender to the receiver). For simplicity, we ignore the "cc" and "bcc" fields. We focus on what we term as a bidirectional clique defined as a subgraph where edges exist in both directions between any pair of nodes. The largest bidirectional clique uniquely existed at the size of $N = 11$ ($M = 110$). For a series of email exchanges within this clique in a given month, we can define $m(\tau)$ as the count of emails representing the various directed edges from the first τ emails, as compared with the number of passes in ball sports. Figure 7 shows $m(\tau)$ plots for August 2000 (upright triangles), October 2001 (inverted triangles), and September 1999 (rhombus), including 288 (maximum), 196, and 117 emails, respectively; the theoretical expression (3) fits with $\alpha = 2.38$, 2.82, and 4.13, respectively.

This result needs more consideration because it is not clear to what extent emails are similar to passes. Social

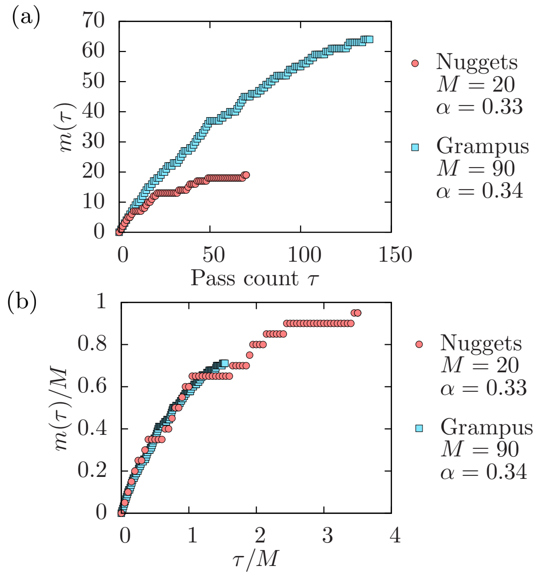

FIG. 6. Comparison of basketball and football. (a) $m(\tau)$ vs τ graphs for Denver Nuggets (circles) and Nagoya Grampus Eight (squares), which are the same as those in Figs. 5 and 3 (game B1 and match F1, respectively). (b) Overlapping in $m(\tau)/M$ vs τ/M plot.

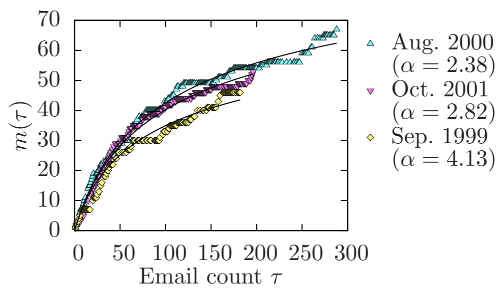

FIG. 7. $m(\tau)$ plots of email exchange within the $N = 11$ clique of the Enron email dataset for August 2000 (upright triangle), October 2001 (inverted triangle), and September 1999(rhombus). The solid curves represents the fitting curves of Eq. (3).

interactions generally possess heterogeneous and multi-layered structures, and the proposed model possibly requires certain extensions or modifications for improved accuracy. Furthermore, the interevent time is bursty in many modes of communication, and the effect of burstiness on the dynamics of the system has been verified [31–33]. (The burstiness is not exhibited in ball-passing networks because opponents continuously attempt to steal the ball. Holding the ball for a long time and executing passes in bursts is highly difficult for the players.) However, we believe that the preferential selection approach provides a good starting point to a mature and realistic or precise modeling.

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[1] R. Rein, SpringerPlus 5, 1410 (2016).
[2] A. Heuer, C. Müller, and O. Rubner, Europhys. Lett. 80, 38007 (2010).
[3] S. Merritt and A. Clauset, EPJ Data Sci. 3, 4 (2014).
[4] R. da Silva, M. H. Vainstein, S. Gonçalves, and F. S. F. Paula, Phys. Rev. E 88, 022136 (2013).

[5] H. V. Ribeiro, S. Mukherjee, and X. H. T. Zeng, Phys. Rev.E 86, 022102 (2012).
[6] E. Ben-Naim and N. W. Hengartner, Phys. Rev. E 76, 026106 (2007).
[7] E. Ben-Naim, S. Redner, and F. Vazquez, Europhys. Lett. 77, 30005 (2007).
[8] J. Gudmundsson and M. Horton, ACM Computing Survey 50, 22 (2017).
[9] P. L. Krapivsky, S. Redner, and E. Ben-Naim, A Kinetic View of Statistical Physics (Cambridge University Press, Cambridge, 2010).
[10] P. M. Chaikin, Principles of Condensed Matter Physics (Cambridge University Press, Cambridge, 2000).
[11] J. Luzuriaga, Eur. J. Phys. 31, 1071 (2010).
[12] B. K. Chakrabarti, A. Chakraborti, and A. Chatterjee, Econophysics and Sociophysics (Wiley-VCH, 2006).
[13] C. Castellano, S. Fortunato, and V. Loreto, Rev. Mod. Phys. 81, 591 (2009).
[14] M. Buchanan, The Social Atom (Bloomsbury, London, 2007).
[15] A. Clauset, M. Kogan, and S. Redner, Phys. Rev. E 91, 062815 (2015).
[16] J. Duch, J. S. Waitzman, and L. A. N. Amaral, PLoS One 5, e10937 (2010).
[17] T. U. Grund, Social Networks 34, 682 (2012).
[18] T. Narizuka, K. Yamamoto, and Y. Yamazaki, Physica A 412, 157 (2014).
[19] K. Yamamoto and T. Narizuka, Phys. Rev. E 98, 052314 (2018).
[20] J. M. Buldú, J. Busquests, I. Echegoyen, and F. Seirul.lo, Sci. Rep. 9, 13602 (2019).
[21] J. H. Fewell, D. Armbruster, J. Ingraham, A. Petersen, and J. S. Waters, PLoS One 7, e47445 (2012).
[22] P. Passos, K. Davis, D. Araújo, N. Paz, J. Minguéns, and J. Mendes, J. Science and Medicine in Sport 14, 170 (2011).
[23] K. Sasaki, T. Yamamoto, M. Miyao, T. Katsuta, and I. Kono, Intl. J. Performance Analysis in Sport 17, 822 (2017).
[24] A.-L. Barabási and R. Albert, Science 286, 509 (1999).
[25] I. Iacopini, S. Milojević, and V. Latora, Phys. Rev. Lett. 120, 048301 (2018).
[26] P. Holme and N. Masuda, PLoS One 10, e0120567 (2015).
[27] N. Seward, nba-movement-data `https://github.com/sealneaward/nba-movement-data`
[28] P. Johnson Rugby Union: Technique Tactics Training (Crowood, Marlborough, UK, 2014).
[29] J. L. Herrera-Diestra, I. Echegoyen, J. H. Martínez, D. Garrido, J. Busquets, F. S. Io, and J. M. Buldú, Chaos, Solitons & Fractals 138, 109934 (2020).
[30] Enron Email Dataset `https://www.cs.cmu.edu/~./enron/`
[31] A. Vazquez, B. Rácz, A. Lukács, and A.-L. Barabási, Phys. Rev. Lett. 98, 158702 (2007).
[32] J. L. Iribarren and E. Moro, Phys. Rev. Lett. 103, 038702 (2009).
[33] M. Karsai, M. Kivelä, R. K. Pan, K. Kaski, J. Kertész, A.-L. Barabási, and J. Saramäki, Phys. Rev. E 83, 025102(R) (2011).